Response of dispersive cylindrical cloaks to a nonmonochromatic plane wave

Blanchard, Cédric; Wu, Bae-Ian; Portí, Jorge Andrés; Chen, Hongsheng; Zhang, Baile;
Morente, Juan Antonio; Salinas, Alfonso

2009

Blanchard, C., Wu, B. I., Portí, J. A., Chen, H., Zhang, B., Morente, J. A., & Salinas, A. (2009). Response of dispersive cylindrical cloaks to a nonmonochromatic plane wave. *Journal of the Optical Society of America B*, 26(11), 2117-2124.

<https://hdl.handle.net/10356/94934>

<https://doi.org/10.1364/JOSAB.26.002117>

© 2009 Optical Society of America. This paper was published in *Journal of the Optical Society of America B* and is made available as an electronic reprint (preprint) with permission of Optical Society of America. The paper can be found at the following official DOI: <http://dx.doi.org/10.1364/JOSAB.26.002117>. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper is prohibited and is subject to penalties under law.

Downloaded on 26 Apr 2023 20:47:38 SGT

Response of dispersive cylindrical cloaks to a nonmonochromatic plane wave

Cédric Blanchard,^{1,2,*} Bae-Ian Wu,² Jorge Andrés Portí,¹ Hongsheng Chen,^{2,3} Baile Zhang,² Juan Antonio Morente,¹ and Alfonso Salinas¹

¹Department of Applied Physics, University of Granada, 18071 Granada, Spain

²Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

³The Electromagnetic Academy at Zhejiang University, Zhejiang University, 310027 Hangzhou, China

*Corresponding author: cedric@ugr.es

Received August 25, 2009; revised August 25, 2009; accepted September 2, 2009;
posted September 21, 2009 (Doc. ID 108771); published October 20, 2009

The transmission line modeling method is used to get the time domain response of a dispersive cylindrical cloak to an electromagnetic (EM) plane wave that is slightly nonmonochromatic. Our objective is to numerically study two important phenomena derived from the dispersive nature of the invisibility shell: frequency shifts and time delays. On one hand, the frequency domain representation of the cloak's response shows that the frequency center is shifted once the EM wave has crossed the cloak; the shift intensity representation spans the entire rainbow spectrum depending on the observation angle. On the other hand, such a full-wave simulation constitutes tangible evidence of the existence of time delays when the EM wave passes through the device. We show that this phenomenon depends on the employed coordinate transformation. © 2009 Optical Society of America

OCIS codes: 230.3205, 050.1755, 160.3918.

1. INTRODUCTION

The possibility of excluding all electromagnetic (EM) fields from a certain region without perturbing the vicinity has been extensively studied since the pioneer work proposed by Pendry *et al.* [1]. Based on a coordinate transformation, the cloak presented in the related original paper must disperse and can be wholly efficient only at a single frequency. For this reason, most of the works dealing with cloaking have long been focused on monochromatic EM waves [2,3], which elude the dispersion effect. However, the authors recently looked into more physical dispersive cloaks [4]. In particular, it has been theoretically shown that the frequency center of a quasi-monochromatic wave is blueshifted in the forward direction after passing through a spherical cloak [5].

The central aim of this paper is to employ a full-wave numerical analysis of the response of a cylindrical cloak to a quasi-monochromatic EM wave in order to illustrate how some phenomena emerge from the dispersive nature of the device. The simulation will be carried out using the transmission line modeling (TLM) method. The TLM method can model the EM field by filling the space with nodes made of intersecting transmission lines [6,7]; as a time domain method, TLM can well describe the physical aspect of the cloaking process [8].

The first part will deal with TLM of cloaking structures. In its original form, TLM exploits the well-known L - C distributed network [9] representation of homogeneous dielectrics and magnetic materials in which the quantities L and C represent positive equivalent permeability μ and permittivity ε , respectively. Simply interchanging the position of L and C leads to an equivalent dispersive material that assumes negative values for both

ε and μ [10]. Therefore, we will focus on the dispersive nature of TLM mesh for metamaterials.

Second, it will be shown that the frequency center of a quasi-monochromatic wave is blueshifted in the forward direction for the cylindrical cloak after passing through the cloaking structure. It will be demonstrated that the frequency shift distribution depends on the observation angle; in particular a redshift occurs in some directions.

Leonhardt first stressed that the cloaking device causes a time delay [11], which has been deeply studied afterward by using ray-tracing simulations [12]. Such a geometric optics description is valuably interesting, but the fact that it constitutes an approximation (reflection is inevitably ignored, for instance) requires deeper analyses. We propose to confirm this physical aspect of the cloaking dynamics in a full-wave simulation. Furthermore, time delays associated with cloaks based on nonlinear coordinate transformations will also be simulated.

2. TRANSMISSION LINE MODELING METHOD

The coordinate transformation,

$$r' = f_\alpha(r) = [(r - R_1)/(R_2 - R_1)]^{1/\alpha} R_2, \quad (1)$$

which can compress the space from $0 \leq r' \leq R_2$ to the concentric annular region $R_1 \leq r \leq R_2$, yields the well-known formulas [2],

$$\varepsilon_r = \mu_r = \frac{r - R_1}{r},$$

$$\varepsilon_\varphi = \mu_\varphi = \frac{r}{r - R_1},$$

$$\varepsilon_z = \mu_z = \left(\frac{R_2}{R_2 - R_1} \right)^2 \frac{r - R_1}{r}, \quad (2)$$

if $\alpha=1$. These r -dependent quantities lead to the ideal cloak at a single functional angular frequency, which we will refer to as ω_0 in this paper.

Dealing with quasi-monochromatic waves is delicate since they do not differ much from perfectly monochromatic waves; consequently, numerical errors in the modeling have to be minimized. In this sense, Cartesian coordinates, which are usually employed [13,14], lead to staircase approximations when one wants to model the curved shape of the layered cloak. To avoid this undesirable phenomenon, we will use TLM cylindrical nodes whose shape assumes the geometry of the cloaking shell [15,16]. Therefore, with such nodes, it is not necessary to approximate the curved geometry with many Cartesian cells anymore. Furthermore, since ε_φ and μ_φ tend to infinity at the inner boundary, numerical simulations use to adopt approximations arising from the truncation of the inner layer given that they cannot deal with infinite values. With the TLM, dielectric and magnetic constants tending to infinity do not have any detrimental consequence since it can be proven that the limits of the TLM scattering matrix elements still remain finite in this case [17]. That is why this extreme value will be used in the modeling, which will contribute to the improvement of the accuracy of our simulation.

As an illustration of the reachable accuracy, let us consider a 2 GHz EM wave incoming on a cloaked perfect electric conductor (PEC) cylinder. The cloaking shell, with an inner radius of $R_1=0.1$ m and an outer radius of $R_2=0.2$ m, is made up of 200 layers. The polarization is transverse magnetic (TM), which means that the magnetic field is oriented along the axis of the cylinder (z direction). The computed magnetic field distribution is displayed in Fig. 1(a). Note that we have used the Huygens surface technique that consists of dividing the mesh into a total-field inner region and a scattered-field outer region; absorbing boundary conditions at the outer boundary of the mesh can thus be employed, while both the total and scattered fields can be visualized in the same picture. It is plain from Fig. 1(a) that the cloaking effect can be achieved with a great precision by using TLM, with cylindrical nodes, proposed here. For instance, the scattering is shown to be almost zero in region 4. Quantitatively, the far-field scattering radiations for the cloaked cylinder and for the bare cylinder are also shown in Fig. 1(b). It turns out that the scattering for the cloaked cylinder has been reduced by 40 dB in the forward and backward directions, and even by more along other directions, compared with the simple cylinder scattering.

The above example involved a monochromatic wave, but the aim of this paper is to deal with nonmonochromatic waves. Therefore, an important issue is in regard to the dispersion of the cloaking material in the numerical modeling. According to Eq. (2), the cylindrical cloaking shell is made up of an anisotropic material that involves

Fig. 1. (Color online) (a) Magnetic field ($\times 10^{-3}$) mapping in the vicinity of a cloaked PEC cylinder at 2 GHz. Four regions are apparent: 1, cylinder; 2, cloaking shell; 3, free space with total field; 4, scattered-field region where incident field is subtracted. (b) Far-field pattern comparison between the cloaked and the bare cylinders.

different categories of EM constants: the azimuth component is always greater than 1, the radial component is always less than unity, while the axial component is either greater than 1 or less than 1 on both sides of the limit value $r=R_2^2/(2R_2-R_1)$. Given that the interconnection between all the TLM nodes constitutes a usual distributed $L-C$ network, TLM cannot process the exotic values of ε and μ in its original form. The problem can be fixed by substituting the $L-C$ network by a dual one, i.e., the positions of inductors and capacitors are simply interchanged [10,13]. Such a left handed transmission line system is dispersive, which is in agreement with the necessary dispersive nature of metamaterials. Thus, the TLM mesh, which is nothing more than the numerical incarnation of these networks, has also a dispersive nature if ε and μ are less than unity. Consequently, it is not necessary to artificially make ε and μ dispersive; both already are given that they are implemented through the dispersive mesh. An important remaining worry is what is the exact frequency behavior of the EM constants in the cylindrical mesh? We can calculate that the dispersion generated by the mesh is

$$\begin{aligned}\mu_z(\omega) &= A_z(r) - \frac{\omega_0^2}{\omega^2} [A_z(r) - \mu_z(\omega_0)], \\ \varepsilon_r(\omega) &= A_r(r) - \frac{\omega_0^2}{\omega^2} [A_r(r) - \varepsilon_r(\omega_0)],\end{aligned}\quad (3)$$

where A_z and A_r are two functions of r that also depend on the parameters of the TLM. They are given by the formulas

$$A_z(r) = \frac{2\Delta t \Delta z Z_0}{r \Delta r \Delta \varphi \mu_0}, \quad A_r(r) = \frac{\Delta t \Delta r Y_0}{r \Delta \varphi \Delta z \varepsilon_0}, \quad (4)$$

in which Δt is the time step of the numerical simulation; Δr , $\Delta \varphi$, Δz represent the size of the nodes; and Z_0 and $Y_0 = 1/Z_0$ represent the impedance and the admittance, respectively, of the connecting transmission lines constituting the node. Let us consider a dispersive medium whose permittivity and permeability, represented by $\chi(\omega)$, follow the Drude model $\chi(\omega) = 1 - \omega_p^2/\omega^2$ [18], where ω_p is the plasma frequency. From the last equation, we find that $\omega_p^2 = \omega_0^2[1 - \chi(\omega_0)]$ so that $\chi(\omega) = 1 - \omega_0^2/\omega^2[1 - \chi(\omega_0)]$; note that, if the dispersive medium under consideration is a cylindrical cloak, $\chi(\omega_0)$ is given by Eq. (2). We conclude that $\mu_z(\omega)$ and $\varepsilon_r(\omega)$ in Eq. (3) are reminiscent of a Drude model [18]; the factor 1 is, however, substituted by the function A_z or A_r . Since A_z and A_r depend on parameters of the numerical simulation, we will have to ensure that $\mu_z(\omega)$ and $\varepsilon_r(\omega)$ are in agreement with the causality condition $d(\omega\chi(\omega))/d\omega \geq 1$. This condition can be derived from the Kramers–Kronig relations, which are a direct consequence of the causality principle [19]. Note that Yaghjian and Maci showed that considerations involving the EM energy conservation are capable of providing the same inequality, but they moreover proved that the relation $d(\omega\chi(\omega))/d\omega \geq \chi(\omega)$ should be employed for metamaterials [20]. Although in some studies the Lorentz model has been employed for the permeability [21], other contributions dealing with left handed metamaterials [22] or cloaking structures [14] do use the Drude model [18]. In this sense, it is worth emphasizing that the Drude model [18] employed in this paper does not alter any conclusions that we will obtain from the numerical simulation.

3. FREQUENCY SHIFT FOR A QUASI-MONOCROMATIC WAVE

For a time domain method as TLM, a quasi-monochromatic EM wave can be modeled by using a modulated sinusoid in terms of time t ,

$$h(t) = \sin[\omega t] \exp[-g^2(t - t_m)^2], \quad (5)$$

where $\omega = 2\pi f$ is the angular frequency (f is the frequency) of the incoming wave, while g and t_m are two real numbers that define the bandwidth of the signal. The Fourier transform of h is depicted in Fig. 2. Obviously, such an incoming wave behaves as a quasi-monochromatic wave given that its frequency domain representation is a Gaussian pulse centered on ω . The quasi-monochromatic nature of the wave is controlled by the parameter g : the smaller the g constant, the narrower the pulse in the fre-

Fig. 2. Frequency domain representation of the incoming wave (function h) for $\omega = 2\pi \times 2$ GHz and $g = 5 \times 10^7$ s⁻¹.

quency domain. Therefore, if g is chosen to be small enough, any scattering due to the dispersion inherent to the cloak is minimized around the cloaking frequency; and, for this reason, the frequency domain representation of the wave that passes through the cloak, i.e., the transmitted wave, should be a Gaussian too.

Let us consider a PEC cylinder surrounded by a cloaking shell whose functional frequency is $f_0 = 2$ GHz. The structure is illuminated by a TM wave. To see exactly how the cloaking effectiveness is affected by a deviation of the frequency, we first compute the scattering width (SW) of the cloaked PEC cylinder in terms of the frequency. The result is depicted in Fig. 3 and compared with the SW of a simple PEC cylinder. As expected, the cloak is effective only for the f_0 working frequency. However, the SW is obviously very low in a certain bandwidth centered on f_0 and for which the object is still undetectable. The size of such a bandwidth depends on the sensitivity of the detector located outside the structure. In this sense, the bandwidth that may be considered as quasi-undetectable contains the whole frequency range of the plane wave plotted in Fig. 2. Consequently, we will use this quasi-monochromatic wave, with $\omega = 2\pi \times 2$ GHz and $g = 5 \times 10^7$ s⁻¹, in the numerical determination of the frequency shift that follows.

Fig. 3. (Color online) SW of a PEC cylinder with/without the cloaking shell around. The scattering is reduced in a narrowband around the 2 GHz working frequency.

While a strictly monochromatic wave would pass the cylindrical cloak without distortion, the situation might change slightly for a quasi-monochromatic wave even if the bandwidth is narrow as is the case here. First, let us examine what happens in the forward direction. The H field is computed at a distance λ_0 (incoming radiation wavelength) from the outer boundary of the cloak. Furthermore, in order to illustrate how the response is sensitive to the assumed dispersion, two different sets of $\{\mu_z(\omega), \varepsilon_r(\omega)\}$ are considered. Concretely, TLM parameters are set such that $A_r(r)=0.10/r$ and $A_z(r)=0.13/r$ in one case, and $A_r(r)=0.21/r$ and $A_z(r)=0.13/r$ in the other case. As an example, we plot $\varepsilon_r(\omega/2\pi)$ in Fig. 4 at the vicinity of the inner boundary of the cloak, precisely at $r=0.11$ m, for the two dispersion profiles. Therefore, the corresponding frequency domain representation, obtained by Fourier transform, is displayed in Fig. 5 for both relations of dispersion. The frequency centers of the new pulses result to be shifted toward higher frequencies; this corresponds to a blueshift of the wavelength. Concretely, it turns out that the frequency centers of the pulses have been shifted by 0.45 and 0.74 MHz depending on the assumed dispersion. This shows that the frequency shift phenomenon is a direct consequence of the dispersion of the optic constants, and higher slope values lead to more intense blueshift effects. To provide some insights on the relationship between the profile of the constitutive parameters and the observed frequency shift, let us consider Fig. 6 in which $\varepsilon_r(\omega/2\pi)$ has been plotted at $r=0.1, 0.11$, and 0.12 m. It is plain from this figure that there exists a frequency for which $\varepsilon_r=0$ for the three curves. Of course, at $r=R_1$, the frequency that makes ε_r equal to zero is the working frequency of the cloak, in agreement with Eq. (2). However, because of the assumed dispersion, when r increases ε_r reaches the zero value for lower frequencies. The same effect is observed for μ_z . These zero values of the optic constants lead to a concentric wall for the EM wave that enables it to go further toward the inner boundary of the cloak. Thus, while the higher frequency (blue light) components of the quasi-monochromatic source are capable of reaching the PEC core, the lower components (red light) are scattered by the wall before,

Fig. 4. (Color online) Two different curves of dispersion for the constitutive parameter ε_r at $r=0.105$ m. Curves 1 and 2 correspond to $A_r(r)=0.10/r$ and $0.21/r$, respectively.

Fig. 5. (Color online) Frequency domain representation of the transmitted wave in the forward direction. The frequency centers of the pulses are blueshifted. The red dashed curve corresponds to $A_r(r)=0.10/r$ (i.e., curve 1 in Fig. 4), while the blue dotted curve corresponds to $A_r(r)=0.21/r$ (i.e., curve 2 in Fig. 4). The frequency shift depends on the relation of dispersion.

i.e., for $r > R_1$. As evinced in Fig. 6, the apparent radius of the wall is different depending on the frequency component under consideration. Accordingly, the scattered and incident fields should interfere differently depending on the frequency. Moreover, each layer of the cloak is described by a different dispersion profile. Therefore, the interactions inside the cloak are complex and a numerical treatment may be useful. To understand how they interfere, the cloak is illuminated by a monochromatic wave whose frequency is slightly deviated from the cloaking frequency (2 GHz). Concretely, the frequencies of the incoming wave are chosen to be 1.99 and 2.01 GHz; the results are displayed in Figs. 7(a) and 7(b), respectively. We observe that scattering does exist in these two cases, but the interaction between the incident and the scattering fields is not the same: (1) for 1.99 GHz, the total field is less intense than the incident field, meaning that the scattered field destructively interferes with the incident field; (2) for 2.01 GHz, the scattering strengthens the in-

Fig. 6. (Color online) Dispersion of the r component of the permittivity at three different points inside the cloak. At $r=R_1$, ε_r takes, of course, the zero value at the working frequency (2 GHz). But when r increases, the frequency that renders $\varepsilon_r=0$ increases.

Fig. 7. (Color online) Incident, scattered, and total fields for a cloak whose working frequency is 2 GHz. It is illuminated by an EM plane wave with frequencies of (a) 1.99 GHz (red light) and (b) 2.01 GHz (blue light). For the red light, a destructive interference is observed, while the interference is constructive for the blue light.

cident field, which leads to a more intense total field. This means that, on one hand, the amount of red light tends to diminish and, on the other hand, the amount of blue light increases. That is why the blueshift effect occurs overall.

We now numerically calculate the frequency shift along other directions; the result is depicted in Fig. 8. The shift

Fig. 8. (Color online) Distribution of the shifted frequency center in terms of the observation angle for a quasi-monochromatic EM wave traveling from left to right.

Fig. 9. (Color online) Ratio between the total and incident fields, for $f=1.99$ and 2.01 GHz, in terms of the direction of observation.

continuously spans a large spectrum, going from negative to positive values. The corresponding redshift occurs around 35° . Such a distribution can be explained in terms of constructive and destructive interferences as well. In that sense, the ratio between the amplitudes of the total and incident fields, for $f=1.99$ and 2.01 GHz, is plotted in Fig. 9. In the forward direction, in agreement with Fig. 7, this ratio is greater than unity for 2.01 GHz while it is less than unity for 1.99 GHz. However, the predominance of the blue light over the red light can be inverted for other angles. In particular, for $f=1.99$ GHz and in the range of 25° – 45° , the scattering constructively interferes with the incident wave given that we observe a greater amplitude of the total field than the amplitude of the incident field. This tendency is reversed for the blue light so that, overall, a redshift of frequency is expected with a maximum at 32° , in agreement with Fig. 8. It is important to insist that it is the specific employed dispersion that leads to this particular angular distribution. Indeed, the dispersion profile is directly related to the apparent size of the EM wall (for $f < f_0$) and, therefore, to the kind of interaction between the scattered and incident fields for each frequency component.

4. TIME DELAYS

Because the EM constants of the cloak are r dependent, the wave velocity is expected to follow a certain distribution: Chen and Chan showed that the velocity decreases when getting closer to the inner boundary of the invisible shell for a spherical cloak [12]. What about the cylindrical cloak? Let us consider a TM wave incident upon a cylindrical cloak; the incoming magnetic and electric fields can be expressed as

$$\begin{aligned}
 H_z^{\text{inc}}(r) &= H_0 \exp \left[i \frac{\omega}{c} r \cos \varphi \right], \\
 E_r^{\text{inc}}(r) &= E_0 \exp \left[i \frac{\omega}{c} r \cos \varphi \right] \sin \varphi, \\
 E_\varphi^{\text{inc}}(r) &= E_0 \exp \left[i \frac{\omega}{c} r \cos \varphi \right] \cos \varphi.
 \end{aligned} \tag{6}$$

From Eq. (6), the EM field in the cloak can be derived using [19]

$$\begin{aligned}
H_z &= H_z^{\text{inc}}[f(r)], \\
E_r &= f'(r)E_r^{\text{inc}}[f(r)], \\
E_\varphi &= \frac{f(r)}{r}E_\varphi^{\text{inc}}[f(r)],
\end{aligned} \tag{7}$$

where $f(r)$ is the transformation function in Eq. (1). From Eq. (7), the total EM energy density can be obtained as [19]

$$\bar{W} = \frac{1}{4} \left[\epsilon_0 \frac{d(\omega \epsilon_{ik})}{d\omega} E_i E_k^* + \mu_0 \frac{d(\omega \mu_{ik})}{d\omega} H_i H_k^* \right], \tag{8}$$

while the mean value of the Poynting vector is

$$\bar{S} = \frac{1}{2} \text{Re}[\mathbf{E} \times \mathbf{H}^*]. \tag{9}$$

From Eqs. (8) and (9), we can get the velocity according to $u = \bar{S}/\bar{W}$ [19]; finally, the calculation gives

$$u = c \frac{2R_2 \sqrt{\left(\frac{r-R_1}{r}\right)^2 \cos^2 \varphi + \sin^2 \varphi}}{(R_2 - R_1) \left[\left(\frac{R_2 \sin \varphi}{R_2 - R_1}\right)^2 \frac{d(\omega \epsilon_r)}{d\omega} + \left(\frac{r-R_1}{r} \frac{R_2 \cos \varphi}{R_2 - R_1}\right)^2 \frac{d(\omega \epsilon_\varphi)}{d\omega} + \frac{d(\omega \mu_z)}{d\omega} \right]}, \tag{10}$$

where c is the speed of light in free space. For the purpose of illustration, the distribution of the velocity in a cloak that is dispersive in accord with the Drude model [18] is depicted in Fig. 10 for $\varphi = 0^\circ$, 45° , and 90° . It is plain from this last figure that the velocity is smaller near the inner boundary of the cloak. Thus, once the EM wave has passed through the cloak, the process of reaching different points located on a plane normal to the direction of propagation should be completed with a time delay.

Equation (10) has been derived in order to make intuitive the fact that time delays must exist. However, it would be incomplete to use such an approach based on geometric optics (ray-tracing analysis) to determine the magnitude of time delays. Only a full-wave analysis can provide rigorous information for the reasons mentioned in the following:

(1) For a monochromatic plane wave there would be matching of the impedance at the interface of two adjacent layers. However, because of the dispersion, it is not

the case for the nonmonochromatic plane wave involved in that study. That is why the cloaking device scatters (even if the scattering is small) if it is illuminated by a nonmonochromatic plane wave. The consequence is that the plane wave interacts with an object, the cloak, whose dimensions are similar to the wavelength. As a result the time delays are mainly due to diffraction, which cannot be treated by geometric optics and rays of light.

(2) Since the cloaking shell is anisotropic and inhomogeneous, the direction of propagation of the EM wave is not necessarily the same as those of the ray of light.

(3) In Section 3, it has been shown that the low frequency components cannot reach the inner boundary of the cloak, while the high frequency components can. Therefore a serious distortion of the signal is expected at the vicinity of the inner boundary. A ray, which is only a theoretical object, cannot take into account such a distortion, while the physical nonmonochromatic wave definitely can.

Time delays cannot be computed if the incoming wave is monochromatic given that the cloak would induce no distortion in this case; in other words, the incident and the transmitted signals would overlap making impossible the distinction between each. Instead, suppose that the incoming wave is quasi monochromatic and is modeled using the modulated sinusoid of Eq. (5), with the cloaking frequency chosen to be $f_0 = 6$ GHz. The permittivity and permeability are given by Eq. (3) with $A_r(r) = 0.10/r$ and $A_z(r) = 0.13/r$. Note that these parameters are those that were used to get the red curve 1 in Figs. 4 and 5. The H field is calculated in the forward direction at the distance $x_0 = \lambda_0$ from the cloak outer boundary. In Fig. 11, we compare the H field in the absence and in the presence of the whole structure in terms of time, normalized by the period of the incident wave, $T_0 = 1/f_0$. Obviously, if the envelope of the signal is considered, the presence of the cloak results in the EM wave to reach x_0 with a time delay. Since the wave frequency is high, oscillations resulting from the sine in Eq. (5) cannot be observed and we have, therefore, enlarged a certain portion of Fig. 11.

Fig. 10. (Color online) Velocity distribution in a dispersive cloak along three different directions. The optics constants $\mu_z(\omega)$ and $\epsilon_r(\omega)$ of the cloak follow a Drude model [18].

Fig. 11. (Color online) H -field plot in terms of the normalized time. The envelope of the signal does not reach x_0 at the same time depending on whether the cloaking structure is present or not.

It is important to study the response of cloaking devices based on nonlinear coordinate transformations [23], i.e., with values of α not equal to 1 in Eq. (1). Let Π^2 be the plane normal to the x axis at point x_0 . Let t_{td} be the deviation between the required times for the EM wave to reach Π^2 with and without the presence of the whole structure. Let us consider the line, contained in Π^2 , which points along the y direction. Depending on the position of the point under consideration, t_{td} is expected to take different values. In the following, we propose to compute t_{td} (normalized by T_0) in terms of the distance y from the x axis. To calculate t_{td} , we look at the EM power associated with the signal. We first calculate the total power P_0 of the incident wave. Then, we admit that the required time for the transmitted wave to reach Π^2 is when the power $P_0/2$ has flowed through Π^2 . The result is depicted in Fig. 12 for $\alpha=1$, 0.75, and 0.5. As expected, t_{td} decreases when deviating from the x axis. Furthermore, the time delay is plainly sensitive to this type of transformation, with the linear one rendering the delay smaller. For completion's sake, we have tried other frequencies for the incoming wave. The obtained curves turn out to be different from those presented in Fig. 12. The reason is that the phenomenon mainly comes from diffraction; thus the time de-

Fig. 12. (Color online) Time delay (normalized by T_0) in terms of the distance y from the x axis.

lay has to depend on the size of the cloak or, equivalently, on the frequency of the incident wave.

We pointed out the deficiency of the ray-tracing model at the beginning of this section. Therefore, discrepancies between Eq. (10) and our numerical results are expected. For example, it is plain from Fig. 10 that the velocity is a monotonically increasing function of the radial component r . On the contrary, Fig. 12 evinces that the time delay is constant in a certain range. Furthermore, if the above parameters are modified so that $A_r(r)=0.21/r$ and $A_z(r)=0.13/r$ (as it has been done in Section 3), the amount of time delay should be altered. Concretely, we have verified that the time delay is increased in the forward direction, which cannot be entirely explained in the limit of geometric optics. Indeed, as pointed out in [12], a ray would take nearly infinite time to propagate along the propagation axis given that the velocity approaches zero at the inner boundary of the cloak (see Fig. 10). These two comments illustrate that the ray approach is unable to take into account all the material effects as diffraction or distortion of the wave.

It is worth noting that our numerical simulations show that there is a slight pulse broadening. This result was expected given that the constitutive parameters of the cloak disperse, which means that each frequency component propagates with a particular velocity. However, the frequency band of the incident wave is narrow. Therefore the constitutive parameters do not suffer an important variation and the pulse broadening cannot be directly observed in Fig. 11. As evinced in Fig. 4, using $A_r(r)=0.21/r$ and $A_z(r)=0.13/r$ increases the slope of the dispersion curve around the working frequency. It has been checked that these parameters lead to a wider pulse broadening.

Finally let us emphasize that the physical origin of both time delays and frequency shifts is the dispersion of the constitutive parameters of the cloak. If the cloak was not dispersive, neither time delays nor frequency shifts would be observed. Nonetheless, they differ in the mechanism. Time delay is related to diffraction while the frequency shift is related to the apparent size of the concentric wall for each frequency component of the incident wave. In addition, the frequency shift reflects the magnitude of the transmission for each frequency while the time delay reflects the phase profile of the transmission for each frequency.

5. CONCLUSION

The interest in the response of a cloaking shell to a non-monochromatic plane wave derives from the dispersive nature of the invisible device; confining the study to one single frequency eludes this aspect. In this sense, the full-wave simulation of a cylindrical cloak illuminated by a quasi-monochromatic EM wave is able to reveal and confirm interesting phenomena.

We report in this paper the TLM of such a configuration by using cylindrical nodes. We showed first that this treatment provides significantly precise results. When passing through a cylindrical cloak, the frequency center of a quasi-monochromatic wave is blueshifted in the forward direction. This phenomenon is known for a spherical

cloak but had not yet been demonstrated for its two-dimensional equivalence. In other directions, shifts with other intensities occur; we found out in particular that a redshift can take place. Another aspect is in regard to time delay. It is shown that the EM wave envelope exhibits a delay in the process of reaching a plane located at a constant distance from the device. Furthermore, time delays, which mainly come from diffraction, strongly depend on the coordinate transformation that yields the parameters of the cloak.

ACKNOWLEDGMENTS

This work has been supported in part by the “Ministerio de Educación y Ciencia” of Spain under research projects FIS2004-03273 and FIS2007-63293, cofinanced with FEDER funds of the European Union (EU). H. Chen acknowledges the support of the National Natural Science Foundation of China (NSFC) under grant 60801005, the Foundation for the Author of National Excellent Doctoral Dissertation of People’s Republic of China under grant 200950, the Zhejiang Provincial Natural Science Foundation under grant R1080320, and the Ph.D. Programs Foundation of MEC under grant 200803351025.

REFERENCES

1. J. B. Pendry, D. Schurig, and D. R. Smith, “Controlling electromagnetic fields,” *Science* **312**, 1780–1782 (2006).
2. S. A. Cummer, B.-I. Popa, D. Schurig, D. R. Smith, and J. B. Pendry, “Full-wave simulations of electromagnetic cloaking structures,” *Phys. Rev. E* **74**, 036621 (2006).
3. H. Chen, B.-I. Wu, B. Zhang, and J. A. Kong, “Electromagnetic wave interactions with a metamaterial cloak,” *Phys. Rev. Lett.* **99**, 063903 (2007).
4. P. Yao, Z. Liang, and X. Jiang, “Limitation of the electromagnetic cloak with dispersive material,” *Appl. Phys. Lett.* **92**, 031111 (2008).
5. B. Zhang, B.-I. Wu, H. Chen, and J. A. Kong, “Rainbow and blueshift effect of a dispersive spherical invisibility cloak impinged on by a nonmonochromatic plane wave,” *Phys. Rev. Lett.* **101**, 063902 (2008).
6. P. B. Johns and R. L. Beurle, “Numerical solution of 2-dimensional scattering problems using a transmission-line matrix,” *Proc. Inst. Electr. Eng.* **118**, 1203–1208 (1971).
7. P. B. Johns, “A symmetrical condensed node for the TLM method,” *IEEE Trans. Microwave Theory Tech.* **35**, 370–377 (1987).
8. C. Blanchard, J. Portí, B.-I. Wu, J. A. Morente, A. Salinas, and J. A. Kong, “Time domain simulation of electromagnetic cloaking structures with TLM method,” *Opt. Express* **16**, 6461–6470 (2008).
9. G. V. Eleftheriades, A. K. Iyer, and P. C. Kremer, “Planar negative refractive index media using L - C loaded transmission lines,” *IEEE Trans. Microwave Theory Tech.* **50**, 2702–2712 (2002).
10. P. P. M. So, H. Du, and W. J. R. Hoefer, “Modeling of metamaterials with negative refractive index using 2-D shunt and 3-D SCN TLM networks,” *IEEE Trans. Microwave Theory Tech.* **53**, 1496–1505 (2005).
11. U. Leonhardt, “Optical conformal mapping,” *Science* **312**, 1777–1780 (2006).
12. H. Chen and C. T. Chan, “Time delays and energy transport velocities in three dimensional ideal cloaking devices,” *J. Appl. Phys.* **104**, 033113 (2008).
13. C. Blanchard, J. Portí, J. A. Morente, A. Salinas, and B.-I. Wu, “Numerical determination of frequency behavior in cloaking structures based on L - C distributed networks with TLM method,” *Opt. Express* **16**, 9344–9350 (2008).
14. Y. Zhao, C. Argyropoulos, and Y. Hao, “Full-wave finite-difference time-domain simulation of electromagnetic cloaking structures,” *Opt. Express* **16**, 6717–6730 (2008).
15. R. Allen and M. J. Clark, “Application of the symmetrized transmission-line matrix method to the cold modeling of magnetrons,” *Int. J. Numer. Model.* **1**, 61–70 (1988).
16. J. A. Morente, G. J. Molina-Cuberos, J. A. Portí, K. Schwingschuh, and B. P. Besser, “A study of the propagation of electromagnetic waves in Titan’s atmosphere with the TLM numerical method,” *Icarus* **162**, 374–384 (2003).
17. C. Blanchard, B. Zhang, B.-I. Wu, J. A. Portí, H. Chen, J. A. Morente, and A. Salinas, “Importance of the singular constitutive parameters of cylindrical cloaks: illustration on the anticloak concept,” *J. Opt. Soc. Am. B* **26** (to be published).
18. J. B. Pendry, A. J. Holden, W. J. Stewart, and I. Youngs, “Extremely low frequency plasmons in metallic mesostructures,” *Phys. Phys. Lett.* **76**, 4773–4776 (1996).
19. L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, 1984).
20. A. D. Yaghjian and S. Maci, “Alternative derivation of electromagnetic cloaks and concentrators,” *New J. Phys.* **10**, 115022 (2008).
21. J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, “Magnetism from conductors and enhanced nonlinear phenomena,” *IEEE Trans. Microwave Theory Tech.* **47**, 2075–2084 (1999).
22. R. W. Ziolkowski and E. Heyman, “Wave propagation in media having negative permittivity and permeability,” *Phys. Rev. E* **64**, 056625 (2001).
23. W. Cai, U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, and G. W. Milton, “Nonmagnetic cloak with minimized scattering,” *Appl. Phys. Lett.* **91**, 111105 (2007).