Composite adaptive fuzzy control for synchronizing generalized Lorenz systems

Pan, Yongping; Er, Meng Joo; Sun, Tairen

2012

Pan, Y., Er, M. J., & Sun, T. (2012). Composite Adaptive Fuzzy Control for Synchronizing Generalized Lorenz Systems. *Chaos*, 22(2), 023144-.

<https://hdl.handle.net/10356/95008>

<https://doi.org/10.1063/1.4721901>

© 2012 American Institute of Physics. This paper was published in *Chaos* and is made available as an electronic reprint (preprint) with permission of American Institute of Physics. The paper can be found at the following official URL: [<http://dx.doi.org/10.1063/1.4721901>]. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper is prohibited and is subject to penalties under law.

Downloaded on 26 Apr 2023 22:56:25 SGT

Composite adaptive fuzzy control for synchronizing generalized Lorenz systems

Yongping Pan,^{1,a)} Meng Joo Er,^{1,b)} and Tairen Sun^{2,c)}

¹*School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore*

²*School of Electrical and Information Engineering, Jiangsu University, Zhenjiang 212013, China*

(Received 16 November 2011; accepted 8 May 2012; published online 18 June 2012)

This paper presents a methodology of asymptotically synchronizing two uncertain generalized Lorenz systems via a single continuous composite adaptive fuzzy controller (AFC). To facilitate controller design, the synchronization problem is transformed into the stabilization problem by feedback linearization. To achieve asymptotic tracking performance, a key property of the optimal fuzzy approximation error is exploited by the Mean Value Theorem. The composite AFC, which utilizes both tracking and modeling error feedbacks, is constructed by introducing a series-parallel identification model into an indirect AFC. It is proved that the closed-loop system achieves asymptotic stability under a sufficient gain condition. Furthermore, the proposed approach cannot only synchronize two different chaotic systems but also significantly reduce computational complexity and implemented cost. Simulation studies further demonstrate the effectiveness of the proposed approach. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4721901>]

In this study, a novel composite adaptive fuzzy controller (AFC) with asymptotic tracking performance is developed to synchronize two uncertain generalized Lorenz systems (GLSs). The composite AFC (CAFC) containing an identification model and an indirect AFC is constructed to achieve fuzzy identification and fuzzy control simultaneously. The closed-loop system achieves asymptotic stability by a single continuous controller under a sufficient gain condition. The proposed approach cannot only synchronize two different chaotic systems with better convergence of tracking errors but also greatly reduce computational complexity and implemented cost.

I. INTRODUCTION

The GLS, which is also known as the unified chaotic system, essentially is a family of chaotic systems.¹ Several chaotic systems, including the classical Lorenz system, Chen system, and Lü system,² are special cases of the GLS. The chaotic system is a complex nonlinear system with many particular characteristics such as an excessive sensitivity to the initial conditions, typically broadband, and fractal properties of the motion in phase space.³ Master-slave synchronization of chaotic systems means that a slave chaotic system is designed so that its outputs follow the outputs of a master system asymptotically. Chaos synchronization is found to be useful in many practical applications such as secure communication, chemical reactions, biological systems, power converters, and information processing,⁴ and has been well investigated in recent years.⁵ Several model-based control

approaches, such as linear feedback control,^{6–10} nonlinear feedback control,^{9,11,12} and impulsive control,⁹ have been applied to synchronize two GLSs.

In practice, chaotic systems contain various types of uncertainties, including unmodeled dynamics, parameter variations, and external disturbances.¹³ Thus, it is a daunting work to establish a model-based controller to synchronize GLSs. To synchronize uncertain GLSs with unknown parameters, sliding mode control (SMC),³ classical adaptive control,^{14–17} active pinning control,¹⁸ and control Lyapunov function-based control^{19,20} were applied in recent years. However, three controllers are needed in Refs. 14–18 and two controllers are required in Refs. 3, 19 and 20. Afterward, a single SMC-based adaptive controller and a single finite-time stability-based adaptive controller were proposed in Refs. 21 and 22, respectively, to reduce computational complexity and implemented cost of synchronizing uncertain GLSs. Note that the bounds of uncertainties are required to be known *a priori* in Refs. 3, 19 and 20, and monotonically positive bound estimation laws that make their estimations unbounded are applied in Refs. 21 and 22. Moreover, Refs. 3, 14–22 only concern the GLSs with exactly known model structures. Recently, a robust adaptive control approach, which contains three controllers with monotonically positive bound estimation laws, was proposed in Ref. 13 to synchronize uncertain GLSs. Note that all the aforementioned approaches can only synchronize two identical chaotic systems.

The approximation-based AFC (Ref. 23) has also been greatly applied to synchronize or control uncertain chaotic systems.^{24–33} Applying the AFC cannot only remove the restriction that chaos model structures must be known *a priori*, but also realize the synchronization of two different chaotic systems. Several AFCs, including the variable universe of discourse-based AFC,²⁴ SMC-based AFC,^{25–29} fuzzy neural network-based AFC,³⁰ Takagi-Sugeno fuzzy-

^{a)}Author to whom correspondence should be addressed. Electronic mail: yppan@ntu.edu.sg. Telephone: +65-65138167. Fax: +65-68968757.

^{b)}Electronic mail: emjer@ntu.edu.sg.

^{c)}Electronic mail: suntren@gmail.com.

based AFC,³⁴ and H^∞ tracking-based AFC,³¹ were developed to synchronize various types of chaotic systems. Moreover, a state feedback AFC and an output feedback AFC were also addressed in Refs. 32 and 33, respectively, to control the uncertain GLS. However, there exist two controllers and the synchronization problem is not considered in Refs. 32 and 33. Moreover, all the aforementioned approaches can only achieve uniformly ultimately bounded (UUB) stability.

This study focuses on asymptotically synchronizing two uncertain GLSs involving unstructured uncertainties, parameter variations, and external disturbances via a single continuous controller. Based on the work of Refs. 35–37, a novel CAFC, which utilizes both tracking and modeling error feedbacks and has the potential to obtain smooth and quick parameter adaptation,³⁸ is developed for the synchronization problem. The design procedure of the proposed controller is as follows: First, the synchronization problem is transformed into the stabilization problem by feedback linearization to facilitate controller design; second, the CAFC structure is established by introducing a series-parallel identification model with a low-pass filter into an indirect AFC; third, a key property of the optimal fuzzy approximation error (FAE) is exploited by the Mean Value Theorem to facilitate asymptotic stability analysis; finally, the composite adaptive laws are derived by the Lyapunov synthesis, where the closed-loop system achieves asymptotic stability in the sense that all involving signals are UUB and both tracking and modeling errors converge to zero.

The structure of this paper is organized as follows. The dynamic model of the GLS, transformation of the synchronization problem, and control objective are formulated in Sec. II. The design procedure of the proposed approach is given in Sec. III. Simulation results are shown in Sec. VI. Concluding remarks are given in Sec. V.

II. PROBLEM FORMULATION

A. System transformation

The model of the GLS is as follows:³⁹

$$\begin{cases} \dot{x}_1 = (25\alpha + 10)(x_2 - x_1) \\ \dot{x}_2 = (28 - 35\alpha)x_1 - x_1x_3 + (29\alpha - 1)x_2 \\ \dot{x}_3 = x_1x_2 - (8 + \alpha)x_3/3, \end{cases} \quad (1)$$

where $\mathbf{x} = [x_1, x_2, x_3]^T \in \mathbb{R}^3$ is a state vector and $\alpha \in [0, 1]$ is an unknown system parameter. When $\alpha \in [0, 0.8)$, $\alpha = 0.8$ and $\alpha \in (0.8, 1]$, Eq. (1) belongs to the classical Lorenz system, Lü system, and Chen system, respectively. Eq. (1) has three unstable equilibria $\forall \alpha \in [0, 1]$ shown as follows:⁴⁰

$$\begin{cases} S_0 = (0, 0, 0) \\ S_- = (-\sqrt{(8+\alpha)(9-2\alpha)}, -\sqrt{(8+\alpha)(9-2\alpha)}, 27-6\alpha) \\ S_+ = (\sqrt{(8+\alpha)(9-2\alpha)}, \sqrt{(8+\alpha)(9-2\alpha)}, 27-6\alpha). \end{cases} \quad (2)$$

Let Eq. (1) be the master system and define the slave system as the perturbed and controlled GLS:²¹

$$\begin{cases} \dot{y}_1 = (25\alpha + 10)(y_2 - y_1) + d_1(t) \\ \dot{y}_2 = (28 - 35\alpha)y_1 - y_1y_3 + (29\alpha - 1)y_2 + d_2(t) + u, \\ \dot{y}_3 = y_1y_2 - (8 + \alpha)y_3/3 \end{cases} \quad (3)$$

where $\mathbf{y} = [y_1, y_2, y_3]^T \in \mathbb{R}^3$ is a state vector, u is a control input, and d_1 and d_2 are mismatched and matched external disturbances, respectively.

Define the original tracking error vector $\mathbf{e}_o := \mathbf{y} - \mathbf{x} = [e_{o1}, e_{o2}, e_{o3}]^T$. Subtracting Eq. (1) from Eq. (3) yields the tracking error dynamics in the affine nonlinear form:

$$\begin{cases} \dot{\mathbf{e}}_o = \mathbf{f}(\mathbf{e}_o, \mathbf{y}) + \mathbf{b}_0 u + \mathbf{d}(t) \\ \mathbf{y} = \mathbf{h}(\mathbf{e}_o), \end{cases} \quad (4)$$

where $\mathbf{f}$, $\mathbf{b}_0$, $\mathbf{d}$, and $\mathbf{h}$ are given by Eq. (11). Let $L_f h$ and $L_g h$ denote the Lie derivatives of h with respect to $\mathbf{f}$ and $\mathbf{g}$, respectively. Since

$$\begin{cases} L_g h(\mathbf{e}_o) = 0 \\ L_g L_f h(\mathbf{e}_o) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T \begin{bmatrix} -(25\alpha + 10) \\ 25\alpha + 10 \\ 0 \end{bmatrix} = 25\alpha + 10 \neq 0, \end{cases} \quad (5)$$

the strong relative degree of Eq. (4) is equal to two.³⁸ Therefore, there exists a diffeomorphism:

$$T(\mathbf{e}_o) = \begin{bmatrix} \mathbf{e} \\ \eta \end{bmatrix} = \begin{bmatrix} e_{o1} \\ (25\alpha + 10)(e_{o2} - e_{o1}) \\ e_{o3} \end{bmatrix} \quad (6)$$

that transforms Eq. (4) into the perturbed controllable canonical form:⁴¹

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f(\mathbf{e}) + g(\mathbf{e})u + d(t), \\ \dot{\eta} = q(\mathbf{e}, \eta) \end{cases} \quad (7)$$

where $\mathbf{e}$ is a tracking error vector defined as $\mathbf{e} := [e_1, e_2]^T = [e_1, \dot{e}_1]^T$, f , g , d , and q are given by Eq. (12). From Eq. (12), one knows that the system in Eq. (7) has the following property as in Ref. 42.

Property 1: There exist a unknown function $\bar{f}(\mathbf{e})$ and unknown constants $\bar{g}$, $\underline{g}$, and $\bar{d}$ such that $|f(\mathbf{e})| \leq \bar{f}(\mathbf{e})$, $0 < \underline{g} \leq |g(\mathbf{e})| \leq \bar{g}$, and $|d(t)| \leq \bar{d}$, $\forall \mathbf{e} \in \mathbb{R}^n$. Moreover, $f(\mathbf{e})$ satisfies $f(\mathbf{0}) = 0$.

From Eq. (12), one also obtains the zero dynamic of the system as follows:

$$\dot{\eta} = q(\mathbf{0}, \eta) = -(8 + \alpha)\eta/3 \quad (8)$$

which is asymptotically stable with $\alpha \in [0, 1]$. Thus, one can only consider the first two equations of Eq. (7) during controller design.

B. Control objective statement

Choose a gain vector $\mathbf{k} = [k_2, k_1]^T \in \mathbb{R}^2$ so that $h(s) = k_1 s + k_2$ is a Hurwitz polynomial, where s is a complex

variable. If f and g are known and $d = 0$ in Eq. (7), one can apply the following ideal control law:

$$u^* = (-\mathbf{k}^T \mathbf{e} - f(\mathbf{e}))/g(\mathbf{e}) \quad (9)$$

to Eq. (7) for obtaining the tracking error dynamics:

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} \quad (10)$$

where $\mathbf{A} = [0, 1; -k_2, -k_1]$. From the selection of $\mathbf{k}$, one knows $\mathbf{A}$ is a stable matrix and $\lim_{t \rightarrow \infty} \|\mathbf{e}(t)\| = 0$. However, since f and g are unknown and $d \neq 0$ in Eq. (7), u^* in Eq. (9) is unrealizable.

$$\begin{cases} f(\mathbf{e}_o, \mathbf{y}) = \begin{bmatrix} (25\alpha + 10)(e_{o2} - e_{o1}) \\ (28 - 35\alpha)e_{o1} + e_{o1}e_{o3} - e_{o1}y_3 - e_{o3}y_1 + (29\alpha - 1)e_{o2} \\ -e_{o1}e_{o2} + e_{o1}y_2 + e_{o2}y_1 - (8 + \alpha)e_{o3}/3 \end{bmatrix}, \\ \mathbf{b}_0 = [0, 1, 0]^T, \mathbf{d}(t) = [d_1(t), d_2(t), 0]^T, h(\mathbf{e}_o) = e_{o1}. \end{cases} \quad (11)$$

$$\begin{cases} f(\mathbf{e}) = (25\alpha + 10)(27e_1 - 6\alpha e_1 + e_1\eta - e_1y_3 - \eta y_1) + (4\alpha - 11)e_2, \\ g(\mathbf{e}) = 25\alpha + 10, d(t) = L_{f+gu+d}^2 L_d e_1 + \sum_{k=1}^2 L_{f+gu+d}^{k-1} L_d L_f^{3-k} e_1, \\ q(\mathbf{e}, \eta) = -e_1(e_1 + e_2/(25\alpha + 10)) + e_1y_2 + (e_1 + e_2/(25\alpha + 10))y_1 - (8 + \alpha)\eta/3. \end{cases} \quad (12)$$

Accordingly, the objective of this study is to synchronize the slave system in Eq. (3) with the master system in Eq. (1) by a single continuous AFC such that the closed-loop system achieves asymptotic stability in the sense that all involving signals are UUB and $\mathbf{e}$ converges to zero.

Remark 1: From the property of the diffeomorphism,³⁸ one knows that if $\mathbf{e}$ converges to zero or a small neighborhood of zero as $t \rightarrow \infty$, then $\mathbf{e}_o$ also tends to zero or a small value, which implies that the slave system in Eq. (3) is synchronous with the master system in Eq. (1).

III. COMPOSITE ADAPTIVE FUZZY CONTROL

A. Indirect adaptive control structure

From the argument in Ref. 43, one knows that d can also be estimated by a function approximator. Thus, to construct a certain control law, one introduces the following FLS:²³

$$\hat{f}(\mathbf{e}|\hat{\boldsymbol{\theta}}_f) = \hat{\boldsymbol{\theta}}_f^T \boldsymbol{\xi}(\mathbf{e}) \quad (13)$$

$$\hat{g}(\mathbf{e}|\hat{\boldsymbol{\theta}}_g) = \hat{\boldsymbol{\theta}}_g^T \boldsymbol{\xi}(\mathbf{e}) \quad (14)$$

to approximate $f_L(\mathbf{e}) := f(\mathbf{e}) + d$ and $g(\mathbf{e})$, respectively, where $\hat{\boldsymbol{\theta}}_f, \hat{\boldsymbol{\theta}}_g \in \mathbb{R}^M$ are adjustable parameter vectors, and $\boldsymbol{\xi}(\mathbf{e}) = [\xi_1(\mathbf{e}), \dots, \xi_M(\mathbf{e})]^T \in \mathbb{R}^M$ is a fuzzy basic function vector. The elements in $\boldsymbol{\xi}(\mathbf{e})$ are defined as follows:

$$\xi_{l_1 l_2}(\mathbf{e}) := \frac{\prod_{i=1}^2 \mu_{A_i^{l_i}}(e_i)}{\sum_{l_1=1}^p \sum_{l_2=1}^p (\prod_{i=1}^2 \mu_{A_i^{l_i}}(e_i))}, \quad (15)$$

where $A_i^{l_i}$ is the linguistic variable of e_i , $\mu_{A_i^{l_i}}$ is the membership function of $A_i^{l_i}$, p is the number of fuzzy partitions, $M = p^2$ is the number of fuzzy rules, $l_1 l_2 = j$, $l_i = 1, \dots, p$, and $j = 1, \dots, M$. Since $f(\mathbf{0}) = 0$ and $g(\mathbf{0}) > 0$ in Eq. (12), $\hat{f}$ and $\hat{g}$ should be designed to satisfy $\hat{f}(\mathbf{0}|\hat{\boldsymbol{\theta}}_f) = 0$ and $\hat{g}(\mathbf{0}|\hat{\boldsymbol{\theta}}_g) > 0$.

Define compact sets $\mathcal{D} := \{\mathbf{e} | \|\mathbf{e}\| \leq M_e\}$, $\Omega_f := \{\hat{\boldsymbol{\theta}}_f | \|\hat{\boldsymbol{\theta}}_f\| \leq M_f\}$ and $\Omega_g := \{\hat{\boldsymbol{\theta}}_g | \|\hat{\boldsymbol{\theta}}_g\| \leq M_g\}$, where $\mathcal{D}$ is the fuzzy approximation region, and $M_e, M_f, M_g \in \mathbb{R}^+$ are finite constants. Then, define the optimal FAE as follows:

$$w(\mathbf{e}) := (f_L(\mathbf{e}) - \hat{f}(\mathbf{e}|\boldsymbol{\theta}_f^*)) + (g(\mathbf{e}) - \hat{g}(\mathbf{e}|\boldsymbol{\theta}_g^*))u, \quad (16)$$

where $\boldsymbol{\theta}_f^*$ and $\boldsymbol{\theta}_g^*$ are H^∞ optimal parameter vectors given by

$$\boldsymbol{\theta}_f^* = \arg \min_{\boldsymbol{\theta}_f \in \Omega_f} \left(\sup_{\mathbf{e} \in \mathcal{D}} |f_L(\mathbf{e}) - \hat{f}(\mathbf{e}|\boldsymbol{\theta}_f)| \right), \quad (17)$$

$$\boldsymbol{\theta}_g^* = \arg \min_{\boldsymbol{\theta}_g \in \Omega_g} \left(\sup_{\mathbf{e} \in \mathcal{D}} |g(\mathbf{e}) - \hat{g}(\mathbf{e}|\boldsymbol{\theta}_g)| \right), \quad (18)$$

respectively. Consequently, one can determine the following certain control law:

$$u = (-\mathbf{k}^T \mathbf{e} - \hat{f}(\mathbf{e}|\hat{\boldsymbol{\theta}}_f))/\hat{g}(\mathbf{e}|\hat{\boldsymbol{\theta}}_g). \quad (19)$$

Substituting Eq. (19) into Eq. (7) and using Eq. (16), one obtains the tracking error dynamics:

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{b}(\tilde{\boldsymbol{\theta}}_f^T \boldsymbol{\xi}(\mathbf{e}) + \tilde{\boldsymbol{\theta}}_g^T \boldsymbol{\xi}(\mathbf{e})u + w), \quad (20)$$

where $\tilde{\boldsymbol{\theta}}_f := \boldsymbol{\theta}_f^* - \hat{\boldsymbol{\theta}}_f$, $\tilde{\boldsymbol{\theta}}_g := \boldsymbol{\theta}_g^* - \hat{\boldsymbol{\theta}}_g$ and $\mathbf{b} := [0, 1]^T$.

From Eqs. (9) and (10), one knows if $\mathbf{e}(0) \in \mathcal{D}_a \subset \mathcal{D}$, then there exists a finite $u = u^*$ in Eq. (9) such that $\lim_{t \rightarrow t_1} \|\mathbf{e}(t)\| = 0$, where $\mathcal{D}_a$ is a domain of attraction.⁴⁴ Since u in Eq. (19) is used to approximate u^* in Eq. (9), u should be designed such that $u \in L_\infty$. Therefore, combining with Property 2, one has the following property as in Ref. 23.

Property 2: There exists a finite constant $\bar{w} \in \mathbb{R}^+$ such that $\bar{w} = \sup_{\mathbf{e} \in \mathcal{D}} |w|$.

B. Identification model

Being motivated by Ref. 23, one introduces the series-parallel identification model with a low-pass filter:

$$\begin{cases} \dot{\hat{e}}_1 = \hat{e}_2 \\ \dot{\hat{e}}_2 = -\alpha \hat{e}_2 + \alpha e_2 + \hat{f}(\mathbf{e}|\hat{\theta}_f) + \hat{g}(\mathbf{e}|\hat{\theta}_g)u + v \end{cases} \quad (21)$$

to achieve composite adaptation, where $\hat{e}_i$ is the estimation of e_i , $i = 1, 2$, α is a user-defined filter parameter, and v is a modeling compensation term. Define the modified modeling error as follows:³⁵

$$\varepsilon := e_2 - \hat{e}_2. \quad (22)$$

Then v in Eq. (21) can be given by

$$v = -\beta \operatorname{sgn}(\varepsilon) \quad (23)$$

in which $\beta \in \mathbb{R}^+$ satisfying $\beta \geq \bar{w}$ is a user-defined finite constant. Using Eqs. (7), (21), and (22), one gets the modified modeling error dynamics:

$$\dot{\varepsilon} = -\alpha \varepsilon + \tilde{\theta}_f^T \xi(\mathbf{e}) + \tilde{\theta}_g^T \xi(\mathbf{e})u + w - v. \quad (24)$$

C. Control law derivation

Before giving the main result of this study, we show a key property of the optimal FAE.

Lemma 1: The optimal FAE w in Eq. (16) satisfies:

$$w(\mathbf{e}) \leq \rho(\|\mathbf{e}\|) \|\mathbf{e}\|, \quad (25)$$

where $\rho(\|\mathbf{e}\|)$ is a positive, globally invertible, and nondecreasing function.

Proof: Making $\mathbf{e}(t) = \mathbf{0}$ in Eq. (7), one has

$$f(\mathbf{0}) + g(\mathbf{0})u + d(t) = 0. \quad (26)$$

Making $\mathbf{e}(t) = \mathbf{0}$ in Eq. (19), one gets

$$u(\mathbf{0}) = -\hat{f}(\mathbf{0}|\hat{\theta}_f)/\hat{g}(\mathbf{0}|\hat{\theta}_g) = 0. \quad (27)$$

Making $\mathbf{e}(t) = \mathbf{0}$ in Eq. (16) and using Eqs. (26) and (27), one obtains

$$\begin{aligned} w(\mathbf{0}) &= (f_L(\mathbf{0}) - \hat{f}(\mathbf{0}|\hat{\theta}_f^*)) + (g(\mathbf{0}) - \hat{g}(\mathbf{0}|\hat{\theta}_g^*))u(\mathbf{0}) \\ &= -\hat{f}(\mathbf{0}|\hat{\theta}_f^*) + \hat{g}(\mathbf{0}|\hat{\theta}_g^*)\hat{f}(\mathbf{0}|\hat{\theta}_f)/\hat{g}(\mathbf{0}|\hat{\theta}_g) = 0. \end{aligned} \quad (28)$$

Then, one applies the Mean Value Theorem to get

$$w(\mathbf{e}) = w(\mathbf{e}) - w(\mathbf{0}) = w'(\mathbf{e})\mathbf{e}, \quad (29)$$

where $w'(\mathbf{e}) = dw(\mathbf{e})/d\mathbf{e}|_{\mathbf{e}=\xi}$, $\xi \in [0, \mathbf{e}]$. From the similar results in Refs. 44–46, there must exist a positive, globally invertible, and nondecreasing function $\rho(\|\mathbf{e}\|)$ such that $w'(\mathbf{e})\mathbf{e} \leq \rho(\|\mathbf{e}\|) \|\mathbf{e}\|$ holds. Combining with Eq. (29), it is easy to get Eq. (25). $\square$

Since A is a stable matrix, there exists a unique positive definite symmetric matrix P for any given positive definite symmetric matrix Q such that

$$A^T P + PA = -Q. \quad (30)$$

Choose a Lyapunov function candidate:

$$V_L = \mathbf{e}^T P \mathbf{e} / 2 + \gamma_e \varepsilon^2 / 2 + \tilde{\theta}_f^T \tilde{\theta}_f / 2\gamma_f + \tilde{\theta}_g^T \tilde{\theta}_g / 2\gamma_g, \quad (31)$$

where $\gamma_e, \gamma_f, \gamma_g \in \mathbb{R}^+$ are learning rates. Now we start the main result of this study.

Theorem 1: For the system in Eq. (7), choose Eq. (19) as the controller and Eq. (21) as the identification model, and design the composite adaptive laws as follows:

$$\dot{\tilde{\theta}}_f = \gamma_f (\mathbf{e}^T P \mathbf{b} + \gamma_e \varepsilon) \xi(\mathbf{e}), \quad (32)$$

$$\dot{\tilde{\theta}}_g = \gamma_g (\mathbf{e}^T P \mathbf{b} + \gamma_e \varepsilon) \xi(\mathbf{e})u. \quad (33)$$

The selection of Q is subjected to

$$\lambda_{\min}(Q) > 2 \|P\mathbf{b}\| \rho(\|\mathbf{e}\|) \quad (34)$$

in which $\lambda_{\min}(Q)$ is the minimal eigenvalue of Q . Then the closed-loop system achieves asymptotic stability in the sense that all involving signals are UUB, $\lim_{t \rightarrow \infty} \|\mathbf{e}(t)\| = 0$ and $\lim_{t \rightarrow \infty} \|\varepsilon(t)\| = 0$.

Proof: Differentiating Eq. (31) along Eqs. (20) and (24) and using Eq. (30) yields

$$\begin{aligned} \dot{V}_L &= -\mathbf{e}^T Q \mathbf{e} / 2 + \mathbf{e}^T P \mathbf{b} (\tilde{\theta}_f^T \xi(\mathbf{e}) + \tilde{\theta}_g^T \xi(\mathbf{e})u) + \mathbf{e}^T P \mathbf{b} w \\ &\quad - \gamma_e \alpha \varepsilon^2 + \gamma_e \varepsilon (\tilde{\theta}_f^T \xi(\mathbf{e}) + \tilde{\theta}_g^T \xi(\mathbf{e})u) + \gamma_e \varepsilon (w - v) \\ &\quad + \tilde{\theta}_f^T \dot{\tilde{\theta}}_f / \gamma_f + \tilde{\theta}_g^T \dot{\tilde{\theta}}_g / \gamma_g \\ &= -\mathbf{e}^T Q \mathbf{e} / 2 - \gamma_e \alpha \varepsilon^2 + \mathbf{e}^T P \mathbf{b} w + \gamma_e \varepsilon (w - v) \\ &\quad + \tilde{\theta}_f^T (\mathbf{e}^T P \mathbf{b} \xi(\mathbf{e}) + \gamma_e \varepsilon \xi(\mathbf{e}) - \dot{\tilde{\theta}}_f / \gamma_f) \\ &\quad + \tilde{\theta}_g^T (\mathbf{e}^T P \mathbf{b} \xi(\mathbf{e})u + \gamma_e \varepsilon \xi(\mathbf{e})u - \dot{\tilde{\theta}}_g / \gamma_g). \end{aligned}$$

Using Eqs. (22), (32), and (33), one obtains

$$\begin{aligned} \dot{V}_L &= -\mathbf{e}^T Q \mathbf{e} / 2 - \gamma_e \alpha \varepsilon^2 + \mathbf{e}^T P \mathbf{b} w + \gamma_e \varepsilon (w - v) \\ &\leq -\mathbf{e}^T Q \mathbf{e} / 2 - \gamma_e \alpha \varepsilon^2 + \mathbf{e}^T P \mathbf{b} w. \end{aligned}$$

Applying Eq. (25) to the above expression leads to

$$\begin{aligned} \dot{V}_L &\leq -\mathbf{e}^T Q \mathbf{e} / 2 - \gamma_e \alpha \varepsilon^2 + \|P\mathbf{b}\| \rho(\|\mathbf{e}\|) \|\mathbf{e}\|^2 \\ &\leq -(\lambda_{\min}(Q)/2 - \|P\mathbf{b}\| \rho(\|\mathbf{e}\|)) \|\mathbf{e}\|^2 - \gamma_e \alpha \varepsilon^2. \end{aligned}$$

Let $\lambda_Q := \lambda_{\min}(Q)/2 - \|P\mathbf{b}\| \rho(\|\mathbf{e}\|)$. Noting Eq. (34), one gets $\lambda_Q \in \mathbb{R}^+$. Thus, one has

$$\dot{V}_L \leq -\lambda_Q \|\mathbf{e}\|^2 - \gamma_e \alpha \varepsilon^2 < 0, \forall \mathbf{e} \neq 0, \varepsilon \neq 0,$$

which implies that the closed-loop system is asymptotically stable.³⁸ Thus, one gets $\mathbf{e}, \mathbf{x}, \varepsilon \in L_\infty$, $\lim_{t \rightarrow \infty} \|\mathbf{e}(t)\| = 0$ and $\lim_{t \rightarrow \infty} \|\varepsilon(t)\| = 0$. From Eq. (19), it is easy to obtain $u \in L_\infty$. Thus, all involving signals are UUB. $\square$

Remark 2: In Eq. (34), Q is user-defined and ρ is unknown. Combining with Eq. (30), one knows that for ρ in Eq. (25), there must exist proper k such that Eq. (34) holds. To enhance the convergence of $\mathbf{e}$, one can increase γ_f, γ_g, Q , and/or k ; to enhance the convergence of ε , one can increase γ_e, α , and/or β .

Remark 3: Since the strong relative degree of the controlled GLS in Eq. (3) is equal to two, the control law in Theorem 1 can also be used in the synchronization of two different chaotic systems in the form of Eq. (3) with different α value.

Remark 4: The latest AFC-based chaotic synchronization approach in Ref. 30 has two major limitations: (1) there are three basic AFCs with total $15M$ adaptive parameters, and three additional robust control terms with total 12 adaptive parameters; (2) the applied monotonically positive bound estimation laws for achieving asymptotic stability make their estimations unbounded. In our approach, there exists only a

single controller with total M adaptive parameters without the additional robust control term; the applied key property of the optimal FAE w in Lemma 1 makes the closed-loop system asymptotically stable under continuous and bounded control input.

IV. NUMERICAL SIMULATION

The procedure of the controller parameter selection is as follows: First, select the MFs of $A_i^{l_i}$ as follows:

$$\mu_{A_i^{l_i}}(x_i) = \exp(-(x_i - 2(l_i - 3))^2 / 0.5^2 / 2),$$

FIG. 1. Tracking performance of case 1.

FIG. 2. Tracking performance of case 2.

where $l_i = 1, \dots, 5$, and $i = 1, 2$, and set $\hat{\theta}(0) = [0, \dots, 0]^T$. Second, let $k_1 = 24$, $k_2 = 144$, and $Q = \text{diag}(10, 10)$; third, let $\gamma_f = 30$, $\gamma_g = 1$, and $\gamma_e = 15$ in Eqs. (32) and (33); finally, let $\alpha = 10$ and $\beta = 1$ in Eq. (21) with Eq. (23).

Case 1: Synchronization of two identical Lorenz systems. Let $\alpha = 0.2$ in Eqs. (1) and (3) to make both the master and the slave systems be Lorenz systems. Let $\mathbf{x}(0) = [1, 1, 1]^T$, $\mathbf{y}(0) = [-0.3, -4, -2]^T$,²¹ $d_1(t) = 0.1\sin(2\pi t)$ and $d_2(t) = 0.2\sin(\pi t)$.³⁰ The tracing performance in this case is shown in Fig. 1. One observes that the proposed approach obtains favorable tracking performance with rapid rise time and small tracing errors.

Case 2: Synchronization of Lorenz and Chen systems. Let $\alpha = 1$ in Eq. (1) to make the master system be a Chen

system and $\alpha = 0.2$ in Eq. (3) to make the slave system be a Lorenz system. The tracking performance of this case is shown in Fig. 2. The qualitative analysis of the tracking trajectories is similar with that of case 1, which demonstrates the effectiveness of the proposed approach for synchronizing two different Chaos systems.

V. CONCLUSION

This paper has successfully developed a continuous CAFC for the asymptotic synchronization of two uncertain GLSs. The overall control scheme is comprised of a series-parallel identification model with a low-pass filter and an indirect AFC. Compared with the previous GLS synchronization approaches, the advantages of our approach are as follows:

(1) both tracking and modeling errors are applied to enhance tracking performance; (2) the asymptotic stability of the closed-loop system is obtained by a continuous controller; (3) computational complexity and implemented cost is greatly reduced due to the use of only a single controller; and (4) it can also be used for synchronizing two different chaotic systems. Simulation results have demonstrated the effectiveness of the proposed approach.

ACKNOWLEDGMENTS

The authors would like to acknowledge reviewers for their useful suggestions that have greatly improved the quality of this manuscript. This work is partially supported by the Singapore Agency for Science, Technology and Research (A*STAR) Science and Engineering Research Council under Grant No. 1122904016.

- ¹S. Celikovskiy and G. Chen, "On a generalized Lorenz canonical form of chaotic systems," *Int. J. Bifurcation Chaos* **12**, 1789–1812 (2002).
- ²J. Lü and G. Chen, "A new chaotic attractor coined," *Int. J. Bifurcation Chaos* **12**, 659–661 (2002).
- ³J. Yan, Y. Yang, T. Chiang, and C. Chen, "Robust synchronization of unified chaotic systems via sliding mode control," *Chaos, Solitons Fractals* **34**, 947–954 (2007).
- ⁴G. Chen and X. Dong, *From Chaos to Order: Methodologies, Perspectives and Applications* (World Scientific, Singapore, 1998).
- ⁵S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, and C. S. Zhou, "The synchronization of chaotic systems," *Phys. Rep.* **366**, 1–101 (2002).
- ⁶X. Wu, G. Chen, and J. Cai, "Chaos synchronization of the master-slave generalized Lorenz systems via linear state error feedback control," *Physica D* **229**, 52–80 (2007).
- ⁷G. Gambino, M. C. Lombardo, and M. Sammartino, "Global linear feedback control for the generalized Lorenz system," *Chaos, Solitons Fractals* **29**, 829–837 (2006).
- ⁸Y. Chen, X. Wu, and Z. Gui, "Global synchronization criteria for a class of third-order non-autonomous chaotic systems via linear state error feedback control," *Appl. Math. Model.* **34**, 4161–4170 (2010).
- ⁹X. Wang and J. Song, "Synchronization of the unified chaotic system," *Nonlinear Anal. Theory, Methods Appl.* **69**, 3409–3416 (2008).
- ¹⁰C. Kim and D. Chwa, "Synchronization of the bidirectionally coupled unified chaotic system via sum of squares method," *Chaos* **21**, 013104 (2011).
- ¹¹X. Liao, F. Xu, P. Wang, and P. Yu, "Chaos control and synchronization for a special generalized Lorenz canonical system—The SM system," *Chaos, Solitons Fractals* **39**, 2491–2508 (2009).
- ¹²J. H. Park, "On synchronization of unified chaotic systems via nonlinear control," *Chaos, Solitons Fractals* **25**, 699–704 (2005).
- ¹³H. R. Kofigar, S. Hosseinnia, and F. Sheikholeslam, "Robust adaptive synchronization of uncertain unified chaotic systems," *Nonlinear Dyn.* **59**, 477–483 (2010).
- ¹⁴G. Chen and J. Lü, "Synchronization of an uncertain unified chaotic system via adaptive control," *Chaos, Solitons Fractals* **14**, 643–647 (2002).
- ¹⁵Y. Yu, "Adaptive synchronization of a unified chaotic system," *Chaos, Solitons Fractals* **36**, 329–333 (2008).
- ¹⁶X. Wu, "A new chaotic communication scheme based on adaptive synchronization," *Chaos* **16**, 043118 (2006).
- ¹⁷W. Yu, J. Cao, K. Wong, and J. Lu, "New communication schemes based on adaptive synchronization," *Chaos* **17**, 33114 (2007).
- ¹⁸L. Pan, W. Zhou, J. Fang, and D. Li, "A novel active pinning control for synchronization and anti-synchronization of new uncertain unified chaotic systems," *Nonlinear Dyn.* **62**, 417–425 (2010).
- ¹⁹H. Wang, Z. Han, Q. Xie, and W. Zhang, "Finite-time synchronization of uncertain unified chaotic systems based on CLF," *Nonlinear Anal.: Real World Appl.* **10**, 2842–2849 (2009).
- ²⁰H. Wang, Z. Han, W. Zhang, and Q. Xie, "Synchronization of unified chaotic systems with uncertain parameters based on the CLF," *Nonlinear Anal.: Real World Appl.* **10**, 715–722 (2009).
- ²¹J. Lin and J. Yan, "Adaptive synchronization for two identical generalized Lorenz chaotic systems via a single controller," *Nonlinear Anal.: Real World Appl.* **10**, 1151–1159 (2009).
- ²²U. E. Vincent and R. Guo, "Finite-time synchronization for a class of chaotic and hyperchaotic systems via adaptive feedback controller," *Phys. Lett. A* **375**, 2322–2326 (2011).
- ²³L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis* (Prentice Hall, Englewood Cliffs, NJ, 1994).
- ²⁴Y. Che, J. Wang, W. Chan, and K. Tsang, "Chaos synchronization of coupled neurons under electrical stimulation via robust adaptive fuzzy control," *Nonlinear Dyn.* **61**, 847–857 (2010).
- ²⁵A. Poursamad and A. H. Davaie-Markazi, "Robust adaptive fuzzy control of unknown chaotic systems," *Appl. Soft Comput.* **9**, 970–976 (2009).
- ²⁶H. Layeghi, M. T. Arjmand, H. Salarieh, and A. Alasty, "Stabilizing periodic orbits of chaotic systems using fuzzy adaptive sliding mode control," *Chaos, Solitons Fractals* **37**, 1125–1135 (2008).
- ²⁷A. Bagheri and J. J. Moghaddam, "Decoupled adaptive neuro-fuzzy (DANF) sliding mode control system for a Lorenz chaotic problem," *Expert Sys. Applic.* **36**, 6062–6068 (2009).
- ²⁸M. Roopaei and M. Z. Jahromi, "Synchronization of two different chaotic systems using novel adaptive fuzzy sliding mode control," *Chaos* **18**, 033133 (2008).
- ²⁹M. Roopaei, M. Z. Jahromi, and S. Jafari, "Adaptive gain fuzzy sliding mode control for the synchronization of nonlinear chaotic gyros," *Chaos* **19**, 013125 (2009).
- ³⁰C. Chen and H. Chen, "Robust adaptive neural-fuzzy-network control for the synchronization of uncertain chaotic systems," *Nonlinear Anal.: Real World Appl.* **10**, 1466–1479 (2009).
- ³¹E. Hwang, C. Hyun, E. Kim, and M. Park, "Fuzzy model based adaptive synchronization of uncertain chaotic systems: Robust tracking control approach," *Phys. Lett. A* **373**, 1935–1939 (2009).
- ³²B. Chen, X. Liu, and S. Tong, "Adaptive fuzzy approach to control unified chaotic systems," *Chaos, Solitons Fractals* **34**, 1180–1187 (2007).
- ³³Y. Liu and Y. Zheng, "Adaptive robust fuzzy control for a class of uncertain chaotic systems," *Nonlinear Dyn.* **57**, 431–439 (2009).
- ³⁴X. Wang and J. Meng, "Observer-based adaptive fuzzy synchronization for hyperchaotic systems," *Chaos* **18**, 033102 (2008).
- ³⁵D. Bellomo, D. Naso, and R. Babuška, "Adaptive fuzzy control of a nonlinear servo-drive: Theory and experimental results," *Eng. Applic. Artif. Intell.* **21**, 846–857 (2008).
- ³⁶M. Hojati and S. Gazor, "Hybrid adaptive fuzzy identification and control of nonlinear systems," *IEEE Trans. Fuzzy Syst.* **10**, 198–210 (2002).
- ³⁷D. Naso, F. Cupertino, and B. Turchiano, "Precise position control of tubular linear motors with neural networks and composite learning," *Control Eng. Pract.* **18**, 515–522 (2010).
- ³⁸J. E. Slotine and W. Li, *Applied Nonlinear Control* (Prentice Hall, Englewood Cliffs, NJ, 1991).
- ³⁹S. Celikovskiy and G. Chen, "On the generalized Lorenz canonical form," *Chaos, Solitons Fractals* **26**, 1271–1276 (2005).
- ⁴⁰J. Lü, G. Chen, D. Cheng, and S. Celikovskiy, "Bridge the gap between the Lorenz system and the Chen system," *Int. J. Bifurcation Chaos* **12**, 2917–2926 (2002).
- ⁴¹B. S. Chen, C. H. Lee, and Y. C. Chang, " H^∞ tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach," *IEEE Trans. Fuzzy Syst.* **4**, 32–43 (1996).
- ⁴²S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable Adaptive Neural Network Control* (Kluwer, Boston, MA, 2001).
- ⁴³J. A. Farrell and M. M. Polycarpou, *Adaptive Approximation Based Control: Unifying Neural, Fuzzy and Traditional Adaptive Approximation Approaches* (John Wiley & Sons, Hoboken, NJ, 2006).
- ⁴⁴B. Xian, D. M. Dawson, M. S. de Queiroz, and J. Chen, "A continuous asymptotic tracking control strategy for uncertain nonlinear systems," *IEEE Trans. Autom. Control* **49**, 1206–1211 (2004).
- ⁴⁵P. M. Patre, S. Bhasin, Z. D. Wilcox, and W. E. Dixon, "Composite adaptation for neural network-based controllers," *IEEE Trans. Autom. Control* **55**, 944–950 (2010).
- ⁴⁶P. M. Patre, W. MacKunis, K. Kaiser, and W. E. Dixon, "Asymptotic tracking for uncertain dynamic systems via a multilayer neural network feedforward and RISE feedback control structure," *IEEE Trans. Autom. Control* **53**, 2180–2185 (2008).