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Modified Bouc-Wen model for hysteresis behavior of RC beam-column joints with limited transverse reinforcement

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Abstract

An analytical approach based on modified Bouc-Wen-Baber-Noori model has been proposed in this paper for predicting the hysteresis behavior of reinforced concrete beam-column joints with limited transverse reinforcement. The analytical model presented in this research is able to capture the characteristics of non-seismic detailed beam-column joints such as stiffness and strength degradation and pinching. Livermore Solver for Ordinary Differential Equations (LSODE) and Genetic Algorithm (GA) have been employed to solve the differential equations and to execute systematic estimation of the parameters associated with the model respectively. The analytical model has been calibrated with the experimental results of old fashioned interior and exterior beam-column joints obtained from the literature. In a bid to examine the influence of variation of each analytical parameter on the model, sensitivity analysis has been performed. Thereafter, an extensive parametric study has been conducted to relate the physical parameters of beam-column joints to the analytical model parameters. The upper and lower bounds of the magnitude of the analytical model parameters have been proposed subsequently with a method to identify the parameters for a specific beam-column joint depending on its physical parameters.

1. Introduction

Reinforced concrete (RC) structures in regions of low to moderate seismicity are designed for gravity loading, not complying with the modern seismic design codes. RC moment resisting frames have little or no transverse reinforcement in the beam-column joint regions which in turn may not perform adequately to withstand earthquake-induced actions. In the Padang Earthquake (2009), several RC buildings experienced extensive damage at the beam-column joints, as shown in Fig. 1. Although substantial experimental research has been undertaken by several researchers [1-5] till date on RC non-seismic detailed beam-column joints, there is still a dearth of analytical studies on prediction of their hysteresis behavior.

Hysteresis models developed in the past includes Elasto-plastic model by Veletsos and Newmark [6], where the primary force-deformation curve is represented by an elastic portion indicating the cracked-section behavior with no incremental stiffness upon

yielding and during unloading. Clough Degrading Stiffness model [7] operates on a bilinear primary curve with ascending post yielding branches and stiffness degradation during load reversals. Takeda model [8] works on a trilinear primary curve representing uncracked, cracked and post-yielding stages where nonlinear deformation initiates after section cracks. Degrading Bilinear model by Imbeault and Neilson [9] is a peak-oriented hysteresis model in which stiffness changes only when the prior maximum is exceeded in any direction. For Q-Hysteresis model by Saidi and Sozen [10] and Otani Hysteresis model [11], the primary curve is a bilinear curve with ascending post-yielding branches with inclusion of stiffness degradation at unloading and load reversal. Hysteresis Shear model by Ozcebe and Saatcioglu [12] is based on statistical analysis of past experimental data. Alath and Kunnath model [13] simulates the joint shear deformation by a rotational spring with degrading hysteresis while Biddah and Ghobarah [14] model comprises of separate rotational springs for the joint shear and bond-slip deformation. Hwang and Lee model [15] predicts the shear strength of exterior beam-column joints for seismic resistance based on softened strut-and-tie model. Youssef and Ghobarah joint element [16] has two diagonal translational springs and twelve translational springs to simulate the joint shear deformations and bond-slip respectively. Lowes and Altoontash joint element [17] consists of eight zero-length translational springs, a zero-length rotational spring and four zero-length shear springs to simulate the bond-slip response of longitudinal reinforcement, the joint shear deformation and the interface shear deformation respectively while simplified Altoontash joint element [18] has four zero-length rotational springs at beam-column interfaces and a rotational spring to simulate the member-end rotations due to bond-slip and the joint shear deformation. Shin and LaFave

Notations			
m	system mass	ε	hysteretic energy
u	relative displacement of mass with respect to base displacement	f_h	hysteretic force
t	time	δ_v	parameter that controls rate of strength degradation
$(\cdot)$	derivatives with respect to time t	δ_η	parameter that controls rate of stiffness degradation
c	viscous damping coefficient	ζ_1	parameter that controls magnitude of initial drop in slope, dz/du
z	hysteretic displacement	$\text{sgn}()$	Signum function
α	a weighing constant that represents relative participation of linear and nonlinear terms, also known as rigidity ratio i.e. the ratio of final and initial tangent stiffness	q	parameter that sets pinching level as a fraction of ultimate hysteretic strength, z_{max}
k	stiffness	ζ_2	parameter that causes the pinching region to spread
k_i	initial tangent stiffness	ζ_s	parameter that measures total slip
k_f	final tangent stiffness	p	parameter that controls the rate of initial drop in slope, dz/du
F_T	total non-damping restoring force	ψ	parameter that contributes to amount of pinching
$F(t)$	time dependent forcing function	δ_ψ	parameter specified for desired rate of pinching spread
ζ_0	linear viscous damping ratio	λ	parameter that controls rate of change of ζ_2 with change of ζ_1
ω_0	pre-yield natural frequency of system	y	vector containing the set of Ordinary Differential Equations
$f(t)$	mass normalized forcing function	f	vector-valued function of y and t
β, γ, n	hysteresis shape parameter	$e_{\pm 0}$	root mean square error
v	strength degradation parameter	N	number of data points for input displacement function
η	stiffness degradation parameter	$ e_{\pm 0} $	maximum root mean square error
A	parameter that regulates the tangent stiffness	f'_c	concrete compressive cylinder strength
$h(z)$	pinching function	A_g	gross area of beam-column joint
z_{max}	ultimate value of z i.e. z at $dz/du = 0$		
α_0	magnitude of α at zero displacement		
D_{max}	maximum positive or negative displacement depending on $u > 0$ or $u < 0$ respectively		

model [19] proposes the joint as rigid elements along the panel edges with a rotational spring embedded in one hinge linking adjacent rigid elements and two rotational springs at beam-joint interfaces to simulate the member-end rotations due to inelastic behavior of the beam longitudinal reinforcement and the plastic hinge rotations due to inelastic

behavior of the beam separately. The analytical model proposed by Favvata et al. [20] assumes the exterior beam-column joint element as a zero length spring element which incorporates stiffness degradation and pinching effect as special rules.

An analytical model of beam-column joint requires a force displacement relationship capable of producing requisite strength and stiffness degradation and pinching at all displacement levels. This is a stringent requirement considering the numerous parameters contributing to the hysteresis behavior of beam-column joints. After exploring the above-mentioned analytical models, an utmost effort has been undertaken to illustrate the hysteresis behavior of RC non-seismic detailed beam-column joints analytically based on modification of Bouc-Wen-Baber-Noori model [21,22]. The efficiency of the proposed model is then verified by calibrating it with the experimental results of interior and exterior beam-column joints, obtained from the literature. Sensitivity of the model due to the variation of each analytical parameter has been investigated and thereafter an extensive parametric study with varying joint physical parameters has been conducted to provide the approximate range of magnitudes for the parameters and to determine an effective way to identify them for any beam-column joint depending on its physical parameters.

2. Proposed analytical hysteresis model

2.1. Background

With an objective of developing a generic, computationally efficient and mathematically tractable hysteresis model, the basis for this model is selected as the Bouc, Wen, Baber and Noori (BWBN) model. Bouc suggested a versatile, smoothly varying hysteresis model for a single-degree-of-freedom (SDOF) system under forced vibration. Baber and Wen [22] extended the model to include stiffness and strength degradation as a function of hysteretic energy while Baber and Noori [21] incorporated pinching in the hysteresis model. This paper modifies the original BWBN model accordingly to assimilate hysteresis behavior of reinforced concrete substandard beam-column joints.

2.2. Equation of motion and constitutive relations

The equation of motion for a single-degree-of-freedom system can be expressed as follows:

$$m\ddot{u} + c\dot{u} + F_T[u(t), z(t), t] = F(t) \quad (1)$$

where u is the relative displacement of mass m with respect to ground motion and dot ($\dot{\cdot}$) signifies the differential with respect to time; c is the linear viscous damping coefficient; $F_T[u(t), z(t), t]$ is the non-damping restoring force consisting of the linear restoring force $\alpha k u$ and the hysteretic restoring force $(1 - \alpha)kz$; α is stiffness ratio i.e. the ratio of final asymptote tangent stiffness k_f to initial stiffness k_i and $F(t)$ is the time-dependent forcing function.

Dividing both sides of (1) by m , the following standard expression is obtained:

$$\ddot{u} + 2\xi_0\omega_0\dot{u} + \alpha\omega_0^2u + (1 - \alpha)\omega_0^2z = f(t) \quad (2)$$

where ξ_0 is the linear damping ratio, $c/2\sqrt{k_i/m}$; ω_0 is the pre-yield system natural frequency, $\sqrt{k_i/m}$; $f(t)$ is the mass-normalized forcing function.

Hysteretic restoring force is a function of hysteretic displacement z and thus the relationship between z and u is shown in the following expression:

$$\dot{z} = h(z) \frac{A\dot{u} - v(\beta|\dot{u}||z|^{n-1}z + \gamma\dot{u}|z|^n)}{\eta} \quad (3)$$

where β, γ and n are hysteretic shape parameters; A determines the tangent stiffness; v and η are the strength and stiffness degradation parameters respectively and $h(z)$ is the pinching function.

For a non-pinching and non-degrading system, it is considered that hysteresis is defined by a continuous function and hysteresis stiffness is always zero at local maximum or minimum. It is the point on the load-slip curve where velocity changes its sign. Hence, at an infinitesimal distance dz away from z_{max} , where velocity is close to but not equal to zero and $\dot{z}_{max} \approx \dot{z}_1$:

$$\begin{aligned} \dot{z}_{max} \approx 0 &= A\dot{u} - v(\beta|\dot{u}||z|^{n-1}z + \gamma\dot{u}|z|^n) \\ z_{max} &= \pm \{A/v(\beta + \gamma)\}^{1/n} \end{aligned} \quad (4)$$

Although inclusion of A grants increased versatility of the model, this parameter is somewhat redundant as both hysteretic stiffness and hysteretic force, a function of hysteretic displacement, can be varied by the stiffness ratio α and the hysteresis shape parameters β, γ and n . Thus, A has been set to unity to remove redundancy.

2.2.1. Stiffness ratio or rigidity ratio

Stiffness ratio or rigidity ratio α is the ratio of final asymptote tangent stiffness to initial stiffness. The magnitude of α is 1 for a linear system and 0 for a complete nonlinear system. In the original BWBN model, α was considered to be of constant magnitude. However, based on experimental results of reinforced concrete beam-column joints under cyclic loading, it can be well perceived that stiffness of the beam-column joints decreases after attaining a certain displacement and thus, stiffness ratio cannot be of constant magnitude. Therefore, α can be expressed as a function of D_{max} :

$$\alpha = \alpha_0 e^{(-0.1D_{max})} \quad (5)$$

Here D_{max} is the absolute value of the maximum positive displacement and maximum negative displacement for $u > 0$ and $u < 0$ respectively. Here α_0 is the magnitude of α at zero displacement, considered to be of constant magnitude. Pinching stiffness

(minimum tangent stiffness of the curve where unloading finishes and reloading begins) is around $\alpha\omega_0^2$ and the stiffness decreases with development of maximum displacement which proves that use of varying α is more accurate. The hysteresis loop with constant and displacement-based α is shown in Fig. 2.

2.2.2. Hysteresis shape parameters

Three hysteresis parameters β , γ and n and their interactions determine the basic hysteresis shape. The absolute values of β and γ inversely influence hysteretic stiffness and strength, as well as smoothness of the hysteresis loops. During loading, parameter n controls sharpness of the transition from initial to asymptotic slope. For $n = 1$, the relationships between β and γ and their effects on hysteresis are described below and shown in Fig. 3.

- $\left. \begin{matrix} \beta + \gamma > 0 \\ \gamma - \beta < 0 \end{matrix} \right\}$ Weak Softening
- $\left. \begin{matrix} \beta + \gamma > 0 \\ \gamma - \beta = 0 \end{matrix} \right\}$ Weak Softening on loading, mostly linear unloading
- $\left. \begin{matrix} \beta + \gamma > \gamma - \beta \\ \gamma - \beta > 0 \end{matrix} \right\}$ Strong Softening on loading and unloading, narrow loop
- $\left. \begin{matrix} \beta + \gamma = 0 \\ \gamma - \beta < 0 \end{matrix} \right\}$ Weak Hardening
- $\left. \begin{matrix} 0 > \beta + \gamma \\ \beta + \gamma > \gamma - \beta \end{matrix} \right\}$ Strong Hardening

2.2.3. Hysteretic energy

Hysteretic energy is an essential term to approximate strength and stiffness degradation. The energy absorbed by the hysteretic element is the continuous integral of hysteretic force, f_h over the total displacement u . The hysteretic energy is expressed as:

$$\begin{aligned} \varepsilon(t) &= \int_{u(0)}^{u(t)} f_h \cdot du = (1 - \alpha)\omega_0^2 \int_{u(0)}^{u(t)} z(u, t) \cdot du \cdot \frac{du}{dt} \\ &= (1 - \alpha)\omega_0^2 \int_0^t z(u, t) \cdot \dot{u}(t) \cdot dt \end{aligned} \quad (6)$$

2.2.4. Strength and stiffness degradation

Strength and stiffness degradation parameters, ν and η respectively, are the functions of total hysteretic energy as shown in the following expressions:

$$\nu(\varepsilon) = 1 + \delta_\nu \varepsilon \text{ and } \eta(\varepsilon) = 1 + \delta_\eta \varepsilon \quad (7)$$

where δ_ν and δ_η are the constants specified for the desired rates of strength and stiffness degradation respectively at different displacement levels. When the magnitudes of δ_ν and δ_η are zero, the structure does not degrade its strength and stiffness. Due to increase in δ_η ,

both hysteretic force and hysteretic stiffness degrade whereas increase in δ_v reduces the hysteretic force without changing the hysteretic stiffness.

2.2.5. Pinching

The expression for pinching function $h(z)$ is as follows:

$$h(z) = 1 - \zeta_1 e^{-(z \operatorname{sgn}(\dot{u}) - q z_{\max})^2 / \zeta_2^2} \quad (8)$$

where ζ_1 determines the severity of pinching or the magnitude of initial drop in slope dz/du and ζ_1 varies from 0 to 1; ζ_2 causes the pinching region to spread and q is a constant that sets the pinching level as a fraction of $z_{\max}$. Both ζ_1 and ζ_2 vary with hysteretic energy (Eq. (6)), as mentioned in the following equations:

$$\zeta_1(\varepsilon) = \zeta_s \{1 - e^{(-p\varepsilon)}\} \text{ and } \zeta_2(\varepsilon) = (\psi + \delta_\psi)(\lambda + \zeta_1) \quad (9)$$

where p is a constant that contributes to the rate of initial drop in slope; ζ_s is the measure of total slip; ψ is a parameter that controls the amount of pinching; δ_ψ is a constant for the desired rate of pinching spread and λ is a parameter that controls the rate of change of ζ_2 with change of ζ_1 .

3. Solving procedure for proposed hysteresis model

The complete hysteresis model can be represented in its analytical form as follows:

$$\ddot{u} + 2\xi_0\omega_0\dot{u} + \alpha\omega_0^2u + (1 - \alpha)\omega_0^2z = f(t) \quad (10)$$

$$\dot{z} = \left\langle 1 - \zeta_s(1 - e^{-p\varepsilon})e^{-(z \operatorname{sgn}(\dot{u}) - q\{1/(1+\delta_v\varepsilon)(\beta+\gamma)\})^{1/n})^2 / (\psi + \delta_\psi\varepsilon)^2 [\lambda + \zeta_s(1 - e^{-p\varepsilon})]^2} \right\rangle \\ \times \left\langle \frac{\dot{u} - (1 + \delta_v\varepsilon)(\beta|\dot{u}||z|^{n-1}z + \gamma\dot{u}|z|^n)}{1 + \delta_\eta\varepsilon} \right\rangle \quad (11)$$

$$\varepsilon(t) = (1 - \alpha)\omega_0^2 \int_0^t z(u, t) \cdot \dot{u}(t) \cdot dt \quad (12)$$

Here all the notations carry their usual significances. In Eqs. (10)-(12), all the derivatives appear in the first power and the variables vary with time at highly different rates. Hence, the hysteresis model consists of a stiff set of Ordinary Differential Equations (ODE), which can be solved numerically by using Gear's backward differential formulae. In the present research, Livermore Solver for Ordinary Differential Equations (LSODE) has been chosen for solving the ODEs involved in the proposed analytical model. LSODE, after determining any problem to be comprising of a stiff set of ODEs, uses the Gear Method for solving the equations. Moreover, the input displacement function required for computation, may not necessarily be continuous. Even discrete data points can be read from an external file to serve the purpose.

LSODE requires the user to convert the system of ODEs into an array of first order ODEs.

$$\frac{dy}{dt} = f(t, y) \quad (13)$$

where y is a vector containing the set of ODEs and f is a vector-valued function of t and y . Subsequently, it can be written as

$$\begin{pmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{pmatrix} = \begin{pmatrix} u(t) \\ \dot{u}(t) \\ z(t) \\ \varepsilon(t) \end{pmatrix} \quad (14)$$

The hysteresis model Eqs. (10)-(12) can be rewritten based on Eq. (14) as follows:

$$\dot{y}_1 = y_2 \quad (15)$$

$$\dot{y}_2 = -2\xi_0\omega_0 y_2 - \alpha\omega_0^2 y_1 - (1 - \alpha)\omega_0^2 y_3 + f(t) \quad (16)$$

$$\dot{y}_3 = \left\langle 1 - \zeta_s(1 - e^{-py_4})e^{-(y_3 \operatorname{sgn}(y_2) - q(1/(1+\delta_v y_4)(\beta+\gamma))^{1/n})^2 / (\psi + \delta_p y_4)^2} [\zeta_s(1 - e^{-py_4})]^2} \right\rangle \\ \times \left\langle \frac{y_2 - (1 + \delta_v y_4)(\beta|y_2||y_3|^{n-1} y_3 + \gamma y_2 |y_3|^n)}{1 + \delta_\eta y_4} \right\rangle \quad (17)$$

$$\dot{y}_4 = (1 - \alpha)\omega_0^2 y_2 y_3 \quad (18)$$

LSODE employs user-specified relative and absolute error control. Satisfactory results have been obtained by turning off the relative error control and keeping the absolute error control at a constant magnitude of 10^{12} . Once the displacement function is known and the parameters are estimated using a system identification technique, LSODE can be used to work out these equations without any difficulty.

4. Estimation of parameters involved in hysteresis model

The hysteretic force for input displacement cannot be computed from the model until the analytical parameters are estimated properly and inserted in the solver subroutine. In order to minimize the difference between the experimental results and the model output for a given input function, estimation of the analytical parameters is requisite so that the hysteresis model can be practical and applicable to a wide range of similar problems. Since the hysteresis model is not only sensitive to the parameters, but also to the interaction between them, it is almost impossible to identify the parameters reasonably without a systematic search. Several methods [23-25] have been used by various researchers to carry out efficient parameter estimation. In this study, a Genetic Algorithm (GA) has been written in Visual Fortran to estimate the parameters of the analytical

model. The structure of GA is characterized by four nested loops [26-29]. The innermost loop (Loop 4) is the actual GA that generates a population, checks solver (LSODE) calculations, as well as selects and mates the pairs to crossover and mutate. Solver checking is necessary because the parameters are generated at random. To prevent the GA from falsely recognizing the erroneous sums of squares as better fit, solver computation is checked after each run. Loop 3 executes GA a user-specified number of times, each time with a different randomly chosen initial population. Loop 2 progressively decreases or shrinks the parameter interval. GA is an adaptive algorithm in the sense that it is able to discover erroneous initial input ranges for the parameters. If a wrong interval is specified and the optimal parameter lies outside the interval, the results tend to be clustered near the side of the interval that should be readjusted. GA subsequently shifts the interval in the direction of clustering and starts over, which is the task of Loop 1. One of the significant benefits of using GA is that the interval selection for each parameter does not affect the end result, but it can make a significant difference in the CPU time needed to reach the ultimate solution. Although GA takes longer time to converge than the calculus-based techniques, but a trend can be recognized relatively faster and quick insight can be gained regarding the problem at hand.

5. Calibration of analytical model with experimental results

To check the appropriateness of selection of the pinching function and the accuracy of the solver and algorithm, the hysteresis model after parameter identification has been calibrated with the experimental results of reinforced concrete (RC) interior and exterior beam-column joints with limited transverse reinforcement obtained from the literature.

RC non-seismic detailed interior beam-column joint specimens, Unit O1 by Hakuto et al. [1], Units 1 and 2 tested by Liu et al. [2], Units PEER-1450, PEER-2250, CD15-1450, CD30-1450 and CD30-2250 tested by Walker [3] and PEER-0995 and PEER-4150 tested by Alire [4] and exterior beam-column joint specimens Units O6 and O7 tested by Hakuto et al. [1], Units EJ2 and EJ3 tested by Liu et al. [2] and Units 1, 2, 3, 4, 5 and 6 tested by Pantelides et al. [5] have been selected for calibration of the analytical model with experimental results in order to verify the effectiveness of the proposed approach. The interior and exterior beam-column joint specimens with limited transverse reinforcement tested under cyclic loading have been selected from literature in such a way that a wide range of variation is covered with respect to the joint aspect ratio, application of column axial load, plain or deformed reinforcing bars, reinforcing bar layout and the grade of concrete and steel. However, the retrofitted beam-column joints or the beam-column joints with transverse beam or with slab have been kept out of the scope of this research.

After selection of the specimens to be calibrated, their load deformation data are retrieved to estimate the analytical model parameters for each specimen using Genetic Algorithm, where the stiffness ratio and pinching function are defined based on Section 2. Then, from the analytical parameters estimated for interior and exterior beam-column joint specimens, analytical shear force versus horizontal deflection plots can be obtained using LSODE. The entire process has been summarized in the form of a flowchart in Fig. 4. Comparison between the experimental and analytical shear force-horizontal deflection

plots of lightly reinforced concrete interior and exterior beam-column joint specimens are presented in Figs. 5 and 6 respectively. In order to maintain a level of accuracy for all the specimens, the analytical parameters have been estimated such that the correlation coefficient of the comparison plots remains 0.98 for all of them.

6. Model sensitivity to parameter variations

In pursuance of judging the sensitivity of the hysteresis model to the variation of associated analytical parameters, a numerical example has been deduced from previous section. Calibration of the analytical model with the experimental result of Unit O1 by Hakuto et al. [1] at a level of their correlation coefficient as 0.98, yields the following parameter magnitudes.

$$\begin{aligned} \alpha_0 &= 0.025, & \omega_0 &= 2.5, & \xi_0 &= 0.02, & \beta &= 0.05, & \gamma &= -0.01, \\ n &= 1.01, & \delta_v &= 0.00005, & \delta_\eta &= 0.0005, & \zeta_s &= 0.93, \\ p &= 0.08, & \psi &= 0.8, & \delta_\psi &= 0.11, & q &= 0.03, & \lambda &= 0.1 \end{aligned}$$

Sensitivity of the hysteresis loop has been investigated by changing the magnitude of each analytical parameter individually one after another, while other parameters are kept constant in magnitude. Fig. 7a-j displays the influences of variations of the analytical parameters α_0 , ω_0 , ξ_0 , β , n , δ_v , δ_η , ζ_s , q , p , ψ , δ_ψ and λ on the hysteresis loop correspondingly. The analytical parameters, stiffness ratio α_0 and system natural frequency ω_0 with their changing magnitudes, affect the stiffness of the hysteresis loop, as depicted in Fig. 7a and b. Altering the magnitude of the linear damping ratio ξ_0 , while keeping all other parameters constant, does not produce any significant changes in the hysteresis loop, as shown in Fig. 7c. Fig. 7d-f illustrates how the magnitudes of hysteresis shape parameters β , γ and n can individually influence the shapes of the hysteresis loops. From Fig. 7g and h, it can be understood that with increase or decrease of degradation parameters δ_v and δ_η , structure experiences more or less degradations respectively. The remaining parameters, being pinching parameters only control the pinching of the hysteresis loop. ζ_s controls the amount of total slip in the hysteresis loop as observed in Fig. 7i. When this parameter is of zero magnitude, no slip can be observed in the hysteresis loop whereas with an increase in its magnitude, greater slip is found in the loop. As q is a constant that sets the pinching level, the influence of its varying magnitude can significantly deviate pinching as shown in Fig. 7j. Variation of p contributes to the rate of drop in the slope, as illustrated in Fig. 7k. Moreover, with an increase or decrease in ψ , the amount of pinching behaves proportionately as depicted in Fig. 7l and due to increase in the magnitude of δ_ψ , the pinching region spreads as shown in Fig. 7m. A change in λ also affects the amount and spread of pinching in the hysteresis loop as demonstrated in Fig. 7n. In brief, system properties (α_0 , ω_0 , ξ_0) and hysteresis shape parameters (β , γ , n) control the skeleton of hysteresis loops; degradation parameters (δ_v , δ_η) determine strength and stiffness deteriorations and pinching parameters (ζ_s , q , p , ψ , δ_ψ , λ) govern the slip and pinching magnitude and pinching spread. However, after varying each parameter up to a definite range and computing the error occurred due to each variation, sensitive ranking of each parameter can be easily deduced.

Let $[Y]$ be the hysteretic force for a given input function and the magnitudes of the analytical parameters are estimated as:

$$\begin{aligned}\alpha_0 &= 0.025, & \omega_0 &= 2.5, & \xi_0 &= 0.02, & \beta &= 0.05, & \gamma &= -0.01, \\ n &= 1.1, & \delta_v &= 0.00005, & \delta_\eta &= 0.0005, & \zeta_s &= 0.9, & q &= 0.03, \\ p &= 0.08, & \psi &= 0.8, & \delta_\psi &= 0.11, & \lambda &= 0.1.\end{aligned}$$

Then, each parameter, excluding ω_0 and ξ_0 , is varied from -10% to +10% of its original magnitude. In this sensitivity ranking determination, the system natural frequency ω_0 and the linear viscous damping ratio ξ_0 have been excluded due to the fact that ω_0 of any structure is invariable and for a given ω_0 , variation of ξ_0 does not affect the hysteresis loop. Therefore, an attempt has been made to relate ω_0 with the physical parameter of the beam-column joint and fix a range for ξ_0 in the next section. Now, if due to the variation of a parameter, say α_0 , the hysteretic force becomes $[Y']$, then the root mean square error e_{α_0} will be as follows:

$$e_{\alpha_0} = \left\langle \sum_{i=1}^N (Y - Y')^2 \right\rangle^{1/2} \quad (19)$$

Here N is the number of data points for input displacement function. The maximum error related to the variation of α_0 , termed as $|e_{\alpha_0}|$ can be obtained by the following expression.

$$|e_{\alpha_0}| = \text{maximum}(e_{\alpha_0}) \quad (20)$$

The maximum root mean square error associated with each parameter variation is summarized in Table 1. The parameter with the highest magnitude of maximum root mean square error is ranked as 1 based on its sensitivity. By plotting the root mean square error for any parameter within the range of its variation, a Spider diagram is obtained as shown in Fig. 8.

Parameter sensitivity analysis is vital when dealing with the system identification techniques. A sensitive parameter when deviated from its sought-after magnitude will show reasonable error. Thus, by changing the magnitude of sensitive parameters, better correlation can be achieved. On the contrary, a less sensitive parameter, even when it is fluctuated from its sought-after magnitude, can produce a reasonable response as its contribution to the final response is relatively less. Therefore, providing narrower ranges for less sensitive parameters can increase simplicity in the procedure without affecting the quality of the results.

7. Parametric study

An extensive parametric study has been conducted in this research in order to relate the physical parameters of the beam-column joints with the hysteresis model parameters. Reinforced concrete interior and exterior beam-column joints with limited transverse reinforcement have been modeled in UC-win/COMD [30] based on the principal

features depicted in Table 2. The analytical parameters estimated for the interior and exterior beam-column joints are summarized in Table 3. Next, the structural performance of reinforced concrete non-ductile beam-column joints has been investigated by varying the key factors based on Table 4. Thus, the structural responses of the beam-column joint models are obtained from simulation and the analytical hysteresis parameters are estimated using Genetic Algorithm accordingly. From the estimation, with the change of each joint physical parameter, the affected mathematical parameters can be recognized and subsequently, the magnitudes of concerned analytical parameters are plotted against that joint physical parameter to elucidate their inter-relation.

7.1. Effect of joint aspect ratio

The joint aspect ratio is defined as the ratio of beam depth and column cross-sectional depth. From the simulation results, it has been observed that the joint aspect ratio plays a pivotal role in determining the joint shear strength. In this parametric study, the joint aspect ratio is changed in two ways, first, by keeping the beam depth constant and varying the column cross-sectional depth and secondly, by keeping the column cross-sectional depth constant and varying the beam depth. Joint shear strength decreases with decrease in the column cross-sectional depth whereas it increases with decrease in the beam depth. As a consequence, the system parameters (α_0 , ω_0) associated with the hysteresis model also suffer perturbation. Figs. 9 and 10 show changes in these two parameter magnitudes due to changes in the joint aspect ratio by varying the column cross-sectional depth and the beam depth respectively.

7.2. Effect of concrete and steel grades

In non-seismic designed reinforced concrete buildings, high strength concrete is generally not used for construction. As per the old practice, concrete with compressive cylinder strengths f'_c of 20 MPa, 30 MPa and 40 MPa has been considered for this parametric study. With decrease in the concrete compressive strength, joint shear strength deteriorates along with reduction in the hysteresis model parameters α_0 and ω_0 as presented in Fig. 11. The influence of usage of deformed and plain round bars as longitudinal reinforcement has been investigated when the same specimen has been modeled once with deformed bars having yield strength of 350 MPa and thereafter with plain bars of yield strength 250 MPa as longitudinal reinforcement. The model with plain bar produces reduced shear strength, but higher slip and profound pinching due to severe slippage of longitudinal reinforcement bars. Pinching parameters (ζ_s , q , ψ) and the system parameters (α_0 , ω_0) also experience significant changes in their magnitudes accordingly as depicted in Fig. 12.

7.3. Effect of column axial load

The column axial load plays a significant role in the hysteresis behavior of beam-column joints. From the simulation results, it has been observed that the effect of the column axial load is more prominent in the exterior beam-column joints than the interior beam-column connections. Due to the presence of the column axial load, the joint strength is enhanced, but with increase in the column axial load ratio, more degradation

and pinching are observed. In this parametric study the column axial load ratio has been varied from 0 to 0.1, 0.15, 0.2, 0.25 and 0.3. The influences of different levels of the column axial load ratio on the system properties (α_0 , ω_0), degradation parameters (δ_v , δ_η) and pinching parameters (ζ_s , q , ψ) have been portrayed in Fig. 13.

7.4. Effect of column and beam longitudinal reinforcement

In order to investigate the effect of column longitudinal reinforcement ratio, reinforced concrete beam-column joint models have been built in UC-win/COMD with variation of the column longitudinal reinforcements, while keeping remaining key features of the model unchanged. The simulation results indicate that change in the column longitudinal reinforcement does not exhibit any influence on the joint shear strength and as a result, the hysteresis model parameters will also remain unchanged as shown in Fig. 14. Similarly, to check the possible influence of the beam longitudinal reinforcement ratio, reinforced concrete beam-column joints have been modeled by varying the beam longitudinal reinforcement only. The simulation results show that with increase in the beam longitudinal reinforcement ratio from 1.0 to 2.0, joint shear strength increases around 30%. With alteration of the beam longitudinal reinforcement ratio, the hysteresis model parameters α_0 and ω_0 varies as presented in Fig. 15.

The entire parametric study has been undertaken to realize the hysteresis behavior of a wide range of beam-column joints and the relationship between the joint physical parameters and the mathematical parameters associated with the analytical model. The only aim underlying this is to provide a comprehensive range of the analytical parameters to enable the model to be user-friendly. Though it is not feasible to include parameter magnitudes for all the cases involved in the parametric study, the generalized upper and lower bounds for each of the parameters are tabulated in Table 5. From this table and the figures denoting the influence of the physical parameters on the analytical parameters, the user can gain a quick impression of the magnitudes of the model parameters for a wide variety of beam-column joints. As soon as the approximate parameter sets are identified for a definite reinforced concrete non-ductile beam-column joint, its hysteresis response under a specific displacement history can be attained using any suitable solver. Two examples have been included in the Appendix on the selection process of the analytical parameters for the interior beam-column joint Unit C2 and the exterior beam-column joint Unit L1 by Pampanin et al. [31] based on their physical parameters according to Figs. 9-15. The experimental versus analytical load-deflection plots for Unit C2 and Unit L1 have been shown in Fig. 16.

8. Conclusions

This paper presents an analytical approach to predict the hysteresis behavior of non-seismic detailed reinforced concrete beam column joints based on modified Bouc-Wen hysteresis model. LSODE and Genetic Algorithm (GA) have been opted for solving the analytical model equations and estimation of the parameters associated with the equations respectively. The efficiency of the analytical model and the accuracy of the solver and the algorithm have been proved by the strong correlations between the experimental and the analytical shear force-horizontal deflection plots for lightly reinforced concrete interior

and exterior beam-column joint specimens from the literature. The sensitivity of the proposed model to the variation of its analytical parameters has been investigated and the sensitivity ranking of the parameters have been finalized. Less sensitive parameters can be kept constant in order to keep the model simple without causing much error to the response. The influence of the joint physical parameters, such as the joint aspect ratio, concrete compressive cylinder strength, plain or deformed bars for longitudinal reinforcement, the column and beam longitudinal reinforcement ratio, the column axial load ratio, on the model parameters has been examined meticulously based on the extensive parametric study. Moreover, the approximate upper and lower bounds for the model parameters have been specified, from which the user can identify the approximate magnitudes of the parameters for any beam-column joint depending on its physical parameters. In this simplified approach, the analytical parameters can be estimated instantly without using any system identification tool and the hysteresis behavior of the beam-column joint can be accomplished using the estimated parameters with the help of any efficient solver.

Appendix A

An illustration on determination of parameter magnitudes for an interior beam-column joint specimen Unit C2 and an exterior beam-column joint specimen Unit L1 [29] is shown hereunder. From the sample parameter set, analytical parameters for both joints will be calculated based on the parametric study conducted in this research.

(a) The physical characteristics of Unit C2 by Pampanin et al. [29] are as follows:

- Joint aspect ratio = 1.65 (beam depth 330 mm and column cross-sectional depth 200 mm).
- Concrete compressive cylinder strength of the specimen $f'_c = 23.9$ MPa.
- Average yield strength of steel for longitudinal reinforcement bars = 365.75 MPa (plain bar).
- Column longitudinal reinforcement ratio

$$= [(6 \times 50.3)/(330 \times 200)] \times 100\% = 0.46\%$$

Beam longitudinal reinforcement ratio

$$= [(4 \times 50.3 + 3 \times 113)/(330 \times 200)] \times 100\% = 0.82\% .$$

- Column axial load ratio = 0.08.

The parameter magnitudes for the interior beam-column joint with physical characteristics as mentioned in Table 2 are as follows:

$$\alpha_0 = 0.03, \quad \omega_0 = 2.85, \quad \xi_0 = 0.02, \quad \beta = 0.05, \quad \gamma = -0.01,$$

$$n = 1.01, \quad \delta_v = 0.00005, \quad \delta_\eta = 0.0005, \quad \zeta_s = 0.91, \quad q = 0.03,$$

$$p = 0.08, \quad \psi = 0.8, \quad \delta_\psi = 0.11, \quad \lambda = 0.1$$

Step 1: Effect of joint aspect ratio

In the present problem, joint aspect ratio is 1.65 with beam depth 330 mm and column cross-sectional depth 200 mm. Hence from Figs. 9 and 10 by linear interpolation, it can be calculated that $\alpha_0 = 0.02$, $\omega_0 = 2.35$ when remaining physical parameters of the interior joint remain unaltered.

Step 2: Effect of grade of concrete and steel

For concrete compressive cylinder strength 23.9 MPa and plain reinforcement bar, based on Figs. 11 and 12 respectively, modified magnitudes of the parameters are:

$$\alpha_0 = 0.01, \omega_0 = 1.60, \zeta_s = 0.95, q = 0.05, \psi = 0.905$$

Step 3: Effect of longitudinal reinforcement ratio

Column longitudinal reinforcement ratio does not influence the analytical parameters. But due to shift of beam longitudinal reinforcement ratio from 2% to 0.82%, magnitudes of the parameters α_0 and ω_0 will experience little change according to Fig. 15 as $\alpha_0 = 0.008$, $\omega_0 = 1.17$.

Step 4: Effect of column axial load ratio

According to Fig. 13, due to presence of column axial load $0.08f'_c A_g$, the final parameter magnitudes of Unit C2 are as follows:

$$\alpha_0 = 0.009, \quad \omega_0 = 1.22, \quad \xi = 0.02, \quad \beta = 0.05, \quad \gamma = -0.01,$$

$$n = 1.01, \quad \delta_v = 0.000052, \quad \delta_\eta = 0.00062, \quad \zeta_s = 0.966,$$

$$q = 0.058, \quad p = 0.08, \quad \psi = 0.934, \quad \delta_\psi = 0.11, \quad \lambda = 0.1$$

(b) The physical characteristics of Unit L1 by Pampanin et al. [29] are as follows:

- Joint aspect ratio = 1.65 (beam depth 330 mm and column cross-sectional depth 200 mm).
- Concrete compressive cylinder strength of the specimen $f'_c = 23.9$ MPa.
- Average yield strength of steel for longitudinal reinforcement bars = 365.75 MPa (plain bar).
- Column longitudinal reinforcement ratio

$$= [(6 \times 50.3)/(330 \times 200)] \times 100\% = 0.46\%$$

Beam longitudinal reinforcement ratio

$$= [(4 \times 50.3 + 4 \times 113)/(330 \times 200)] \times 100\% = 0.99\%$$

- Column axial load ratio = 0.

The parameter magnitudes for an exterior beam-column joint with physical characteristics as mentioned in Table 2 are as follows:

$$\begin{aligned}\alpha_0 &= 0.009, \quad \omega_0 = 1.22, \quad \xi = 0.02, \quad \beta = 0.05, \quad \gamma = -0.01, \\ n &= 1.01, \quad \delta_v = 0.000052, \quad \delta_\eta = 0.00062, \quad \zeta_s = 0.966, \\ q &= 0.058, \quad p = 0.08, \quad \psi = 0.934, \quad \delta_\psi = 0.11, \quad \lambda = 0.1\end{aligned}$$

Step 1: Effect of joint aspect ratio

Here, joint aspect ratio is 1.65 with beam depth 330 mm and column cross-sectional depth 200 mm. Hence from Figs. 9 and 10 by linear interpolation, it can be calculated that $\alpha_0 = 0.01$, $\omega_0 = 1.49$, when remaining physical parameters of the exterior joint remain unchanged.

Step 2: Effect of grade of concrete and steel

For concrete compressive cylinder strength 23.9 MPa and plain reinforcement bar, based on Figs. 11 and 12 respectively, the modified parameter magnitudes are:

$$\alpha_0 = 0.006, \omega_0 = 1.01, \zeta_s = 0.93, q = 0.05, \psi = 0.9$$

Step 3: Effect of longitudinal reinforcement ratio

Due to change in beam longitudinal reinforcement ratio from 2% to 0.99% according to Fig. 15, magnitudes of α_0 and ω_0 will be changed into $\alpha_0 = 0.005$, $\omega_0 = 0.17$.

Step 4: Effect of column axial load ratio

According to Fig. 13, due to presence of column axial load $0.08f'_cA_g$, the final parameter magnitudes of Unit L1 are as follows:

$$\begin{aligned}\alpha_0 &= 0.005, \omega_0 = 0.77, \xi_0 = 0.02, \beta = 0.05, \gamma = -0.01, n = 1.01, \\ \delta_v &= 0.00006, \delta_\eta = 0.0008, \zeta_s = 0.93, q = 0.05, p = 0.05, \psi = 0.9, \\ \delta_\psi &= 0.15, \lambda = 0.1\end{aligned}$$

The experimental and analytical hysteresis loops for Unit C2 and Unit L1, are shown in Fig. 16 which depicts the accuracy of the estimated parameters.

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Parameter	Maximum root mean square error	Rank
α_0	1.15	7
β	5.08	3
γ	0.95	8
n	10.1	2
ζ_s	52.15	1
q	0.65	11
p	0.27	12
ψ	3.09	4
δ_ψ	0.81	9
λ	0.78	10
δ_v	1.34	6
δ_η	2.90	5

Table 1

Type of Joint	Interior and exterior joints
Joint aspect ratio	1.67 (column cross sectional depth: 300 mm and beam depth: 500 mm)
Total height of the beam-column joint	2900 mm
Total span of the beam-column joint	3500 mm for interior joint and 1900 mm for exterior joint
Concrete grade	$f'_c = 40$ MPa
Steel grade	$f_y = 350$ MPa and deformed bars
Column longitudinal reinforcement ratio	2%
Beam longitudinal reinforcement ratio	2%
Displacement history	1–5% drift ratio
Axial load ratio	0

Table 2

Parameter	Interior joint	Exterior joint
α_0	0.03	0.01
ω_0	2.85	1.85
ξ_0	0.02	0.02
β	0.05	0.05
γ	-0.01	-0.01
n	1.01	1.01
ζ_s	0.91	0.91
$\mathbf{q}$	0.03	0.03
p	0.08	0.05
ψ	0.8	0.8
δ_ψ	0.11	0.15
λ	0.1	0.1
δ_v	0.00005	0.00006
δ_η	0.0005	0.0008

Table 3

Factors Type of joint	No.	Description Interior and exterior joints
Joint aspect ratio	10	(a) Varying column depth: 1, 1.11, 1.25, 1.43, 1.67 (b) Varying beam depth: 1, 1.17, 1.33, 1.5, 1.67
Concrete grade	3	$f'_c = 20, 30$ and 40 MPa
Steel grade	2	$f_y = 250$ MPa for plain bars, $f_y = 350$ MPa for deformed bars
Column longitudinal reinforcement ratio	5	1.0%, 1.5%, 2%, 2.5%, 3%
Beam longitudinal reinforcement ratio	5	1.0%, 1.5%, 2%, 2.5%, 3%
Axial load ratio	6	0, 0.1, 0.15, 0.2, 0.25, 0.3

Table 4

Parameter	Lower bound	Upper Bound
α_0	0.005	0.07
ω_0	0.5	6.0
ξ_0	0.01	0.03
β	0.04	0.06
γ	-0.02	-0.01
n	1.0	1.1
ζ_s	0.85	0.98
q	0.03	0.08
p	0.04	0.09
ψ	0.75	0.95
δ_ψ	0.1	0.2
λ	0.1	0.12
δ_v	0.00005	0.00008
δ_η	0.0005	0.002

Table 5

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

Fig. 16