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2012

Xu, H., Zhang, B., Yu, T., Barbastathis, G., & Sun, H. (2012). Dielectric waveguide bending adapter with ideal transmission: practical design strategy of area-preserving affine transformation optics. *Journal of the Optical Society of America B*, 29(6), 1287-1290.

<https://hdl.handle.net/10356/95134>

<https://doi.org/10.1364/JOSAB.29.001287>

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Dielectric waveguide bending adapter with ideal transmission: practical design strategy of area-preserving affine transformation optics

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Received December 22, 2011; revised February 28, 2012; accepted March 4, 2012;
posted March 6, 2012 (Doc. ID 160332); published May 17, 2012

Transformation optics has inspired numerous conceptual devices since the first theoretical model of invisibility cloaks. However, the transition of these concepts into industrial applications is now widely acknowledged as a challenging bottleneck. A practical design strategy that can be adopted by industrial engineers is thus desirable at this stage. Here we propose an integrated design strategy imposing practical constraints on the area-preserving affine coordinate transformation as a general practical method to solve the problem of nonmagnetism. Practical concerns related to fabrication, such as anisotropy degree and bending angles, serve as additional constraints to the transformation. As a specific example, we illustrate how to apply this practical strategy to the design of a two-dimensional electromagnetic waveguide bending adapter. Our study is a significant step toward practical use of ideal transformation optics devices that can be implemented with natural dielectric materials. © 2012 Optical Society of America

OCIS codes: 160.3918, 160.1190, 230.7390.

1. INTRODUCTION

Transformation optics, first proposed in the context of invisibility cloaking [1,2], is also applicable to a broad variety of electromagnetic wave converters [3–5]. The original conception of implementing transformation optics necessitates the utilization of media that are both inhomogeneous and anisotropic. To push the implementation into the optical regime, an inhomogeneity-only quasiconformal mapping technique was proposed [6], but the manufacturing cost remains high and undesired distortion inevitably appears in the output rays [7]. To address this challenge, homogeneous transformation methods [8–12] were recently discussed and experimentally demonstrated [13–15]. Their utility became immediately clear after the first two attempts of invisibility cloaking [13,14] were realized at the macroscopic scale and visible wavelengths.

Despite the success of “proof-of-concept” invisibility cloaks, experimental implementations of electromagnetic wave converters with practical utility (e.g., a bending waveguide adaptor) are much fewer to date. While there have been many conceptual designs of bending waveguide adaptors, how to implement them for practical use (or even just for the purpose of experimental verification) is generally not known so far. Therefore, a practical design strategy that can help to get across the final main barrier in transformation optics from scientific concepts to engineering applications will be of great value at this stage.

To achieve this goal, we need to solve two problems with the premise that the ideal transformation photonic functions

cannot be sacrificed: first, how to realize nonmagnetism (i.e., $\mu = 1$), and second, how to adopt dielectric materials (i.e., limit the design values of ϵ to those of nonconducting materials that are available to industry). Two approaches have been proposed to tackle these problems. The first approach, which is commonly used in the literature to date, is to renormalize both the permittivity and the permeability such that the permeability is forced to be unity [16–18]. However, renormalization introduces impedance mismatch at interfaces between regions with different transformation kernels. That is, the basic transformation optics premise of invariance in Maxwell’s equations is violated, resulting in reflection and scattering. The second possible approach, which, on the other hand, has received much less attention [16], is to maintain the transformation Jacobian at the value of 1 throughout space. This was first suggested in the context of curved transformation design [19] and then utilized implicitly in a special case in [8,20] with designed material parameters subject to the intrinsic limitation of loss [8]. Although this approach has ingeniously solved the first problem of nonmagnetism, the second problem of how to adopt dielectric materials with available permittivity still remains vague. As a result, the final barrier toward experimental implementations still has not been overcome, nor has any experimental implementation been reported in the literature to date, to our knowledge.

Here, we introduce an integrated transformation optical design strategy based on both conditions of nonmagnetism and dielectric materials simultaneously. This new approach can

potentially be used as an industrial guideline for practical designs of transformation optical devices. Our approach admits both macroscale and nanoscale fabrication. In the macroscopic regime, our design can be achieved using natural birefringence materials such as calcite [13,14]. Nanoscale realizations are available via subwavelength anisotropic patterning (i.e., form birefringence [21,22]). The homogeneous implementation is much less challenging than the inhomogeneous case, due to the proximity effect correction (PEC) in electron beam lithography [23]. Moreover, the geometrical approach simplifies the problem to one of graphical design that in many cases can be carried out analytically.

2. METHOD

Consider a general two-dimensional (2D) affine coordinate transformation $x' = ax + by + c$ and $y' = dx + ey + f$ from triangle AOB to $AO'B'$ in the x - y plane, as shown in Fig. 1(a). The associated Jacobian matrix is $\bar{J} = [a, b; d, e]$. For transverse magnetic (TM) modes where the H field is perpendicular to the x - y plane, we obtain the transformed relative dielectric permittivity and permeability as $\bar{\epsilon}' = \bar{J}\bar{J}^T / \det(\bar{J})$ and $\mu' = 1 / \det(\bar{J})$, respectively.

We can achieve unitary permeability μ' by imposing a unitary Jacobian $\det(\bar{J}) = 1$. The geometric interpretation is that the transformation is area-preserving, such that the area of the triangle AOB is always equal to that of $AO'B'$.

Next we consider permittivity. Analytically, the permittivity tensor $\bar{\epsilon}$ can be solved given the position of AOB and $AO'B'$. The solution can be simplified under the assumption that the transformation takes place only along the x axis [i.e., $y' = y$, as shown in Fig. 1(b)]. In this case the heights of $AO'B'$ and AOB are equal. To guarantee area preservation, we set the bottom length of the two triangles equal, i.e., $|A'B'| = |AB|$. The transformation function now becomes a horizontal shear: $x' = x + by$, $y' = y$, where $b = d_o/h$; d_o and h are labeled in Fig. 1(b). The relative permittivity and permeability become

$$\bar{\epsilon}' = \begin{bmatrix} b^2 + 1 & b \\ b & 1 \end{bmatrix}, \quad \mu' = 1. \quad (1)$$

We define the anisotropy ratio as $R \equiv n_2/n_1 = \sqrt{\epsilon_2/\epsilon_1}$, where n_1 and n_2 ($n_1 > n_2$) are the two principal refractive indices along two orthogonal directions, and ϵ_1 and ϵ_2 are the corresponding eigenvalues of the permittivity tensor $\bar{\epsilon}'$. R can be obtained analytically as

$$R = \left[\frac{(b^2 + 2)/2 - \sqrt{(b^2 + 2)^2/4 - 1}}{(b^2 + 2)/2 + \sqrt{(b^2 + 2)^2/4 - 1}} \right]^{1/2}. \quad (2)$$

Fig. 1. (Color online) (a) Area-preserving transformation from triangle AOB (blue) to $AO'B'$ (green) along both x and y directions, (b) special case of horizontal shear along the x direction only.

Although several reports have discussed the practical concerns of achievable values for either permittivity or permeability [24–26], there has been much less discussion, which is generally focused on cloaking applications (e.g., [15]), on the practically achievable values of R .

As a case study of a 2D TM beam bending adapter, we will illustrate the comparison between a boundary-preserving transform (BPT) method [Fig. 2(a)] and the area-preserving transformation (APT) method [Fig. 2(b)]. As shown in both figures, the rectangular region $AOBC$ is a part of a rectangular planar waveguide. To form one arm of the bending adapter, we transform ΔAOB to the new triangle $\Delta AO'B$, where the angle α is the half-bending angle of the adapter. The final bending angle is 2α , formed by mirroring the structure with regard to axis OB' , as shown in the inset figure. Without loss of generality, we set length $|OB| = 1$ and fix the point A on the horizontal axis at location $(-L, 0)$. Besides these common parts, the difference between the two figures is the position of O' .

We first discuss the BPT method in Fig. 2(a), where the location of O' is determined from the boundary-preserving requirement, i.e., that the geometrical conditions $O'A//BC$ and $\angle OAB$ may not be violated; thus, O' should remain on the horizontal axis. Figure 2(c), corresponding to BPT, shows that the permeability indeed varies dramatically with the bending angle 2α (blue curves) and cannot equal one except in the trivial case $O = O'$. The variance is in accordance with the area change ratio (red curves), defined as $(S_a - S_b)/S_a$, where S_a and S_b denote the areas after and before transformation, respectively. The area change is directly related to the amount of nonunit permeability μ' . Interestingly, if the permeability is renormalized to unity, the resulting anisotropic medium is equivalent to an isotropic medium with impedance η_0/μ' , where η_0 is the impedance of the original medium before transformation. Therefore μ' also measures the impedance mismatch in the renormalization.

In contrast, the APT design is shown in Fig. 2(b). Here we require that the area of triangle AOB equals that of $AO'B$. Thus, O' should be placed such that $OO'//AB$, i.e., $x_o + Ly_o' = 0$. The Jacobian is thus unitary, leading to $\mu' = 1$. It is a special case ($\angle AOB = 90^\circ$) of the horizontal shear APT [Fig. 1(b)].

Next we proceed to consider the practical limit of achievable anisotropy in APT. R depends on the bending angle 2α ($0 < 2\alpha < 180^\circ$) given a certain arm length L according to Eq. (2), as shown in Figs. 3(a) ($L < |OB|$) and 3(b) ($L > |OB|$). For fixed arm length $L = |OA|$, increasing 2α results in larger shear, and thus smaller R . On the other hand, for fixed 2α , varying L also results in increased R when

Fig. 2. (Color online) (a) BPT (from AOB to $AO'B$) for one arm of the bending adapter, (b) APT for one arm of the bending adapter, (c) in the BPT case, the permeability (blue) and the area changing ratio (red) as functions of bending angle 2α and the arm length L .

Fig. 3. (Color online) (a) Anisotropy ratio $R = n_2/n_1$ as a function of bending angle 2α and arm length L ($L < |OB|$). The dashed line corresponds to the value of $n_{\perp}/n_{\parallel}$ for the silicon-air material system. (b) Same as (a), except $|OB| < L < 10|OB|$. (c) Maximum bending angle $2\alpha_{SA}$ as a function of L for the silicon-air material system. (d) Maximum bending angle $2\alpha_{max}$ as a function of anisotropy ratio R .

$L < |OB|$, but decreased R when $L > |OB|$ for certain values of 2α . The amount of allowable anisotropy can thus be determined from these plots, taking into the account the available anisotropy of our chosen optical material or nanofabrication method. For the consideration of industrial fabrication based on dielectric materials, we focus on materials with permittivity $\epsilon > 1$. Thus free space ($\epsilon = 1$) applications are out of the scope of this paper.

3. RESULT

As an example, we analyze the largest bending angle that can be achieved using a silicon-air layered structure with sub-wavelength period $\lambda/10$. The refractive indices of silicon and air are $n_{Si} = 3.48$ and $n_{Air} = 1$, respectively, at $\lambda = 1550$ nm. From the effective medium theory, the parallel and perpendicular effective permittivities are $\epsilon_{\parallel} = r\epsilon_{Si} + (1-r)\epsilon_{Air}$ and $\epsilon_{\perp} = \epsilon_{Si}\epsilon_{Air}/[r\epsilon_{Air} + (1-r)\epsilon_{Si}]$, respectively. The filling factor r is set as 0.5 to achieve maximum anisotropy, i.e., the smallest ratio of $n_{\perp}/n_{\parallel} = \sqrt{\epsilon_{\perp}/\epsilon_{\parallel}} = 0.53$, marked as the dashed line crossing Figs. 3(a) and 3(b). If the required value of R (the anisotropy ratio) is larger than $n_{\perp}/n_{\parallel}$, it can be achieved by adjusting the filling factor r . Otherwise, the silicon-air structure would not possess sufficient anisotropy to realize this bending angle. Setting $n_1/n_2 = n_{\perp}/n_{\parallel}$, we obtain the largest achievable bending angle $2\alpha_{SA_MAX} = 72^\circ$ [Fig. 3(c)]. In

Fig. 4. (Color online) (a) Magnetic field distribution of the BPT case with scaled μ and ϵ , (b) the magnetic field distribution of the APT case. The inset figure demonstrates the silicon-air layered structure.

Figs. 3(a) and 3(b), this equivalence is shown as crossing points between the line $n_{\perp}/n_{\parallel}$ and the curves of n_2/n_1 with certain L value. Maximum bending is obtained when $L = 2.53|OB|$, as shown by the rightmost crossing point in Fig. 3(b). Besides the example of the silicon-air layered structure, the maximum bending angles $2\alpha_{max}$ that can be realized by various materials such as calcite and calomel are calculated and marked on the curve of $2\alpha_{max}$, varying as a function of the anisotropy ratio R in Fig. 3(d).

To confirm the effectiveness of the wave-bending adapter, we performed numerical simulations using the commercial FEM solver, COMSOL Multiphysics. As an example, we chose a 60° adapter (i.e., $\alpha = 30^\circ$). The lengths of OB and OA were set as 1.2 and 1.5 μm , respectively. Flint glass with refractive index $n_d = 1.87$ was chosen as the waveguide material. A TM planar wave at wavelength 1550 nm is incident from the left waveguide. Figure 4(a) shows the distribution of the magnetic field H in the BPT adapter, with μ' normalized to unity and $\epsilon_1(\epsilon_2)$ scaled for nonmagnetism. Figure 4(b) shows H in the APT adapter. The silicon-air interfaces have certain angles (-2.64° for the left arm and -57.36° for the right arm) with respect to the x axis, creating the desired permittivity tensor. The transmission in the BPT and APT cases was found to be 92.37% and 100%, respectively. In the BPT adapter it will decrease dramatically if the bending angle is increased further. The energy transmission loss of 60° BPT is similar to that through a glass slab. However, the beam profile has been seriously distorted, which will affect the signal delivery in potential optoelectronic application. In contrast, the beam profile is well preserved in the APT adapter. It should be noted that similar nonmagnetic designs in [8,17] can be treated as a special case in our analysis, but they are not practically implementable solutions without going through the above design procedure as a general practical standard.

4. CONCLUSION

In summary, we have proposed an integrated design strategy by imposing practical constraints on the area-preserving affine coordinate transformation as a general practical strategy for designing functional transformation optical devices. We also illustrated a practical design of the 2D TM bending adapter that is implementable with existing dielectric materials. The beam profile is well preserved with almost ideal transmission in a nonmagnetic realization. Our detailed design strategy integrating practical limitations will provide guidance for future large-scale industrial fabrication in both macroscopic and nanoscale regimes.

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