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Shot noise of low energy electron field emission due to Klein tunneling

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This paper investigates the property of shot noise for low energy electron field emission from a single-layer vertically aligned graphene sheet assuming the emission process is due to Klein tunneling. In our model, we use two different methods (relativistic WKB and transfer matrix) to calculate the transmission coefficient and thus obtain the Fano factor (γ or suppression of shot noise) as a function of temperature T, Fermi energy E_f, and local electric field F. It is found that a universal maximum value of about $\gamma = 1/3$ can be reached at low temperature limit within a certain range of local electric field. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4733349]

Graphene, first isolated in 2004,[1] has attracted extraordinary interests due to the unusual electronic properties and applications.[2] Recently, many experiments have confirmed that graphene or graphene composite is promising to be an efficient electron emission source, because of low turn on voltage,[3] high current density,[3,4] good stability,[4,5] long lifetime,[6] and compactness in a nanogap.[7] Compared to the conventional bulk semiconductors, despite the reduction on the dimensionality, the low energy charge carriers near the corner of the hexagonal Brillouin zone are described as massless, chiral Dirac fermions, which exhibit angle dependent Klein tunneling behavior.[8,9] Our recent model[10] has suggested that once the Klein tunneling effect is taken into account, the field emission characteristic of graphene is different from the traditional Fowler-Nordheim (FN) theory.[11] Under the same geometrical field enhancement factor, our model predicts a much higher emitted current as compared to the FN law at low voltages.

Shot noise is the fluctuation in the electrical signal due to the discreteness of electron charges, which is useful to obtain information not available through the standard conductance measurement. The general expression between the spectral power density S and the mean value of current I is $S = 2\gamma qI$, where q is the unit charge. Here, γ is the Fano factor, which represents the deviation from the uncorrelated (or full) shot noise $2qI$.[12] Due to the correlation among the electrons such as the Coulomb correlation, and the effects of quantum partitioning due to the Pauli exclusion principle,[13] suppression of shot noise (with $\gamma < 1$) can be realized.

For graphene, Tworzydlo[14] first predicts that the Fano factor of a single barrier structure has a maximum value of $\gamma = 1/3$ at the Dirac point for a short and wide graphene strip, which is coincidently the same as that of diffusive metals.[15] Recent shot noise measurements in single- and multi-layer graphene samples[16,17] have shown good agreement with this theoretical predication, and the model has been extended to different systems, such as double[18] and multi-barrier structures.[19] It is generally accepted that the sub-Poisson shot noise of graphene is due to the unique band structure at the low energy states near the Dirac point. In other words, the charge carriers are governed by the Dirac equation instead of the Schroedinger equation.

Compared to the charge transport inside the material, it was first proposed that similar quantum partitioning effect that induces shot noise suppression may also be found in the field emission process.[12] During the electron tunneling process from the surface of the electrode, there is a quantum partitioning effect due to which the shot noise is no longer uncorrelated, and its corresponding quantum shot noise suppression (or Fano factor) has been calculated by using a simplified one-dimensional (1D) model with an arbitrarily fixed field enhancement factor. Subsequent models have been developed to include more realistic potential profiles,[20] space charge effects,[21] and 2D model including the consistent field enhancement calculation.[22]

Motivated by these studies, in this paper, we will investigate the field emission shot noise behavior of a vertically aligned single layer graphene sheet. We show later that the resultant Fano factor is closely related to temperature T, Fermi energy E_f, and local electrical field F. Furthermore, the universal maximum value of the Fano factor ($\gamma = 1/3$) is recovered at low temperature limit. Note that vertically well-aligned graphene single layer considered here has been recently fabricated in arrays with height H from 50 to 800 nm.[23]

Our graphene field emission model is exactly same to our previous works illustrated in the Fig. 1(a) in the paper.[10] In this model, a single-layer graphene with height H is assumed to be vertically aligned (along x-axis) inside a dc gap of spacing $D \gg H$ with an applied voltage of V_g, and it is infinitely long along y-axis to neglect the boundary condition (zig-zag or armchair) on the graphene edge. To calculate the Fano factor γ, the main difficulty is that unlike metals or semiconductors, graphene is a 2D superlattice with linear energy-momentum dispersion at the low energy level. Thus, the conventional formulas[12,21,22] used to calculate the mean emitted current I and the power spectra density S for a 3D bulk material can no longer be applied directly. It is necessary for us to re-derive the respected formulas starting from the original definition.

By consider the effects of linear energy dispersion $E = \hbar v_F k$ as well as the Klein tunneling, our previous work has already established the equation to calculate the emission line current density J_L [see Eq. (1) in Ref. 10]. It is straightforward to obtain the equation for the total emitted current I, which is

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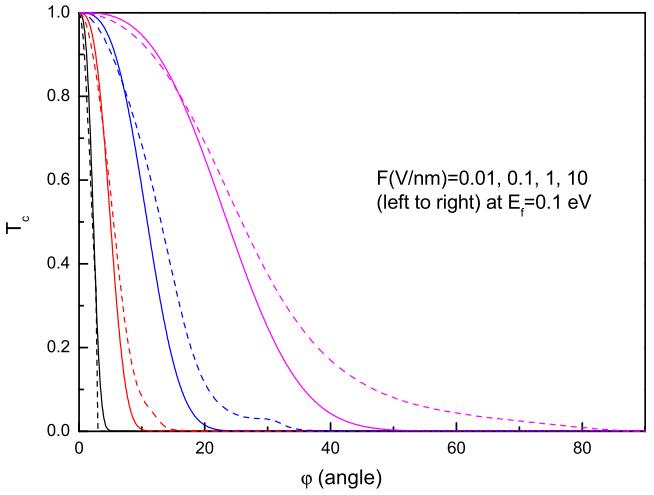

FIG. 1. Transmission coefficient T_c computed by using the relativistic WKB method (solid line) and the transfer matrix method (dash line). From left to right, F (V/nm) = 0.01, 0.1, 1, and 10 at $E_f = 0.1$ eV.

$$I = L \times J_L = L \times \frac{q}{2\pi^2\hbar^2 v_F}\int_0^{E_0}\int_{-\pi/2}^{\pi/2} T_c(E,\phi)f(E)E\cos\phi dEd\phi. \quad (1)$$

The power spectra density S can be obtained from the scattering and correlation theory,[24,25] given by

$$S = L \times \frac{2q^2}{2\pi^2\hbar^2 v_F}\int_0^{E_0}\int_{-\pi/2}^{\pi/2} T_c(E,\phi)f(E) \times [1 - T_c(E,\phi)f(E)]E\cos\phi dEd\phi, \quad (2)$$

where L is the emission length on the graphene edge, q is the electron charge, $\hbar$ is the reduced plank constant, $v_F = 1\times10^6$m/s is the Fermi-velocity of graphene, $f(E)$ is the Fermi-Dirac distribution, and T_c is the Klein tunneling coefficient, which is a function of electron energy E, and incident angle ϕ (in x-y plane). Note Eqs. (1) and (2) remain valid as long as the electron energy E is small so that the linear energy dispersion is maintained, and the Klein tunneling effect is important. In our calculations shown below, we have assumed $E_f = 0.1$ eV (unless it is specified), and the upper limit of the integration E_0 (=0.5 eV) is set to 5 times higher than E_f to avoid numerical errors.

Based on Eqs. (1) and (2), the Fano factor γ can be calculated by

$$\gamma = \frac{S}{2qI} = \frac{\int\int T_c f(1-T_c f)E\cos\phi dEd\phi}{\int\int T_c f\, E\cos\phi dEd\phi}, = \frac{\int\int T_c f(1-f)E\cos\phi dEd\phi}{\int\int T_c f\, E\cos\phi dEd\phi} + \frac{\int\int T_c(1-T_c)f^2\, E\cos\phi dEd\phi}{\int\int T_c f\, E\cos\phi dEd\phi}, = \gamma_T + \gamma_p \quad (3)$$

which clearly indicates two sources of correlations.[12] The first term γ_T is mainly affected by the probabilistic occupation of states in the emitter (through the Fermi Dirac distribution $f(E)$). It is very sensitive to the intrinsic thermal agitations of the emitter since $f(1-f) = -k_BT(\partial f/\partial E)$ and it vanishes at zero temperature. The second term γ_p is closely related to the quantum partitioning and the fact that charge is carried by discrete portions. It only contributes at finite transmission probability other than zero and perfect transmission ($T_c \neq 0, 1$) and does not vanish at zero temperature. Note γ_p term is exactly equal to the one used in previous works[18,19] in their calculation of the Fano factor for double- and multi-barrier structures at the low temperature condition.

To obtain the transmission coefficient T_c, we solve the Dirac equation $\hat{H}\Psi = E\Psi$ numerically, where $\hat{H}$ is the Hamiltonian of graphene expressed as $\hat{H} = v_F \times (\delta_x p_x + \delta_y p_y) + V(x)$, δ_x and δ_y are Pauli's matrices, $p_x = -i\hbar\frac{\partial}{\partial x}$ and $p_y = -i\hbar\frac{\partial}{\partial y}$ are the momentum operators, and $V(x)$ is the potential barrier. For field emission, the potential barrier is in a form of $V(x) = V_0 - F\times x - Q/x$, where $F = \beta F_0$ is the local electric field at the graphene edge (already contains the geometrical effect), F_0 is the applied field ($=V_g/D$) and β is the corresponding enhancement factor, $V_0 = 4.66$ eV is set to be the work function of graphene, and Q/x is the image-charge potential with $Q = q^2/4\pi\varepsilon_0$ is a constant. Note that the assumption of using the same classical image charge potential for graphite field emission will require further investigation, which will not be discussed here.

In our previous results[10] that angle-dependence T_c was calculated by a relativistic WKB method.[26,27] Here, we will compare them (solid lines) with the transfer matrix method (dashed line),[28,29] which are plotted in Fig. 1 for various local electric field F (V/nm) = 0.01, 0.1, 1, and 10 (left to right) at $E_f = 0.1$ eV. The comparison shows that the difference between the two methods is small at small angle and large transmission ($T_c > 0.8$). In general, the transfer matrix method is suited better for the rectangular potential or trapezoid potential. For the smoothly varying potential $V(x)$ used here, it will require a large number of the elementary matrices in order to get the accurate results. The relativistic WKB method is, however, simple and fast to implement, and can still generate a reasonable good accuracy. In the following figures, we will compare the calculated Fano factor of both methods. Note that different numerical methods will only change slightly the quantitative value, and have little effect on the overall behavior of the shot noise calculations due to the fact that the transmission probability is small and the temperature T appears, linear to leading order, in both the numerator and denominator.

Figure 2 demonstrates the temperature dependence of the Fano factor γ as a function of the local electrical field F at various temperatures $T = 5$, 100, 300, 500, and 1000 K (bottom to top, solid lines), and the Fermi energy E_f is fixed at 0.1 eV. For comparison, we also calculate γ using the transfer matrix method at $T = 5$ K and $E_f = 0.1$ eV (dash line). At low temperature condition ($T = 5$ K), γ approaches zero at very small electrical field ($F < 10^{-8}$ V/nm) as well as at very high electrical field ($F > 0.1$ V/nm). The

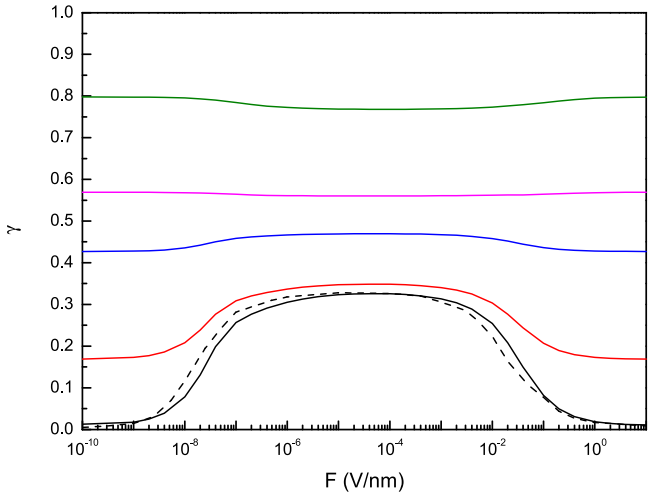

FIG. 2. The Fano factor γ as a function of the local electric field F at various temperatures T (K) = 5, 100, 300, 500, and 1000 (bottom to top, solid lines) at $E_f = 0.1$ eV. The dashed line is the result computed by using the transfer matrix method at $T = 5$ K and $E_f = 0.1$ eV.

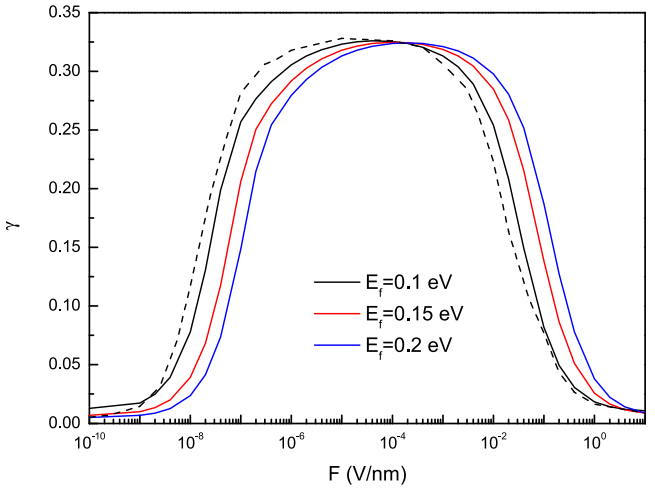

FIG. 3. The Fano factor γ as a function of the local electric field F at various Fermi energies E_f (eV) = 0.1, 0.15, and 0.2 (left to right, solid lines) at $T = 5$ K. The dashed line is the result computed by using the transfer matrix method at $T = 5$ K and $E_f = 0.1$ eV.

maximum value of γ is around 0.326 at $F = 3 \times 10^{-5}$ V/nm which is indeed the expected universal limit value predicted before. This finding can be explained by the following reasoning. At low temperature, we have the Fermi distribution $f(E) \approx 1$, $\gamma_T \approx 0$ and thus the shot noise is determined mainly by the second term γ_p. At very small and high electrical field, the transmission coefficient is, respectively, $T_c \approx 0$ and $T_c \approx 1$, for which the correlation due to the quantum partition is minimum and thus $\gamma_p \approx 0$. At the intermediate vicinity of $F \approx 10^{-5}$ V/nm, such as $F = 3 \times 10^{-5}$ V/nm, the correlation is maximum and we recover the universal maximum value of $\gamma = 1/3$.

By increasing the temperature T, the first term (γ_T) becomes more dominant due to the change in the Fermi distribution, and the overall Fano factor $\gamma = \gamma_T + \gamma_p$ will exceed 1/3. In the range of $T = 300$ to 1000 K, γ tends to be uniform regardless of the electrical field F, which clearly indicates that the quantum partition effect (γ_p) is only significant at low temperature condition.

Figure 3 demonstrates the Fermi energy dependence of the Fano factor γ as a function of the local electrical field F at $T = 5$ K at different Fermi energies $E_f = 0.1$, 0.15, and 0.2 eV (left to right, solid lines). The comparison with the transfer matrix method is also plotted (dashed line) at $T = 5$ K and $E_f = 0.1$ eV. From the figure, we can easily observe that the results shift to high F as the Fermi energy increases, while the maximum value of γ remains about the same ($\approx 1/3$) regardless of the Fermi energy level. As mentioned previously, at $T = 5$ K, $\gamma_T \approx 0$ and γ_p plays the major role that the shot noise is mainly due to the quantum partition ($\gamma_p \approx 0$ at both very low and high electric fields).

It is worth to mention that as long as the charge carrier in graphene is described by the Klein tunneling coefficient $T_c(E, \phi)$ and governed by the Dirac equation, the maximum value of γ shall never exceed $1/3$ for a static scalar potential barrier, such as $V(x)$ studied here at low temperature regime. Unless a vector potential is added (by applying a magnetic field) and γ shall possibly exceed this limit.[30]

In conclusion, this paper investigates the effects of the Klein tunneling on the shot noise property of electron field emission from a vertically aligned single layer graphene sheet. The temperature and the Fermi energy dependence of the Fano factor γ are clearly illustrated. At low temperature condition ($T = 5$ K), the universal maximum value of 1/3 is recovered. This work could provide some interests and inspirations in the graphene electron emission experiment and its noise analysis.

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