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Optical force on a cylindrical cloak under arbitrary wave illumination

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The optical force distribution in the cylindrical cloak under arbitrary incident waves is presented. We show that on the inner surface of the cloak both the induced surface currents and polarization charges interact with the waves and give opposite radiation pressure onto the inner surface. The Lorentz force in the cloak can contribute to change the trajectory of the rays, while in some cases it may only reflect the rays having a tendency to decrease the total energy it carries. The force is symmetric and in balance. Therefore the total momentum transfer from the waves to the cylindrical cloak is zero. © 2010 Optical Society of America

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The coordinate transformation based cloaks, which can make the object inside perfectly invisible, have received much attention [1–7]. They can be mainly grouped into three types: the point transformed cloak, e.g., the spherical cloak [1]; the line transformed cloak, e.g., the cylindrical cloak [3]; and the plane transformed cloak [4,5]. Compared with the plane transformed cloak, which can be invisible only under certain incident direction for transverse magnetic modes [4] or can be invisible only when the hidden object is on a background of conducting plane [5], the other two types of cloak can be invisible under any wave incident direction for any kind of modes. In particular, it is interesting to see that although the transformation procedure on the cylindrical cloak is in a two-dimensional plane, both the ray tracing [6] and the full wave analysis [7] show that the cloak is invisible to any arbitrary incidence waves, reflecting a three-dimensional invisible behavior of the cloak. Owing to the extreme material parameters at the cloak's interior interface, electric and magnetic surface displacement currents will be induced to prevent the power from penetrating into the core [7]. These induced surface currents, as well as the induced polarization charges, will interact with the electromagnetic waves and give electromagnetic forces upon the matter. Such phenomena are very interesting and worthy to be revealed.

In this Letter, we analytically calculate the optical force distribution in the cylindrical cloak under arbitrary incident wave illumination. We show that the Lorentz force in the cloak can contribute to change the trajectory of the rays, while in some cases it may only reflect the rays having a tendency to decrease the total energy it carries. The force on the cloak is symmetric and in balance. Therefore there is no net momentum transfer from the waves to the cylindrical cloak. Our results are helpful to understand the mechanical behavior of the cylindrical cloak under arbitrary wave illumination.

We consider a cylindrical cloak with inner radius R_1 and outer radius R_2 . The cloak layer within $R_1 < \rho < R_2$ is an anisotropic and inhomogeneous me-

dium with permittivity tensor $\bar{\epsilon} = \epsilon_\rho \hat{\rho}\hat{\rho} + \epsilon_\phi \hat{\phi}\hat{\phi} + \epsilon_z \hat{z}\hat{z}$ and permeability tensor $\bar{\mu} = \mu_\rho \hat{\rho}\hat{\rho} + \mu_\phi \hat{\phi}\hat{\phi} + \mu_z \hat{z}\hat{z}$. Assuming $f(\rho)$ to be an arbitrary transformation function between the original cylindrical coordinate system (ρ', ϕ', z') and the physical cylindrical coordinate system (ρ, ϕ, z) , we can get the constitutive parameters ϵ_ρ , ϵ_ϕ , ϵ_z , μ_ρ , μ_ϕ , and μ_z based on the transformation method proposed in [1]: $\epsilon_\rho/\epsilon_0 = \mu_\rho/\mu_0 = f(\rho)/[\rho f'(\rho)]$, $\epsilon_\phi/\epsilon_0 = \mu_\phi/\mu_0 = [\rho f'(\rho)]/f(\rho)$, $\epsilon_z/\epsilon_0 = \mu_z/\mu_0 = [f(\rho)f'(\rho)]/\rho$, where ϵ_0 and μ_0 are the permittivity and permeability of free space. The time-harmonic incident wave is $\bar{E}_i = (\hat{v}_i E_{vi} + \hat{h}_i E_{hi}) e^{i\bar{k} \cdot \bar{r}}$, where $\bar{k} = \hat{x}k_x + \hat{z}k_z$, $k^2 = \omega^2 \mu_0 \epsilon_0$, $\hat{h}_i = (\hat{z} \times \hat{k})/|\hat{z} \times \hat{k}|$, and $\hat{v}_i = \hat{h}_i \times \hat{k}$ [8]. Here we consider the case of vertical polarization, i.e., $E_{vi} = 1$ and $E_{hi} = 0$. The horizontal polarization, i.e., $E_{vi} = 0$ and $E_{hi} = 1$, can be considered as the dual case. The incident angle is $\alpha = \tan^{-1}(k_z/k_x)$. Based on a full wave scattering analysis, we can get the electromagnetic field inside the cloak layer ($R_1 < \rho < R_2$),

$$E_\rho^c = -f'(\rho) \sin \alpha \cos \phi e^{ik_x f(\rho) \cos \phi + ik_z z},$$

$$H_\rho^c = -\sqrt{\frac{\epsilon_0}{\mu_0}} f'(\rho) \sin \phi e^{ik_x f(\rho) \cos \phi + ik_z z},$$

$$E_\phi^c = \frac{f(\rho)}{\rho} \sin \alpha \sin \phi e^{ik_x f(\rho) \cos \phi + ik_z z},$$

$$H_\phi^c = -\sqrt{\frac{\epsilon_0 f(\rho)}{\mu_0 \rho}} \cos \phi e^{ik_x f(\rho) \cos \phi + ik_z z},$$

$$E_z^c = \cos \alpha e^{ik_x f(\rho) \cos \phi + ik_z z}, \quad H_z^c = 0. \quad (1)$$

The fields in the core are zero as shown in [7], while the fields outside of the cloak (air region) can be expressed using Eq. (1) with the transformation function determined by $f^{air}(\rho) = \rho$. The time-average Lor-

entz force density due to the harmonic wave excitation is [9]

$$\begin{aligned} \langle \bar{f} \rangle = & \frac{1}{2} \text{Re} \{ (-\nabla \cdot \bar{P}) \bar{E}^* + (-\nabla \cdot \mu_0 \bar{M}) \bar{H}^* - i\omega \bar{P} \times \mu_0 \bar{H}^* \\ & + i\omega \mu_0 \bar{M} \times \epsilon_0 \bar{E}^* \}. \end{aligned} \quad (2)$$

The polarization and magnetization are defined as $\bar{P} = \bar{D} - \epsilon_0 \bar{E}$ and $\mu_0 \bar{M} = \bar{B} - \mu_0 \bar{H}$, respectively. Substituting Eq. (1) into Eq. (2), we can see that the final two terms in {} in Eq. (2) are pure imaginary numbers, indicating that in the cloak layer of an ideal lossless cloak the force density on the electric and magnetic currents is zero. Therefore, the force density in the cloak layer is equal to the bulk force density on the bound electric and magnetic charges, i.e., the first two terms in Eq. (2). The force in the region of $R_1 < \rho < R_2$ is given by

$$\begin{aligned} \langle \bar{f}_b \rangle = & \frac{\epsilon_0}{4} \left[\frac{f'(\rho)}{\rho} + f''(\rho) - \frac{f(\rho)}{\rho^2} \right] \left[-\hat{z} \sin 2\alpha \cos \phi \right. \\ & + \hat{\rho} 2f'(\rho)(1 - \cos^2 \alpha \cos^2 \phi) \\ & \left. + \hat{\phi} \frac{f(\rho)}{\rho} \cos^2 \alpha \sin 2\phi \right]. \end{aligned} \quad (3)$$

Similar to [10], we can get the surface force density on the bound electric and magnetic surface charges at $\rho = R_2$,

$$\begin{aligned} \langle \bar{f}_{\rho=R_2} \rangle = & \frac{\epsilon_0}{4} [1 - f'(R_2)] \{ -\hat{z} \sin 2\alpha \cos \phi + \hat{\rho} [1 + f'(R_2)] \\ & \times (1 - \cos^2 \alpha \cos^2 \phi) + \hat{\phi} \cos^2 \alpha \sin 2\phi \}. \end{aligned} \quad (4)$$

On the inner surface of the cylindrical cloak, magnetic and electric surface currents are induced by the incoming waves [7], which is different from that of the spherical cloak. The surface force density is

$$\begin{aligned} \langle \bar{f}_{\rho=R_1} \rangle = & \frac{\epsilon_0}{4} \{ \hat{\rho} [f'^2(R_1)(1 - \cos^2 \alpha \cos^2 \phi) - \cos^2 \alpha] \\ & - \hat{z} f'(R_1) \sin 2\alpha \cos \phi \}. \end{aligned} \quad (5)$$

The above expression of force will give automatically zero field density if we assume $f(\rho) = \rho$, in which case the cloak material will be free space and there will be no force density in the cloak. In addition, it is easily seen from Eqs. (3)–(5) that due to the symmetry, the total integration of the Lorentz force is zero.

For simplicity of demonstration, we consider normal incidence ($\alpha = 0$), which is widely considered as a two dimensional case. Assuming that the cloak is linearly transformed with $f(\rho) = [R_2/(R_2 - R_1)](\rho - R_1)$ and $R_2 = 2R_1 = 2\lambda_0$ ($\lambda_0 = 0.1$ m), we can plot the force distribution inside the cylindrical cloak as shown in Fig. 1. The black arrows show the bulk force density with a unit of N/m³, while the gray arrows show the surface force density with a unit of N/m². The maximum bulk force density is 1.77×10^{-10} N/m³, while the maximum surface force density is 6.63

Fig. 1. (Color online) Lorentz force density (arrows) on the ideal linearly transformed cylindrical cloak under normal incidence ($\alpha = 0$). The background pattern is the electric field with unit amplitude (volt/meter) propagating along the x direction. The axes are normalized by λ_0 .

$\times 10^{-12}$ N/m². It is seen that the force on the outer surface ($\rho = R_2$) points inward, while the force in the cloak layer ($R_1 < \rho < R_2$) points outward. Different from that in the spherical cloak [10], the force on the inner surface ($\rho = R_1$) of the cylindrical cloak does not always point outward. In the front region where $-\sin^{-1}[1/f'(R_1)] < \phi < \sin^{-1}[1/f'(R_1)]$ or in the rear region where $\pi - \sin^{-1}[1/f'(R_1)] < \phi < \pi + \sin^{-1}[1/f'(R_1)]$, the force points inward, while in other regions (the lateral regions) the force points outward. This is because the force on the inner surface is composed of two parts: the force on the induced magnetic surface current equaling $-\hat{\rho}(\epsilon_0/4)$ and the force on the polarized surface charges equaling $\hat{\rho}(\epsilon_0/4)f'^2(R_1)\sin^2 \phi$. They are in opposite directions.

Because the total radiation force exerted by the electromagnetic fields upon matter is zero, we can conclude that the total momentum transfer to the cloak is zero. However, the force density in the cloak media is nonzero. Likewise, equal and opposite stresses exist upon the electromagnetic fields. As an example, we consider obliquely incident case and show in Fig. 2 how a ray feels the forces (represented by arrows) when obliquely incident into the cylindrical cloak with $\alpha = \pi/6$; i.e., the ray is incident along the $3\hat{x} + \sqrt{3}\hat{z}$ direction. Figures 2(a) and 2(c) show the forces on the ray in the xy and xz planes, respectively; Fig. 2(b) shows from the propagation direction of the incident wave (in this angle of view, each ray will look as a point if the cylindrical cloak is removed, since the incident ray is perpendicular to the paper), while Fig. 2(d) shows the three dimensional plot of the ray and the accompanied forces. Note that the arrows shown in Fig. 2 represent the forces on the electromagnetic waves exerted by the matter, while the arrows shown in Fig. 1 represent the forces on the matter exerted by the waves. They are in opposite directions in their corresponding cases. We see in Fig. 2 that the ray feels a surface force when impinging onto the cloak and deviated from its original path. When propagating inside the cloak, it feels a centripetal force, which produces a circulating motion. After

Fig. 2. (Color online) The forces the ray feels when obliquely incident into an ideal linearly transformed cylindrical cloak with $\alpha = \pi/6$: (a) in the xy plane, (b) viewed from the incident wave direction, (c) in the xz plane, and (d) in three-dimensional view. The blue arrows at the outer surface of the cloak are surface force density with unit of N/m^2 , while the red arrows distributed inside the cloak layer are bulk force density with unit of N/m^3 . The dashed circle represents the inner surface of the cloak.

propagating out of the cloak, the ray feels a surface force pushing it back to its original path. It is interesting to see that, within the cloak, the bulk forces are perpendicular to the direction of ray propagation (Poynting vectors). This is because the forces on the bulk polarization charges are always in the EH plane, perpendicular to the Poynting vectors.

However, not all of the forces contribute to change the trajectory of the rays; some parts of the forces only reflect the rays having a tendency to decrease the total energy it carries. For example, in the cloak medium, the central ray at $\phi = 0$ is a straight line along the $\sqrt{3}\hat{x} + 2\hat{z}$ direction (not shown in Fig. 2), but it still feels a force along the $2\hat{x} - \sqrt{3}\hat{z}$ direction from the calculation of Eq. (3). Since the density of ray reflects the energy density, we can see from Fig. 2(a) that the forces tend to push the ray from a region with a high energy density to a region with a low energy density. Equivalently, the material boundary feels an opposite force exerted by the wave and tends to move in an opposite direction under the Lorentz force. For the convenience of illustration, we consider a simple case with $\alpha = \pi/2$, which corresponds to the case that the wave is propagating along the z direction. The Lorentz force distribution inside the cylindrical cloak is shown in Fig. 3, where the background pattern represents the amplitude of the Poynting vector along the z direction. We see that the inner surface feels a force to expand the core region since there is no energy inside, while the outer surface feels a force to shrink the cloak since the energy density in the free space is smaller than the outer part of the cloak. All of the rays are straight lines, and the Lorentz forces here therefore do not contribute to change the trajectory of the rays; they only reflect the

Fig. 3. (Color online) Lorentz force density (arrows) on the cloak in the xy plane when the wave vector is along the z direction ($\alpha = \pi/2$). The background pattern represents the amplitude of the Poynting power.

rays having a tendency to decrease the total energy it carries. This phenomenon exists in most of the inhomogeneous media. Since the cloak is solid and also due to the symmetric pattern of the force distribution, we can see that the stresses in the cloak material are in balance, affecting a stationary cloak.

In conclusion, the Lorentz force distribution in the cylindrical cloak under arbitrary incident waves is presented. We show that the Lorentz force in the cloak can contribute to change the trajectory of the rays, while in some cases it may only reflect the rays having a tendency to decrease the total energy it carries. Owing to the symmetric pattern of the force distribution, there is no net momentum transfer from the waves to the cloak. Our results are helpful to understand the mechanical behavior of the cylindrical cloak under wave illumination.

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