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Cohen, Avis H.; Li, Fei; Basu, Arindam; Chang, Chip Hong

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Dynamical Systems Guided Design and Analysis of Silicon Oscillators for Central Pattern Generators

Fei Li, Arindam Basu, *Member, IEEE*, Chip-Hong Chang, *Senior Member, IEEE*, Avis H. Cohen

Abstract— In this paper, a dynamical systems (DS) approach is proposed for the analysis and design of bio-inspired silicon central pattern generator systems. Based on this approach, a new leaky-integrate-and-leaky-discharge oscillator circuit is proposed that has dynamical properties closer to biological half-center oscillators while being power and area efficient. The membrane potential charges and discharges through a single resistor eliminating mismatch in charging and discharging phases. Switched-capacitor (SC) and floating-gate wide input linear range operational transconductance amplifier (FGOTA) based approaches have been proposed to implement the resistor. Both approaches enable controllable and large resistances in a small area. Oscillation frequency can be easily controlled by the frequency of switching in SC based and bias current in FGOTA based implementations, which are very useful for global change of oscillation frequency in an array of oscillators. Dynamical systems analysis has shown that when it is used as a single oscillator, the proposed circuit is able to produce a phase response curve (PRC) close to that of a lamprey central pattern generator (CPG) system. Applying averaging theory to a system of coupled oscillators, we obtain averaged H and G functions for unidirectional and bidirectional coupling cases. Analysis of these functions shows our circuit's superior capability to achieve entrainment when driven by a periodic input (e.g., from sensory feedback) and reach equilibrium even with high frequency mismatch.

Index Terms—Central pattern generators, half-center oscillators, dynamical systems, silicon oscillators, switched-capacitor, floating-gate.

I. INTRODUCTION

Central pattern generators (CPGs) are neural networks that have been found to be the basis for generating essential rhythmic patterns of breathing, swimming, walking, digesting, etc. in a wide variety of animals [1], [2]. Such circuits help in coordinating the rhythms at the low level by integrating command signals from higher control centers with feedback inputs from sensory neurons and producing rhythmic outputs to the motor neurons that control locomotion. However, the CPG

alleviates the need for higher control centers to generate high dimensional signals to micro-manage the motor neuron's activation patterns as descending inputs are not required for the CPG's basic functionality [2], [3]. Typically, the CPG is comprised of a chain of oscillator units interconnected for coordinated activity along the chain. The basic operation of each unit oscillator is modeled by the simple half-center oscillator (HCO) model or a pair of mutually inhibitory coupled oscillators. The mutually inhibitory coupling within a HCO guarantees that one oscillator is active while the other is silent. The two oscillators are forced to produce alternating activities between two opposing centers representing flexors and extensors in locomotion. This is necessary to guarantee stable locomotory gaits (e.g., the left and right side muscles of a lamprey will not contract simultaneously or the left and right feet of a biped will not move forward at the same time).

The theory of dynamical systems (DS) has been used earlier to analyze the properties of the computational models of a biological CPG [4], [5]. Here we propose to use DS for guiding the design of silicon circuits for CPGs by providing a link to the biological differential equations. DS has earlier been used to design or extend silicon neurons that belong to type I [6], [7] by [8]–[10]. To the best of our knowledge, this is the first time DS is used to design silicon CPG systems, except for a preliminary work by us [11]. DS enables us to formulate a criterion to evaluate different candidate oscillators for use as an HCO. For example, phase noise based figure-of-merit used to evaluate oscillators for use in communication applications is not applicable here; instead we can use a figure-of-merit based on the oscillator's ability to synchronize with other oscillators or a forcing sensory input. We also use DS to analyze the properties of a biological HCO and design a silicon HCO with similar dynamical properties but much less power and area overhead than other reported designs.

Several "neuromorphic" silicon implementations of these neural circuits have been attempted in the past with varying degrees of success [12]–[17]. Simoni *et al.* have tried to mimic biology in great detail by implementing multiple conductance neuron based oscillators [12]; such systems, though invaluable for studying biology, are not so easy to use in practical applications because of their large parameter set and sensitivity to variations. Arena *et al.* have proposed a six neuron chip designed by using operational amplifier (opamp) based

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Fei Li, Arindam Basu, and Chip-Hong Chang are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, 639798. (e-mail: life0003@e.ntu.edu.sg, arindam.basu@ntu.edu.sg, echchang@ntu.edu.sg)

Avis H. Cohen is a member of the Department of Biology and Institute for Systems Research, University of Maryland, MD 20742, USA. (e-mail: avis@umd.edu)

switched-capacitor (SC) circuits using the concept of cellular neural network (CNN) [18], [19] for locomotion control of a hexapod robot [13]. These circuits are quite area and power hungry due to the usage of opamps and cannot be used in ultra-low power applications. A good implementation of an artificial CPG is extremely beneficial in biped robotics [14]–[16] for quick and robust control of locomotion in uncertain conditions as well as prosthetics [17]. Lewis *et al.* have created a neural chip that contains electronic analogs of a biological neuron with two neurons based on simple integrate-and-fire (IF) model using bursts of spikes for coupling [14]. Tenore *et al.* proposed an improved CPG chip that contains ten neurons based on similar neurons but with modified coupling [15], [16]. All of these approaches use two oscillators for a HCO. Also, using bursting neurons in the HCO causes the membrane capacitance to be charged and discharged multiple times in every slow oscillation cycle (which is the desired locomotion frequency) - this leads to excessive power dissipation compared to our method which can be thought to model the envelope of these bursts.

The paper is organized as follows. Section II introduces the proposed oscillator topology and discusses two types of circuit implementations of the proposed topology. In Section III, the mathematical fundamentals of the coupled oscillator model are introduced. The analysis methods adopted to obtain the key synchronization parameters from the oscillator and coupling circuits are also discussed. Section IV discusses the obtained simulation and measurement results from the two types of circuit implementations and compares the results with existing circuits. Finally, conclusions are presented in Section V.

II. PROPOSED OSCILLATOR CIRCUIT

A. Biological HCO vs. Silicon Oscillators

In this paper, we concentrate on implementations that use some form of abstraction of biology (in particular, the CPG of the lamprey system) that may result in a practical, usable system with easily controlled properties. Our ultimate goal is to use this artificial CPG to control a fish-like autonomous underwater vehicle (AUV) with a bio-inspired fin for swimming. Some details about a possible implementation of this robot are provided in the appendix. The lamprey system was chosen as a reference because it is a swimmer and its CPG has been extensively studied in [3], [20] and [21]. Also, we can directly use the results derived for sensory feedback [22] and body-fluid interactions [23] for the lamprey system in our swimming robot.

The concept of a phase response curve or PRC from DS is used to analyze if a single oscillator can model the dynamics observed in the HCO of a lamprey. Typically two mutually coupled unit oscillators are used to implement a silicon HCO [13]–[16]. Being able to use just one oscillator reduces the power consumption and silicon area of each unit neuron circuit and enables low-power VLSI implementation of CPG systems for complex locomotion control. In a dynamical neural system, the phase of an oscillator $\phi(t)$ is a periodic function of

oscillation time t and is defined as $t \bmod T$, where T is the period of oscillation. The PRC of such a system can be measured by perturbing the neuronal oscillator at different phases with an impulse and observing the change of phase produced as a result. The PRC is the function that maps the amplitude of the perturbation and the oscillator phase at which it is applied to the change of phase it produces:

$$PRC(\phi, A) = \{\phi_{new}(t) - \phi(t)\} \quad (1)$$

where $\phi_{new}(t)$ and $\phi(t)$ represent the phases of the perturbed and unperturbed oscillation profiles, respectively at time t and A is the strength of the impulse. Positive (negative) values of the PRC correspond to phase advances (delays). In this paper, the change of phase is calculated based on the phases of the peaks of the oscillation profile. In all plots of PRC in this paper, we use a normalized phase variable defined as $\phi_n = (t \bmod T) / T$ that has a maximum value of 1. Similarly, in other plots of dynamical systems analysis (H and G functions), all normalized variables are represented with a subscript ‘ n ’. The PRC normalized with respect to the perturbation strength A is denoted as $NPRC(\phi, A)$ and defined as $PRC(\phi, A) / A$. The limiting value of the NPRC for $A \rightarrow 0$ is denoted as $Q(\phi)$. It will be used in the dynamical systems analysis in Section III.A and the steps to obtain $Q(\phi)$ are detailed in Section III.C.

The essential features of the experimentally observed PRC of a lamprey CPG system have been captured in a model proposed by Cohen *et al.* [4] and it is shown in Fig. 1. It can be observed that the PRC has both positive and negative values depending on different stimulating phases. The phase shift also increases or decreases gradually with the stimulating phases, which means that the lamprey HCO is able to produce various levels of phase shifts with respect to a forcing oscillator. A single oscillator can model a whole HCO of a lamprey CPG system only when it is able to produce a PRC similar to that of lamprey.

Fig. 1. Simplified model of lamprey's PRC proposed by Cohen in [4].

Two popular simple silicon oscillators are analyzed to verify if they can produce a NPRC similar to the one in Fig. 1. Fig. 2(a) depicts the schematic diagram of the IF oscillator circuit which consists of three transistors, two inverters, and two capacitors [24]. Transistors M_1 and M_2 , being driven by two biases V_1 and V_2 , act as constant current sources that charge and discharge the membrane capacitor C_{MEM} , respectively. Capacitor C_{FB} creates a hysteresis window due to positive feedback and sets the amplitude of oscillation on the membrane node. The charging and discharging phases take place alternatively as V_{MEM} is below or above the switching threshold of inverter 1. Transistor M_3 works as a switch that enables the discharging current to pass through. During the charging phase, V_{OUT} goes to ground and turns off M_3 ; therefore, C_{MEM} is

Fig. 2. (a) Schematic diagram, (b) simulation results and (c) NPRC plotted together with one oscillation cycle of the IF oscillator; (d) schematic diagram, (e) simulation results and (f) NPRC plotted together with one oscillation cycle of Still's differentiator oscillator circuit.

charged up by I_1 . During the discharging phase, V_{OUT} goes to V_{DD} and turns on M_3 , which allows discharging current I_2 to flow through. Therefore, C_{MEM} is discharged by the difference current $I_2 - I_1$. The circuit functionality is simulated using TSMC 0.35 μ m models by setting C_{FB} to 2 pF, C_{MEM} to 8 pF, V_1 to 2.8085 V ($I_1 = 12.12$ pA), and V_2 to 0.208 V ($I_2 = 24.5$ pA) and the results of V_{MEM} and V_{OUT} are shown in Fig. 2(b). It can be observed that for a 50 percent duty cycle of V_{OUT} , the two bias voltages should be chosen such that the discharging current is twice the charging current. Besides, the charging and discharging profiles of C_{MEM} follow a linear relationship with time due to the constant charging and discharging currents.

Fig. 2(d) depicts the schematic diagram of Still *et al.*'s differentiator oscillator [25], which consists of two transistors, two inverters and two capacitors inspired by the early work of Hasslacher and Tilden [26]. The two transistors act as constant current sources. When V_{MEM1} is high, M_1 discharges till it reaches the switching threshold of the inverter 1. At that time, the inverter forces V_{OUT1} to go high quickly which couples to V_{MEM2} through the capacitor C_{MEM2} . This in turn forces V_{OUT2} to drop to a low voltage, which couples to V_{MEM1} through C_{MEM1} and forces it to go low. The same process continues at V_{MEM2} . Fig. 2(e) shows the simulation results of the node voltages when C_{MEMi} is set to 10 pF and V_{Bi} to 0.2138 V ($I_{Bi} = 24.5$ pA). It can be observed that the two membrane potential profiles only have discharging phases during which V_{MEMi} decreases linearly with time until they reach the switching threshold of the inverters.

The membrane node of each oscillator is perturbed at different phases of oscillation in the simulation with a pulsed

current of 1 μ A for 100 ns and the obtained NPRCs for the two oscillators are plotted in Figs. 2(c) and 2(f). (The exact method used to compute the NPRC from SPICE simulations is described in details in Section III.C) It can be observed that the differentiator oscillator has only negative phase shifts. Both NPRCs are very close to square waveform, implying that both oscillators can only produce two levels of phase shifts. Thus, neither oscillator can be considered as a good candidate to model a whole HCO.

Leaky integrate-and-fire (IF) oscillators [27] charge the membrane capacitor through a resistor. They are able to produce a PRC with gradually changing phase shifts at the positive cycle. However, once it reaches a threshold, the membrane gets reset - hence, it only has positive phase shift in its PRC. To model a whole HCO, another resistor can be added to discharge the membrane capacitor to generate the other half cycle of the PRC. This is the basic philosophy behind our proposed design. However, as the resistors occupy the largest area, the area overhead would be doubled. Besides, the mismatch between the two resistors for the positive and negative half cycles will cause an imbalance in the PRC and oscillation waveforms. To solve these issues, a new type of oscillator circuit is proposed next, where the same resistor is reused for both phases of oscillation.

B. The Leaky-Integrate-and-Leaky-Discharge Oscillator Topology

As shown in Fig. 3(a), the proposed leaky-integrate-and-leaky-discharge oscillator circuit consists of an ordinary differential input transconductor g_m , two inverters $inv1$ and $inv2$, one resistor R , one membrane capacitor C_{MEM} , one

positive feedback capacitor C_{FB} , and a multiplexer MUX . The transconductor acts like a comparator switching its output V_{GM} when V_{MEM} crosses V_{REF} . Based on V_{OUT2} (which is a digitally buffered version of V_{GM}), MUX connects the resistance to either V_{LOW} or V_{UPP} , which are two bias voltages smaller and larger than V_{REF} , respectively. There is only one resistor to be matched across an array of oscillators, which results in a better matching of oscillation frequencies. In this oscillator circuit, the transconductor is designed using the topology shown in Fig. 3(b). Being the only source of static power consumption in the oscillator, the transconductor is biased to operate in the subthreshold regime [24]. The main advantages of using a transconductor instead of a simple inverter is the flexibility to choose the switching threshold and the elimination of short circuit power dissipated in the inverter when V_{MEM} is close to the switching threshold.

Fig. 3. (a) The general circuit architecture of the proposed leaky-integrate-and-leaky-discharge oscillator. (b) The circuit schematic of the ordinary differential input transconductor circuit.

Assuming a zero initial potential at C_{MEM} , the transconductor output V_{GM} will be pulled up to V_{DD} , which sets the output of $inv1$ and $inv2$ to low and high, respectively. Consequently, C_{MEM} will start to charge up towards V_{UPP} through R . Once the membrane potential V_{MEM} is larger than V_{REF} , the voltages V_{GM} , V_{OUT1} and V_{OUT2} will be toggled to low, high and low, respectively. The sudden change of V_{OUT1} from low to high will sweep V_{MEM} to a potential much higher than V_{REF} due to the hysteresis effect of the feedback capacitor C_{FB} . The multiplexer

connects V_{LOW} to R , and C_{MEM} starts to discharge. When V_{MEM} is lower than V_{REF} , the opposite process takes place and the charging phase starts. The two phases repeat alternatively.

The discharging and charging time of the oscillator can be expressed as:

$$t_{dis} = RC_{MEM} \ln \left(1 + \frac{\Delta V_{MEM}}{\alpha} \right) \quad (2a)$$

and

$$t_{ch} = RC_{MEM} \ln \left(1 + \frac{\Delta V_{MEM}}{\beta} \right) \quad (2b)$$

where

$$\alpha = V_{REF} - V_{LOW} \quad (3a)$$

$$\beta = V_{UPP} - V_{REF} \quad (3b)$$

and

$$\Delta V_{MEM} = \frac{C_{FB}}{C_{MEM} + C_{FB}} V_{DD} \quad (4)$$

Therefore, the oscillation frequency of the proposed oscillator circuit is expressed as:

$$f_{osc} = \frac{1}{RC_{MEM}} \cdot \frac{1}{\ln \left(1 + \frac{\Delta V_{MEM}}{\beta} \right) + \ln \left(1 + \frac{\Delta V_{MEM}}{\alpha} \right)} \quad (5)$$

Equation (2) reveals the governing parameters of the discharging and charging cycles. Both phases have the same time constant, which are equal to RC_{MEM} and are affected by ΔV_{MEM} . As the V_{REF} value is normally fixed at $V_{DD} / 2$, the discharging process is controlled by an external input V_{LOW} through the parameter α , while the charging process is controlled by V_{UPP} through the parameter β . To obtain a digital output with 50% duty cycle, α and β are set to be the same.

C. Two Implementations of the Resistor in the Proposed Oscillator

For locomotion application, the nominal oscillation frequency of the proposed oscillator is set to 1 Hz. In order to achieve such a large time constant with a reasonably sized capacitor (~10 pF), a resistor of up to 20 GΩ has to be used. Traditional polysilicon implementation of such resistor is not feasible as it takes up too much silicon area and is not tunable. A compact oscillator circuit design is desirable for an area efficient locomotion controller. Besides, the oscillation frequency of an oscillator circuit used in a CPG is required to be tunable to suit different locomotion behaviors such as crawling, walking and running. These requirements can be fulfilled by the following two implementations.

C1. Floating-Gate Transconductor Based Implementation

The first implementation method makes use of floating-gate (FG) technology [28]. The schematic and layout of a pFET FG is shown in Fig. 4. The gate terminal of the pFET is floating as it is not resistively connected to any voltage source. The value of gate potential is determined by the charge stored on the floating gate. As shown in the layout, a floating gate is a polysilicon gate surrounded by silicon dioxide, a high quality insulator, allowing the charge to be stored permanently.

Floating gates can be programmed using Fowler-Nordheim [29] tunneling or hot-electron injection processes [30].

Fig. 4. Circuit schematic and layout of a pFET floating-gate transistor.

The resistor is realized by a FGOTA [31] connected in a voltage buffer configuration as shown in Fig. 5(a). The wide input linear range is achieved through the capacitive division constructed by the two capacitors at the two input nodes of an ordinary n-type differential input pair transconductor as shown in Fig. 5(b). The resulting input linear range is $n + 1$ times larger than the input linear range of an ordinary transconductor input stage. Within the linear range, the resistance value is equal to $(n+1)V_T / (\kappa I_{bias})$, where V_T is the thermal voltage, κ is the coupling of a gate to the channel of a subthreshold transistor and I_{bias} is the bias current. However, one disadvantage of using FG devices is that it will need additional high voltage circuits to program the charge on the floating nodes.

C2. Switched-Capacitor Based Implementation

An alternative method for obtaining a large tunable resistor is to use a switched-capacitor as has been implemented in [13]. As will be shown, the frequency of oscillation in this case depends only on the ratio of capacitors which is well controlled. Also, there are no standard simulation models for FG devices making it difficult to rely on simulations for these devices - switched capacitors provide an easier alternative. Fig. 6 shows the circuit schematic of a simple SC implementation of a resistor. The simple circuit consists of one capacitor and two switches, which are controlled by a pair of non-overlapping clock ϕ_1 and ϕ_2 with switching frequency of f_{clk} . The equivalent resistance of this SC circuit, as shown in (6), depends on the capacitor value and the switching frequency of the

non-overlapping clocks.

$$R = \frac{V_1 - V_2}{I} = \frac{1}{C f_{clk}} \quad (6)$$

A smaller capacitor and slow f_{clk} are needed to achieve a large resistance. However, using a small capacitor increases the charge injection error, which greatly affects the accuracy of the effective capacitance [32], while reducing f_{clk} causes error induced by the leakage current through the OFF switches. In addition, the value of the equivalent resistance is limited by the parasitic capacitances of the switches.

Fig. 6. Simple SC implementation of a resistor.

To overcome the shortcomings of the implementation in Fig. 6, a novel circuit topology to implement the SC resistor is proposed. The circuit schematic of this topology is depicted in Fig. 7. The circuit consists of n ($n = 4, 6, 8, \dots$) capacitors of the same capacitance and $(2n - 1)$ switches which are controlled by a pair of non-overlapping clocks ϕ_1 and ϕ_2 . The circuit diagram at the top right of Fig. 7 is the equivalent circuit when ϕ_1 is switched on and that at the bottom right is the equivalent circuit when ϕ_2 is switched off. It can be observed that when ϕ_1 is switched on, half of the capacitors are connected in series between V_1 and V_2 ; while the other capacitors are discharged as their top and bottom plates are connected together to V_2 . When ϕ_2 is switched off, the capacitors that are in series connection during ϕ_1 are now in parallel connection between V_{NEW} and V_2 while the other capacitors are in series connection between V_{NEW} and V_2 .

During ϕ_1 , the charges stored in the capacitor network and the capacitor C_1 are given by:

Fig. 5. (a) Schematic of the resistor implemented by FGOTA connected as a voltage buffer. (b) Schematic of the FGOTA.

Fig. 7. Circuit topology of the proposed SC implementation of a resistor (left). Equivalent circuit when ϕ_1 is switched on (top right). And equivalent circuit when ϕ_2 is switched on (bottom right).

$$Q_{tot,\phi_1} = C_{SC} (V_1 - V_2) \quad (7a)$$

$$Q_{C_1,\phi_1} = \frac{2}{n} C_{SC} (V_1 - V_2) \quad (7b)$$

where C_{SC} denotes the nominal capacitance value of all the capacitors in Fig. 7. During ϕ_2 , due to the conservation of charges, the total charge stored in the capacitor network does not change and the two values in (7) now become:

$$Q_{tot,\phi_2} = \left(\frac{n}{2} + \frac{2}{n} \right) C_{SC} (V_{NEW} - V_2) = C_{SC} (V_1 - V_2) \quad (8a)$$

$$Q_{C_1,\phi_2} = \frac{2n}{n^2 + 4} C_{SC} (V_1 - V_2) \quad (8b)$$

Therefore, the charges transferred from V_1 to V_2 through capacitor C_1 equals:

$$\Delta Q_{C_1} = \left(\frac{2}{n} - \frac{2n}{n^2 + 4} \right) C_{SC} (V_1 - V_2). \quad (9)$$

The resulting equivalent resistance of this implementation is then obtained as:

$$R = \frac{n^3 + 4n}{8C_{SC} f_{clk}} \quad (10)$$

Equation (9) shows that the effective charge transferred from the voltage terminal to one capacitor is $2/n - 2n/(n^2 + 4)$ times smaller than the value in the simple SC resistor. In this way, the effective capacitance of the SC circuit is reduced without physically reducing the capacitor value. As compared to the approach of connecting n capacitors in series where the effective capacitance is C/n , this topology provides $(n^2 + 4)/8$ times smaller effective capacitance. The equivalent resistance of this architecture is derived in (10). It can be observed that a much larger capacitor can be used to implement a large resistance with greatly reduced charge injection effect, making the equivalent resistance much more accurate and stable. By substituting (10) into (5), the final expression for the oscillation frequency of the proposed oscillator circuit is obtained.

$$f_{osc} = f_{clk} \frac{8C_{SC}}{(n^3 + 4n)C_{MEM}} \frac{1}{\ln\left(1 + \frac{\Delta V_{MEM}}{\beta}\right) + \ln\left(1 + \frac{\Delta V_{MEM}}{\alpha}\right)} \quad (11)$$

III. DYNAMICAL SYSTEMS ANALYSIS

A. Mathematical Fundamentals of Coupled Oscillators

The theory of dynamical systems has been used as a tool to analyze and compare the proposed oscillator circuit with two other existing oscillators. Besides the PRC, which describes the property of the oscillator alone, we can derive other measures, which describe how the oscillator behaves when driven by external forcing through different coupling functions. This is important to understand how HCOs can be coupled together to form a CPG or how the CPG will react when the sensory input is fed back to it. The oscillator phase model theory used for this purpose is based on the assumption of weak coupling between oscillators. Though this assumption is restrictive and inaccurate for even the lamprey system, it has been shown [5], [33] that the trend of results found from the weak coupling assumption still

hold true for strong coupling in some cases. To avoid the ambiguity, we present simulation results of our oscillator's synchronization properties for strong perturbations in Section III.B and demonstrate the qualitative insight provided by the weak coupling theory in this case. A detailed analysis of oscillator phase models and reduction of coupled oscillatory differential equations by averaging are presented in [27]. For the sake of completeness, we briefly describe the method in the following section.

Consider ' n ' weakly coupled oscillators of the form:

$$\dot{x}_i = f_i(x_i) + \varepsilon \sum_{j=1}^n g_{ji}(x_i, x_j), \quad x_i \in \mathbb{R}^m \quad (12)$$

where x_i represents the state variables of the i -th oscillator, ε denotes the strength of coupling and g_{ij} are the coupling functions between oscillators i and j . By applying the phase reduction method [27], the corresponding phase model can be obtained:

$$\dot{\phi}_i = 1 + \varepsilon Q_i(\phi_i) \sum_{j=1}^n g_{ij}(x_i(\phi_i), x_j(\phi_j)) \quad (13)$$

where $x_i(\phi_i)$ is the state variable represented in terms of the phase along the limit cycle and $Q_i(\phi_i)$ is the NPRC of the oscillator with phase ϕ_i as $A \rightarrow 0$.

To study the collective properties of the network, each $\phi_i(t)$ can be expressed as $t + \varphi_i$, with t representing the fast free running natural oscillation and φ_i representing the slow network induced phase deviation from the natural oscillation. Assuming the same nominal frequency for the oscillators, one can consider only the phase difference of each oscillator from its nominal frequency. By rewriting the above equations in terms of this phase difference φ_i , and assuming ε is small, one can apply the averaging theory to obtain:

$$\dot{\varphi}_i = \varepsilon \omega_i + \varepsilon \sum_{j \neq i}^n H_{ij}(\varphi_j - \varphi_i) \quad (14)$$

where

$$H_{ij}(\varphi_j - \varphi_i) = \frac{1}{T} \int_0^T Q_i(t) g_{ij}(x_i(t), x_j(t + \varphi_j - \varphi_i)) dt \quad (15)$$

and ω_i is the frequency deviation of the i -th oscillator from the nominal frequency $1/T$. In (15), g_{ij} is the coupling function between the two oscillators.

In the case of two coupled oscillators (e.g., HCO), (14) can be further simplified by taking the phase difference between the two oscillators as our variable of interest. Hence, the simplified equation becomes:

$$\dot{\phi}_1 - \dot{\phi}_2 = \dot{\chi} = \omega + G(\chi) \quad (16)$$

where $\chi = \phi_1 - \phi_2$, $\omega = \omega_1 - \omega_2$ and $G(\chi) = H_{12}(-\chi) - H_{21}(\chi)$.

The H function indicates how well an oscillator can achieve phase locking with a driving oscillator or external sensory inputs. In the case of two coupled oscillators or an oscillator with external inputs, the condition for phase-locking can be found by setting (14) to zero.

$$(\omega_{osc} - \omega_m) + H_{osc}(\varphi_{osc} - \varphi_m) = 0 \quad (17)$$

where the subscript ' m ' refers to either the forcing oscillator or the external input signal. The equilibria of (14) are solutions to

$H_{osc}(\varphi_{osc} - \varphi_{in}) = \omega_{in} - \omega_{osc}$, and they are the intersections of the H function and the horizontal line of $\omega_{in} - \omega_{osc}$ as shown in Fig. 8. The dashed line represents $\omega_{in} - \omega_{osc}$ and equilibrium only exists in the range between the maximum and minimum values of the H function. The two peak values of the H function indicate the maximum frequency deviation between the oscillator and the driving signal, over which the phase locking can be achieved. The polarity of the H function indicates the oscillator's ability to advance or retard phase shifts to achieve phase-locked oscillation in response to external inputs.

Fig. 8. Geometrical interpretation of equilibria of the H function

Similar analysis can be applied to the G function. By setting (16) to zero, the condition for phase shift between the two mutually coupled oscillators can be solved. The solution is the intersection of the graph of the G function and the horizontal line $\omega_2 - \omega_1$. Therefore, the G function indicates how well an oscillator could achieve synchronized but phase-shifted oscillation when it is connected bi-directionally with a similar oscillator. The maximum and minimum values of the G function determine the tolerance of the network for the eigen oscillation frequency mismatch. This is useful in understanding the synchronization of two oscillators forming a HCO or between oscillators connected in a chain.

For each oscillator type, its H and G functions are computed for the following types of commonly used coupling functions (CF).

CF1: Resistive coupling [27], defined as:

$$g_{ij} = g_0 (v_{memi} - v_{memj}) \quad (18)$$

where g_0 represents a constant conductance or resistance connected between the membrane potentials v_{memi} and v_{memj} of the driving and driven oscillators. The simplest circuit implementation of this coupling function is by connecting two membrane potentials with one resistor. A more area-efficient implementation is shown in Fig. 9, where two membrane potentials are connected by two OTAs.

CF2: Pulsed constant current coupling [14], defined as:

$$g_{ij} = I_0 u(v_{memi} - V_{th}) \quad (19)$$

where I_0 represents a constant current, V_{th} is a threshold voltage and $u(\cdot)$ is a unit step function. One possible circuit implementation of this coupling function is also depicted in Fig. 9. In this circuit, V_{th} can be interpreted as the switching voltage of an inverter or the reference voltage of an OTA inside the oscillator. The output V_{outi} turns ON and OFF a switch which emulates the unit step function and allows a constant current to flow into the other oscillator.

CF3: Pulsed conductance coupling [34], defined as:

$$g_{ij} = I_0 u(v_{memi} - V_{th}) (V_{syn} - v_{memj}) \quad (20)$$

where V_{syn} refers to the potential of the synapse cell. It is obtained by multiplying (19) with a variable term $(V_{syn} - v_{memj})$, which represents the potential difference between the synapse cell and the driven oscillator. The circuit implementation of this coupling function is slightly different from the circuit of coupling function CF2 as the two constant current sources are replaced by two resistors with the common node biased at V_{syn} . The resistors could again be implemented either by FGOTA or SC techniques.

Fig. 9. Possible circuit implementations of the three coupling functions: CF1: resistive coupling, CF2: pulsed constant current coupling, and CF3: pulsed constant conductance coupling.

B. Theoretical PRC, Weak and Strong Perturbation, Synchronization

Consider the following equation for the proposed oscillator circuit with a pulsed input coupling:

$$C_{MEM} \frac{dv}{dt} = f(v) + I_{ptb} p(t) \quad (21)$$

where v is the membrane potential, $f(v)$ is the vector field and $p(t)$ describes a pulse with unit amplitude and duration h_{ptb} . The change in voltage, Δv , induced on the membrane capacitor after each impulse can be written as $I_{ptb} h_{ptb} / C_{MEM}$. The PRC of this oscillator can be analytically calculated from the time difference Δt between the state $v(t)$ before perturbation and the state $v(t) + \Delta v$ after perturbation.

The instantaneous voltage of the membrane potential during charging and discharging can be expressed as:

$$v_{ch}(t) = V_{REF} - \Delta V_{MEM} + (\Delta V_{MEM} + \beta)(1 - e^{-t/\tau}) \quad (22a)$$

$$v_{dis}(t) = V_{LOW} + (\Delta V_{MEM} + \alpha)e^{-t/\tau} \quad (22b)$$

where $\tau = RC_{MEM}$ is the time constant of the circuit. Incorporating Δv and Δt , (22) can be rewritten as:

$$v_{ch}(t) + \Delta v = V_{REF} - \Delta V_{MEM} + (\Delta V_{MEM} + \beta)(1 - e^{-(t+\Delta t_{ch})/\tau}) \quad (23a)$$

$$v_{dis}(t) + \Delta v = V_{LOW} + (\Delta V_{MEM} + \alpha)e^{-(t+\Delta t_{dis})/\tau} \quad (23b)$$

By solving for Δt_{ch} and Δt_{dis} , we obtain:

$$\Delta t_{ch} = \tau \ln \left(\frac{\Delta V_{MEM} + \beta}{\Delta V_{MEM} + \beta - \Delta v e^{t/\tau}} \right) \approx \frac{\tau e^{t/\tau} \Delta v}{\Delta V_{MEM} + \beta} \quad (24a)$$

$$\Delta t_{dis} = \tau \ln \left(\frac{\Delta V_{MEM} + \alpha}{\Delta V_{MEM} + \alpha + \Delta v e^{t/\tau}} \right) \approx -\frac{\tau e^{t/\tau} \Delta v}{\Delta V_{MEM} + \alpha} \quad (24b)$$

The approximate expressions in equations (24a) and (24b) are obtained by weak coupling assumptions and explicitly show how the functional form of the PRC changes by assuming weak perturbations.

Consider one cycle of oscillation which starts the charging process at $t = 0$. The PRC can be obtained by:

$$PRC(\phi, \Delta v) = \begin{cases} \Delta t_{ch}, & (0 \leq \phi < t_{ch}, v(t) + \Delta v < V_{REF}) \\ t_{ch} - t, & (0 \leq \phi < t_{ch}, v(t) + \Delta v \geq V_{REF}) \\ \Delta t_{dis}, & (t_{ch} \leq \phi < T, v(t) + \Delta v > V_{REF}) \\ T - t, & (t_{ch} \leq \phi < T, v(t) + \Delta v \leq V_{REF}) \end{cases} \quad (25)$$

where t_{ch} is the charging time. Eq. (25) describes the full PRC of the proposed oscillator. For weak coupling ($\Delta v \rightarrow 0$), the second and fourth equations are not important (since they are valid only on sets of measure zero) while the first and third equations assume the approximate values in (24a) and (24b). The NPRC of the oscillator is given by:

$$NPRC(\phi, \Delta v) = PRC(\phi, \Delta v) / \Delta v \quad (26a)$$

$$Q(\phi) = \lim_{\Delta v \rightarrow 0} (NPRC(\phi, \Delta v)) \quad (26b)$$

To validate our analytical model of (25), the simulated and theoretical NPRCs at different perturbation currents I_{ptb} of 0.1, 1, 2, 4 and 8 μA are plotted in Fig. 10 (corresponding to Δv ranging from approximately 1.25 mV to 100 mV), from which an insight into the perturbation strength level at the transition into weak coupling can be gained. For this simulation, an ideal resistor is used while the actual implementations of the resistor are considered in the next sections. It can be observed that the simulated and theoretical NPRCs match well for all perturbation strengths. From the plots, it is estimated that the weak coupling approximation is valid for $\Delta v < 6$ mV ($I_{ptb} < 0.5 \mu A$) where the effect of the second and fourth equations in (25) is negligible. Similar analysis can be carried out for the IF oscillator.

Next, to verify if the weak coupling theory presented earlier can be used to make predictions about the synchronization properties of this oscillator across a wide range of coupling strengths, we used one oscillator to drive another (unidirectional coupling) using the second coupling function CF2 shown above. The reason for choosing CF2 is its simple VLSI implementation, which requires just a current source and a switch. This external periodic input could represent another oscillator in a chain or sensory feedback input. In this experiment, the frequency of the driving oscillator is different from the nominal frequency of the driven oscillator and the aim is to find the maximum frequency deviation at which the oscillators can still synchronize for a fixed coupling strength. The same experiment is also repeated for different coupling strengths varying from weak coupling to strong coupling. The

Fig. 10. Simulated and theoretical NPRCs of the proposed oscillator obtained at different perturbation strengths with one oscillation cycle.

theoretical maximum frequency deviations, $\max(\omega_{ptb} - \omega_{osc})$ over which the phase locked solution exists are obtained for both the proposed and IF oscillators using the H function approach described earlier. The equation of H function in (15) can be rewritten as:

$$H_1(\chi) = \frac{I_{ptb}}{C_{MEM}T} \int_{\chi}^{\chi+T/2} Q(t)dt \quad (27)$$

However, as the PRC depends on the perturbation strength for strong perturbations, we used a second function, H^* as a candidate to better predict the phase locking.

$$H_1^*(\chi) = \frac{I_{ptb}}{C_{MEM}T} \int_{\chi}^{\chi+T/2} NPRC\left(t, \frac{I_{ptb}}{C_{MEM}} \frac{T}{2}\right) dt \quad (28)$$

Finding the maximum frequency deviation is equivalent to finding the maximum value of the H or H^* function at different perturbation strengths. Fig. 11 plots the simulated maximum frequency deviation versus coupling strength for both oscillators. Two different theoretical predictions are made based on H and H^* .

Fig. 11. Plot of the simulated and theoretical maximum frequency deviation between the train of input pulses and the natural frequency of the oscillator with phase locked solutions at different perturbation strengths.

It can be observed from Fig. 11 that the plot obtained by using H^* matches simulations better for both our proposed and the IF oscillators. The proposed oscillator is able to phase-lock with external inputs at a maximum frequency mismatch of at least 1.6 times larger than that of the IF oscillator at all perturbation strengths. Hence, we can conclude that for these oscillator topologies, the DS analysis results obtained with the weak coupling theory can be extended and hold true qualitatively even for strong coupling.

C. Analysis Method

The PRC and NPRCs of the two existing oscillators and the proposed SC based oscillators are obtained from SPICE simulations using CADENCE (San Jose, CA, USA) and MATLAB (MathWorks, Natick, MA, USA). The procedure is given as follows:

1. In CADENCE, record unperturbed waveform V_{MEM} of each oscillator.
2. Perturb each oscillator throughout one period of oscillation T in n evenly distributed steps and record the simulation data.
3. Detect local peaks of both unperturbed and perturbed waveforms. For all n steps, compute the time difference between the second peaks from the first perturbation of both unperturbed and perturbed waveforms in MATLAB.

Fig. 12. Simulation results of the oscillator circuit based on SC resistor based implementation (a) oscillation profile. Simulated NPRC plotted together with one unperturbed oscillation cycle for the (b) SC resistor based and (c) FGOTA resistor based implementation. Dynamical systems analysis of the three oscillator circuits of (d) $H1$ function, (e) $H2$ function, (f) $H3$ function, (g) $G1$ function, (h) $G2$ function and (i) $G3$ function.

4. PRC is obtained by plotting n time differences against the perturbation step number normalized to T . Normalize the PRC with respect to $I_{ptb}h_{ptb} / C_{MEM}$ to obtain NPRC.

The procedures to calculate H and G functions are:

1. Extract one unperturbed oscillation cycle from the waveform before the time of perturbation.
2. Perform linear interpolant fitting of both the obtained NPRC function and the extracted waveform with an identical set of equally spaced time variables with resolution $\Delta t = T/m$. The two resulting fitting functions are denoted as $Q_i(t)$ and $x_i(t)$ or $x_j(t)$.
3. For the m -th data point, shift $x_i(t)$ circularly m times since $x_j(t + m\Delta t)$ can be interpreted as the shifted version of $x_j(t)$.
4. Perform dot product of $Q_i(t)$ and all $m + 1$ numbers of $x_j(t + \chi)$ and take the summation of the $m + 1$ products to realize the integration.

TABLE I. CIRCUIT AND ANALYSIS PARAMETERS FOR THE SIMULATION AND ANALYSIS OF THE PROPOSED OSCILLATOR CIRCUIT

V_{DD}	V_{REF}	α, β	C_{FB}	C_{MEM}	C_{SC}
3.3 V	1.65 V	0.05 V	2 pF	8 pF	0.1 pF
f_{clk}	$I_{bias}(\text{ord})$	$I_{bias}(\text{lin})$	n	I_0	g_0
1 kHz	10 nA	23 pA	8	0.1 mA	0.1mS

IV. RESULTS AND DISCUSSION

A. Simulation of the Two Implementation Approaches to the Oscillator Circuit

The simulation results presented under this section are obtained with the circuit and analysis parameters shown in Table I. $I_{bias}(\text{ord})$ and $I_{bias}(\text{lin})$ are the biasing currents of the ordinary and wide linear range transconductors. n is the capacitor ratio of the wide linear range transconductor. I_0 and g_0 are the constant current and conductance, respectively, in the three CFs. To have a fair comparison among the three oscillators, the total capacitance (which dominates the area of the oscillator layout) and the oscillation frequencies are kept the same in all cases.

Fig. 12(a) depicts the simulation results of the membrane potential profile of the oscillator circuit based on SC implementation together with the two digital outputs V_{OUT1} and V_{OUT2} . The two digital outputs can for example drive motor neurons and represent the flexors and extensors in locomotion. Figs. 12(b) and (c) plot the simulated and fitted NPRCs together

with one oscillation cycle of both the SC resistor and FGOTA resistor based oscillator circuits. It can be observed that the NPRCs have both positive and negative values and the phase shifts change gradually with stimulating phases. The obtained NPRCs of the two approaches are much closer to the PRC of the lamprey CPG system than the IF and differentiator oscillators, which demonstrates that our oscillator can be used as an artificial HCO. The advantage of bipolar PRC is that the H function, which determines the efficacy of a control input on the phase of the oscillator, is bipolar for all common coupling types as well (implying control inputs can both advance and retard the oscillator phase). Also, for brief impulsive perturbations, the H function can be shown to be a scaled version of the PRC. In that case, if the PRC has only two values (like the IF case), the phase dependence of the perturbation is almost non-existent. Also, mimicking the biological system is useful since this is a control strategy used by nature and is expected to be effective. Having an artificial system behave in the same way is not only useful to make a better robot, but also a good test-bed to understand more about the biological control strategies and how they are different from the traditional control theory.

Figs. 12(d) to (f) compare the three H functions ($H1$, $H2$, and $H3$) of the four oscillator circuits based on resistive (CF1), pulsed constant current (CF2), and pulsed constant conductance coupling (CF3) functions, respectively. All three H functions of the two implementations of our proposed oscillator have similar shapes with both positive and negative polarities. The SC resistor based implementation has the widest H function range among the four designs for all three coupling functions and the FGOTA based design has wider range than the other two oscillators in $H2$ and $H3$ functions. The results indicate that our oscillator is able to achieve phase-locking with external input or driving oscillator over a larger range of frequency deviation. This is also proved by the simulations in Section III.C.

Figs. 12(g) to (i) show the three G functions ($G1$, $G2$ and $G3$) of the four oscillator circuits based on the resistive, pulsed constant current, and pulsed constant conductance coupling functions, respectively. It can be observed that the amplitudes of the G functions of both implementations of our proposed oscillator are at least 2.2 times those of the other two designs for all three coupling functions. Thus the proposed oscillator can achieve synchronized oscillations when connected bi-directionally with another oscillator over a wider range of eigen frequency mismatch.

Table II summarizes the power consumption, the maximum and minimum values of NPRC, H and G functions and the polarities of the H functions for the IF [14] and differentiator [25] oscillator circuits as well as the SC and FGOTA resistor based implementations of the proposed oscillator. In the two contending oscillators, inverters are used to switch between the charging and discharging phases. This incurs a great short circuit current because the membrane potential slowly varies around the switching threshold keeping both transistors in the inverter on for a long time. For a fair comparison, the two inverters in the IF oscillator [14] and the first inverter in the

TABLE II. PERFORMANCE COMPARISON OF THE FOUR OSCILLATOR CIRCUITS BASED ON SPICE SIMULATION

	IF [14]	Still [25]	SC based	FGOTA based
Technology	TSMC 0.35μm			
Supply (V)	3.3			
Power (nW)	49	92	51.5	
Perturbation Strength	1uA for 100ns			
NPRC (Max/Min)	0.634 -0.648	0.0405 -0.258	2.584 -2.583	1.628 -2.236
PRC Range	1.282	0.298	5.166	3.865
H1/H2/H3 (Maximum)	0.0405	0.0198	0.0328	0.0269
	0.0375	-9.4e-5	0.0489	0.0183
	0.0275	-0.00028	0.0366	0.0129
H1/H2/H3 (Minimum)	0.00017	-0.0008	-0.0137	-0.0114
	-0.0311	-0.00881	-0.05	-0.0584
	-0.00596	-0.00842	-0.0278	-0.0318
H1/H2/H3 (Range)	0.0403	0.0206	0.0464	0.0383
	0.0685	0.00872	0.0989	0.0767
	0.0335	0.00977	0.0643	0.0447
H1/H2/H3 (Polarity)	Positive	Dual	Dual	Dual
	Dual	Negative	Dual	Dual
	Dual	Negative	Dual	Dual
G1/G2/G3 (Maximum)	0.0169	0.0028	0.0453	0.0379
	0.0076	0.00041	0.0551	0.0422
	0.0042	0.00278	0.0361	0.0422
G1/G2/G3 (Minimum)	-0.0169	-0.0028	-0.0453	-0.0379
	-0.0076	-0.00041	-0.0551	-0.0422
	-0.0042	-0.00278	-0.0361	-0.0422
G1/G2/G3 (Range)	0.0338	0.0056	0.0906	0.0758
	0.0152	0.00082	0.11	0.0844
	0.0084	0.0056	0.0722	0.0844

differentiator oscillator [25] are replaced by the same transconductor used in our design for power measurement. It can be seen that all four circuits consume very little power and it is primarily determined by the number of transconductors used in each circuit. The power consumption of the wide input linear range transconductor in the FGOTA resistor based design is negligible as its biasing current is very low (23 pA). The power consumption of the transconductor used is 40 nW, which can be further reduced by reducing the biasing current given the low requirement on slew rate and bandwidth. The effect due to noise is also limited due to the low signal bandwidth requirement of this application.

B. Implementation of the FG Resistor based Oscillator on a Field Programmable Analog Array (FPAA)

To verify the concept of the design architecture in a realistic hardware environment where noise and parasitic effect could affect the operation and dynamics of the oscillator, the FGOTA resistor based approach has been implemented on an FPAA chip [31]. Recently, large scale programmable analog arrays have been reported for signal processing [31] and neural modeling applications [35]. FPAAs have an array of computational analog blocks (CABs) immersed in a sea of routing elements or switches. Each CAB has a variety of analog sub-circuits. By programming selected switches, a desired larger circuit can be implemented using the sub-circuits in

TABLE III. CIRCUIT PARAMETERS OF THE FG RESISTOR BASED OSCILLATOR CIRCUIT ON THE FPAA PLATFORM

V_{DD}	C_{FB}	C_{MEM}	I_b (ord)	gm (ord)	I_b (lin)	gm (lin)	I_0	g_0	V_{th}	V_{syn}	ΔV_{ptb}	h_{ptb}
2.4 V	1.5 pF	6.54 pF	5 nA	140 nS	0.3 nA	0.9 nS	0.1 mA	0.1 mS	0.472 V	0.7 V	0.352 V	560 μ s

individual CABs. The FPAA presented in [31] is used, which has transistors, OTAs, FGOTAs, floating-gate transistors, buffers, capacitors and gilbert multipliers in its 32 CABs. All of the OTAs are biased with floating gate (FG) transistors, which provide options to adjust bandwidth, noise and power. Two of the OTAs in each CAB have FG differential pairs, which enable the offset of the amplifier to be programmable as well as a wide input linear range that is essential to reduce the distortion in Gm-C filters and oscillators. The switches in this FPAA are also implemented using FG devices. To configure a particular circuit, certain set of switches need to be turned ON and some floating gate elements need to be programmed to generate the biases. A software tool chain has been developed to enable this task [30].

Fig. 13. (a) Circuit schematic of the FGOTA resistor based oscillator implementation configured on a FPAA platform. (b) PCB for the FPAA test setup.

The circuit schematic of the proposed oscillator topology implemented on FPAA platform is depicted in Fig. 13(a). It can be observed that the two inverters in Fig. 3(a) are realized by two OTAs while the resistor is implemented by a FGOTA connected as a voltage buffer. The reason for using OTAs instead of inverters is the abundance of OTAs on this chip compared to NMOS or PMOS transistors. The PMOS transistor M_P can generate the perturbation current to facilitate the measurement of the oscillator's PRC. During normal operation, V_{ptb} is equal to V_{DD} ; and during perturbation, it is set to $V_{DD} - \Delta V_{ptb}$ to generate a current injection into the membrane potential, where ΔV_{ptb} is the voltage drop applied at the gate of M_P . The following experiments were carried out on the FPAA board shown in Fig. 13(b). A USB connection serves as the

communication link with the host PC and also provides power to the printed circuit board. The measured data are recorded by an oscilloscope and transmitted to the host PC through a GPIB port. Table III lists some circuit and analysis parameters of the FPAA based oscillator circuit. I_b (ord) and I_b (lin) refer to the bias current for the ordinary and wide linear input range transconductors, respectively. I_0 and g_0 have the same meanings as explained earlier. V_{th} and V_{syn} are the threshold and synaptic voltages used to calculate the H function with the measured data. ΔV_{ptb} and h_{ptb} are the voltage drop of V_{ptb} and the duration of perturbation, respectively.

As opposed to simulations, it was not possible in the measurements to create a perturbation at a predefined phase of the oscillation. Instead, the oscillator was perturbed at random times and the full PRC is obtained by making sufficiently large number of measurements (which spreads the observations across the entire oscillation period by virtue of its randomness). The method of computing the NPRC from these measurements is also slightly different from the earlier one and is described next. After obtaining the NPRCs, the same method described in Section III.C is used to calculate the H and G functions.

The following procedure is adopted to calculate the NPRC from the FPAA measurements as illustrated in Fig. 14.

1. In MATLAB (MathWorks, Natick, MA, USA), extract the peaks of the obtained experimental waveforms using peak detection algorithm.
2. Take the differences between the peaks before perturbation and calculated the average oscillation period T .
3. Find the two peaks t_{pk3} and t_{pk4} between which the perturbation takes place and normalize the time of perturbation to phase $\phi_n = (t_{pk3} - t_{ptb}) / T$ with respect to one unperturbed period.
4. Find the time difference $t_{pk4} - (t_{pk2} + 2T)$ between the second peak after perturbation and the time of the same peak if the perturbation is not present. Normalize this time difference to phase difference and repeat for other waveforms.
5. Plot all phase differences against the phase of perturbation to obtain the PRC. Normalize the PRC with respect to $I_{ptb}h_{ptb} / C_{MEM}$ to obtain the NPRC. (I_{ptb} is obtained from ΔV_{ptb})

Fig. 14. Illustration of the PRC measurement algorithm

Fig. 15 plots the measured membrane potential of the oscillator circuit implemented on FPAA platform. The measured waveform displays close similarity to the simulated membrane potential. Due to the constraint of the capacitor value C_{MEM} , the oscillation frequency is 72 Hz with $V_{REF} = 1.2$ V, $\alpha = -0.165$ V ($V_{LOW} = 1.365$ V) and $\beta = 0.172$ V ($V_{UPP} =$

Fig. 15. Measured oscillatory membrane potential of the FGOTA based implementation.

1.372 V). However, all the results presented are qualitatively similar to those that would be obtained at a lower oscillation frequency of 1 Hz. In terms of dynamical systems, the implemented oscillator is a time scaled version of the desired design. It should be noted that α is negative in this experiment. This is due to a level shift V_{SH} on the output node of the FGOTA as a result of an effective input referred offset voltage V_{OFF} [31] of the transconductor. This effective level shift equals $9V_{OFF}$ due to the capacitive division at the input nodes of the FGOTA. V_{SH} can be calculated by modifying V_{LOW} / V_{UPP} to obtain the same charging and discharging times ($t_{ch} = t_{dis}$) of the oscillation. Taking V_{SH} into consideration, (2) becomes:

$$t_{dis} = RC_{MEM} \ln \left(1 + \frac{\Delta V_{MEM}}{\alpha'} \right) \quad (29a)$$

and

$$t_{ch} = RC_{MEM} \ln \left(1 + \frac{\Delta V_{MEM}}{\beta'} \right) \quad (29b)$$

where α' and β' are the actual values of (3), given by

$$\alpha' = \alpha + V_{SH} \quad (30a)$$

$$\beta' = \beta - V_{SH} \quad (30b)$$

As t_{ch} equals t_{dis} when α' equals β' , the values of V_{SH} and V_{OFF} are calculated to be 0.1685 V and 18.7 mV, respectively.

Experiments have been carried out to verify the dependence of f_{osc} , t_{ch} and t_{dis} on α' and β' . Both experimental and theoretical results are presented in Fig. 16. As the values of the resistor and membrane capacitor are subjected to process variation and parasitic effect, the first points of the measured t_{ch} and t_{dis} in both dependence studies are used to calibrate the value of RC_{MEM} in (29). Fig. 16(a) plots the extracted average f_{osc} , t_{ch} and t_{dis} from the measured oscillation waveforms over a 100 ms run time at different values of α' while β' is kept at 3.5 mV. It can be observed that as α' increases, t_{ch} stays almost constant while t_{dis} decreases as reflected by (29a). Consequently, the oscillation period decreases while f_{osc} increases. Fig. 16(b) plots the same quantities at different values of β' while α' is kept at 3.5 mV. It can be observed that as β' increases, t_{dis} stays constant while t_{ch} decreases as reflected by (29b). As a result, the oscillation period decreases while f_{osc} increases. The measurement results match well with theory in both experiments.

The next experiment measures the PRC of the oscillator with the parameter set for oscillation with 50% duty cycle. Fig. 16(c) plots one cycle of the unperturbed membrane potential profile with the perturbed membrane potentials during the discharging and charging phases from the results of the FPAA measurement. It can be observed that the membrane potentials of the unperturbed case coincide with the perturbed potentials during discharging and charging phases before the first and second arrows, respectively in Fig. 16(c). During the

Fig. 16. Plots of the extracted oscillation frequency, charging and discharging times for various values of (a) α' and (b) β' as well as the measurement results of (c) the unperturbed membrane potential profile together with the perturbed membrane profile during the discharging and charging phase, and (d) the unperturbed membrane potential profile and the calculated PRC. Plots of the (e) H functions and (f) G functions for the three CFs for the oscillator implemented on FPAA.

perturbation (indicated by the two arrows), a positive spike, which increases the membrane potential by an amount ΔV_{PTB} is induced on the membrane capacitor. The oscillation is delayed when the oscillator is perturbed during the discharging phase and advanced during the charging phase. Fig. 16(d) depicts the measured and fitted PRCs plotted together with one oscillation cycle of the membrane potential over which the oscillator is perturbed. It can be observed that the measured PRC is very close to the NPRC at strong perturbation strength in Fig. 10 and agrees with the theoretical calculation of (25) and (26a). The reason for using strong perturbations in the experiment is because it is difficult to measure the effect of weak perturbations in the presence of noise with the limited sampling resolution of our oscilloscope. Figs. 16(e) and (f) show the calculated H and G functions, respectively, for the three CFs based on the measurement data obtained from the FPAA. Similar to the simulation results of the two design approaches, the measured H functions for the three CFs are bipolar.

V. CONCLUSION

A new type of oscillator circuit topology based on the charging and discharging of a capacitor has been proposed. Circuit implementations of the proposed topology based on SC and FGOTA techniques have been presented and discussed. Simulation results show that both the SC and FGOTA resistor based circuit implementations consume similar power as the IF oscillator and halve the power of the differentiator oscillator. Both approaches offer wide range of frequency tuning through programmable resistance value to model different locomotion or other rhythmic behaviors. DS has been used as a tool to analyze the circuit's ability to model HCO and the feasibility to couple this type of oscillator in a chain to achieve synchronization. PRC, H and G functions of both design approaches for this oscillator have been analyzed and compared with the other two existing oscillator designs through SPICE and MATLAB simulations. Our analysis shows that the proposed oscillator topology can produce a PRC closer to the lamprey CPG system and can be used as an artificial HCO. The H and G function analysis results prove the superior capability of our proposed oscillator circuit to achieve fast entrained oscillation with sensory feedback of wider frequency range. Indeed, they can reach synchronization even with higher eigen frequency mismatch in a chain of oscillators. Measurement results from the FGOTA resistor based circuit implementation have proved the basic circuit operation principle of the proposed oscillator topology and verified the circuit's capability to model the whole HCO.

APPENDIX

We plan to use our central pattern generator hardware to drive a swimming robot with a bio-inspired, undulatory fin similar to the ones described in [36], [37]. The undulatory propulsor is typically made of parallel connected finrays covered by a flexible membrane. Each biomimetic fin ray is driven by an actuator and typically a number of degrees of freedom (dof) are involved in the fin motion. The basic

requirement of the CPG for these designs is to have co-ordinated periodic outputs from a set of oscillators that can drive these multi-dof fins. Sensory feedback from the water environment (flow, pressure etc) and the mechanical actuators (e.g., amplitude of deflection) can be used to improve the performance of the system [12]. A generic block diagram for such a system is shown in Fig. 17.

Fig. 17. Generic block diagram of the CPG system.

The diagram is quite generic since we expect our oscillators to be able to work with different sensors and actuators. Typically, the output of the inverters from the oscillator can be processed by the signal conditioning block (e.g., integrator, pulse-width modulator, H-bridge etc.) before feeding it to an actuator. We expect the actuator to move in one direction when the output is high and the other direction when low. Also, the sensory signals might need to be conditioned (e.g., low-pass filter, PID controller) before being applied to the CPG for robust gait generation. Independent of what the actual feedback circuit is, our proposed oscillator will be able to synchronize better due to its higher sensitivity to perturbations compared to the other simple oscillators explored in this paper.

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