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Comparative Study of Electromechanical Impedance and Lamb Wave Techniques for Fatigue Crack Detection and Monitoring in Metallic Structures

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ABSTRACT

Fatigue cracks often initiate at the weld toes of welded steel connections. Usually, these cracks cannot be identified by the naked eyes. Existing identification methods like dye-penetration test and alternating current potential drop (ACPD) may be useful for detecting fatigue cracks at the weld toes. To apply these non-destructive evaluation (NDE) techniques, the potential sites have to be accessible during inspection. Therefore, there is a need to explore other detection and monitoring techniques for fatigue cracks especially when their locations are inaccessible or cost of access is uneconomical.

Electro-mechanical Impedance (EMI) and Lamb wave techniques are two fast growing techniques in the Structural Health Monitoring (SHM) community. These techniques use piezoelectric ceramics (PZT) for actuation and sensing. Since the monitoring site is only needed to be accessed once for the instrumentation of the transducers, remote monitoring is made possible. The permanent locations of these transducers also translate to having consistent measurement for monitoring.

The main focus of this study is to conduct a comparative investigation on the effectiveness and efficiency of the EMI technique and the Lamb wave technique for successful fatigue crack identification and monitoring of welded steel connections using piezoelectric transducers. A laboratory-sized non-load carrying fillet weld specimen is used in this study. The specimen is subjected to cyclic tensile load and data for both techniques are acquired at stipulated intervals. It can be concluded that the EMI technique is sensitive to the crack initiation phase while the Lamb wave technique correlates well with the crack propagation phase.

Keywords: Electromechanical Impedance, Lamb waves, Fatigue crack detection, Crack monitoring, Structural Health Monitoring

1. INTRODUCTION

When a welded steel connection is subjected to cyclic load, fatigue cracks usually initiate at the weld toe. To prevent undesirable consequences when the connection fractures and fails, detection and monitoring of these cracks are crucial. Traditional non-destructive evaluation (NDE) techniques like the dye-penetration test and alternating current potential drop (ACPD) may be useful for detecting fatigue cracks in welded steel connections. However, access to the inspection site has to be made accessible for each inspection cycle. This would inevitably increase the time and cost for inspection of inaccessible sites like the soffit of a bridge for example. Therefore, there is a need to explore remote sensing and active sensing as time and cost savings could be substantial.

Structural Health Monitoring (SHM) of fatigue crack has caught much attention in the recent years^{1 - 8}. Electro-mechanical Impedance (EMI) and Lamb wave techniques are two fast growing techniques in the SHM community. By adopting piezoelectric transducers as actuators and sensors, remote monitoring is made possible. Access to inspection site will only be required for the first time for instrumentation of the transducers. Thereafter, detection and monitoring

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can be conducted at a convenient location, where the length of wires permits. With the on-going research in self-powered sensors and wireless technology, wireless remote monitoring may be the new protocol of future SHM.

This paper investigates the effectiveness and efficiency of the EMI technique and the Lamb wave technique for fatigue crack identification and monitoring of welded steel connections using piezoelectric transducers. A laboratory-sized non-load carrying fillet weld specimen is used in this study. The specimen is subjected to cyclic tensile load and experimental data from both techniques are acquired at stipulated intervals. The damage indicators adopted for each technique increases gradually with increasing load cycles. However, the sensitivity and trend of the damage indicators from the two techniques differs. In comparison, the EMI technique provides earlier warning for the onset of a fatigue crack while the Lamb wave technique has a better correlation with the fatigue crack length.

2. SHM TECHNIQUES USING IN-SITU PIEZOELECTRIC TRANSDUCERS

2.1 Piezoelectric transducers as actuators and sensors

Based on linear theory of piezoelectricity, the constitutive equations, in compact form, can be written as⁹,

$$S_p = s_{pq}^E T_q + d_{kp} E_k \quad (1)$$

$$D_i = d_{iq} T_q + \varepsilon_{ik}^T E_k \quad (2)$$

where S_p is the strain vector, s_{pq}^E is the elastic compliance at constant electric field, T_q is the stress vector, d_{kp} and d_{iq} are the piezoelectric constants, E_k is the electric field vector, D_i is the electric displacement vector and ε_{ik}^T is the dielectric permittivity at constant stress. Equation (1) and Equation (2) describe the converse piezoelectric effect and direct piezoelectric effect, respectively. Actuator applications are based on the converse effects, whereby a strain field is induced in the presence of an electric field. On the other hand, sensors applications are based on the direct effects, whereby a charge is generated in response to a stress field.

Depending on the application and the technique adopted, either the steady state mechanical response of the structure (EMI technique) or the transient wave propagation (Lamb wave technique) information can be used to assess the integrity of the structure. These techniques are discussed in detail in the following sections.

2.2 Electro-mechanical impedance (EMI) technique in damage characterization

The EMI technique couples the mechanical impedance of the structural system with the electrical impedance at the piezoelectric transducer terminals. With the ability to convert mechanical energy into electrical energy (direct effect) and vice versa (converse effect), piezoelectric transducers make a good candidate both as an actuator and as a sensor. By applying an alternating voltage to the piezoelectric transducers, the resulting mechanical oscillations actuate the host structure through the bonding layer. The steady state mechanical response is then captured by the same piezoelectric transducer through a complex electrical admittance output. By varying the frequency, one can then obtain a spectrum of peaks and troughs in the complex admittance that reflects the mechanical resonance spectrum of the host structure. The 1-D model of the electrical admittance was first presented by Liang et al.¹⁰, and is given as,

$$Y = i\omega \frac{w_A l_A}{h_A} \left(\frac{d_{31}^2 \bar{Y}_{11}^E Z_A \tan(kl_A)}{Z + Z_A} + \frac{-T}{\varepsilon_{33}} - d_{31}^2 \bar{Y}_{11}^E \right) \quad (3)$$

where ω is the angular frequency, w_A , h_A and l_A are the width, height and half-length of the PZT respectively, d_{31} is the piezoelectric constant, $\bar{Y}_{11}^E$ is the complex modulus of the PZT at zero electric field, Z and Z_A are the mechanical impedance of the host structure and the PZT respectively, k is the wave number and $\frac{-T}{\varepsilon_{33}}$ is the complex dielectric constant at zero stress field. Equation (3) only considers half of the PZT. Thus a factor of 2 is to be included to represent the electrical admittance for the system being considered.

From the expression given by Equation (3), it can be seen that any change in the mechanical impedance of the host structure would result in a change in the electrical admittance. If all other boundary condition remains constant, a change in mechanical impedance could be contributed by damage in the host structure. It has been illustrated that damage, either a hole or a crack, will cause a reduction in resonance frequency and this change is relative to the degree of damage. Conductance (real part of the admittance) is more sensitive to such changes as compared to the susceptance (imaginary part of the admittance)¹¹. Researchers have also explored the use of this technique to detect and monitor fatigue cracks⁵⁻⁸. However, due to the limitations of the EMI technique, it is only sensitive to damage that is near the PZT and the sensitivity reduces as the distance increases¹².

2.3 Lamb wave technique in damage characterization

Lamb wave is a form of guided wave which propagates through the medium guided by the boundaries. The Lamb wave technique was first utilized by Worlton for damage characterization¹³. In general, there are two modes of Lamb waves, namely the symmetric waves and the anti-symmetric waves. The velocity of these waves is a function of the thickness of the specimen and the frequency.

When a PZT is subjected to electrical (AC) excitation, it undergoes oscillatory expansion and contractions. This oscillatory motion is transferred to the bonding layer and then to the host structure's surface. In other words, the PZT generate Lamb waves in the structure by pinching the surface of the structure¹⁴.

The presence of a crack or defect would result in scattering of the incident Lamb waves. These scattered waves can be categorized as forward-scattered waves and backward-scattered waves. As fatigue crack propagates, the energy of the forward-scattered waves change. This change in energy can be used as a damage indicator to monitor fatigue cracks^{2,15}. For a surface crack or through thickness crack, it has been found that the symmetric waves are more sensitive to the damage¹⁶. Anti-symmetric waves, on the other hand, are more sensitive to delamination damages^{17,18}. Therefore, depending on the damage of concern, one can select a frequency where the mode of interest is the dominant wave mode through tuning of the Lamb wave¹⁴.

3. EXPERIMENTAL INVESTIGATION ON FATIGUE CRACK DETECTION AND MONITORING

3.1 Test Specimen and Experimental Setup

In this paper, the experimental investigation is conducted on a non-load carrying welded steel specimen, as shown in Figure 1. A mean stress of approximately 45% (123.75 MPa) of the yield stress is applied to the specimen. Prior to any cyclic loading, this initial state while holding the load at the mean stress is defined as the "baseline" state in order to simulate the specimen under working conditions. Cyclic axial load is applied using a dynamic testing rig (Schenck HydroPULS PSB) to induce and propagate fatigue crack. Subsequent acquisition is conducted at stipulated load cycles while holding the specimen at mean stress. The amplitude of the alternating stress is 101.25 MPa. This translates to a minimum and maximum stress of 22.5 MPa and 225 MPa, respectively, with a load ratio of 0.1. The frequency of the cyclic load used is 15Hz. The piezoelectric transducers used in this study are PIC151 piezoelectric ceramics (PZTs) from PI ceramics. The PZTs are surface bonded to the specimen using a two part industrial epoxy. The entire experiment setup is shown in Figure 2.

3.2 Electro-mechanical impedance (EMI) technique

An impedance analyzer (Wayne Kerr 6420) is used to send an alternating voltage across the PZT which then transmit a harmonic sine wave to the host structure. At the same instance, the impedance analyzer is also used to acquire the steady state response in the form of complex electrical admittance. The bonding layer and ambient temperature may contaminate the signature for frequency beyond 100kHz^{19,20}. Therefore, the frequency range selected for this study is limited to 100 kHz. A frequency range between 45 kHz to 55 kHz is adopted in this study since there are two prominent peaks located within this range. The resolution used for the frequency sweep is 0.01 kHz.

Figure 1. Sketch of test specimen with piezoelectric transducers

Figure 2. Experimental Setup

3.3 Lamb wave technique

An arbitrary function generator (Tarbor Electronics WW5061) is used to send alternating voltage across the PZT which transmits the actuation signal to the host structure. At the sensor location, a digital oscilloscope (Yokogawa DL1620) is used to capture the signal waveform. At the same instance, the impedance analyzer is also used to acquire the steady state response in the form of complex electrical admittance. A 64-cycles averaging acquisition mode is adopted to reduce the random noise in the signal.

In order to concentrate a large amount of energy at the actuation frequency with minimal side loops, a Hanning windowed sine tone-burst is used. The distance between the actuator and sensor is only 200mm, which is relatively short. Therefore, to prevent the possibility of overlapping signals between the electro-magnetic interference and the arrival of the first wave packet, the time window has to be kept short. On the other hand, if the time window is too short, it will result in a broader bandwidth in the frequency domain and leads to more dispersion. To have a balance between the factors involved, a 5-cycle Hanning windowed sine burst is used as the actuation signal.

Since higher order Lamb waves are present at higher frequency, it is possible to determine the cut-off frequency whereby only the fundamental modes exist, according to the dispersion curves. This is to minimize the signal complications due to the presence of higher order Lamb modes. In this experiment, the actuation frequency is selected at 200 kHz. At this frequency, neither overlapping of electro-magnetic interference nor presence of higher order Lamb waves will occur.

4. CRACK DETECTION AND MONITORING RESULTS

4.1 EMI Technique

Figure 3(a) shows the sensor (PZT2) conductance signature for the test specimen at selected load cycles. In general, a leftward shift in the peak is observed. This observation is in agreement with that illustrated by previous researchers^{7,8}, where the reduction in resonance frequency is used as a damage quantifier in their study on an aluminum specimen. The shift in frequency is plotted against the number of cycles and is presented in Figure 3(b). There is a slight reduction in the resonance frequency in the first 40,000 load cycles, and remains constant until 100,000 load cycles. The early reduction in resonance frequency is caused by a “settling-in” period which will need further investigation. This “settling-in” period is also observed by Ryles et al.²¹ in their study on an aluminum specimen. After the “settling-in” period, the response stabilized and remains constant for the next 60,000 cycles. The constant resonance frequency implies that there is no change in the mechanical response of the system, thus crack is unlikely to have developed. At 120,000 cycles, the resonance frequency started to reduce. At this point, there is a change in the mechanical response of the system, implying damage in the system. This damage could be in the form of micro-cracks which is not visible to the naked eye, but changes the stiffness of the system. Therefore, cracks could have physically developed between 100,000 cycles and 120,000 cycles. Adopting this approach, one can assumed that crack has initiated from the previous load cycle when the current acquisition reveals a reduction in resonance frequency (i.e. 100,000 cycles).

Beyond 120,000 cycles, the resonance frequency reduces gradually (almost linearly) with the load cycles. This observation differs from the understanding of a typical fatigue crack growth curve. For a part-through thickness fatigue crack which eventually breakthrough and developed as a through crack, the rate of crack growth increases as load cycle increase.

Figure 3. (a) Sensor conductance signature at selected load cycles; (b) Leftward frequency shift against number of cycles

4.2 Lamb Wave Technique

Figure 4 shows examples of Lamb wave response at selected load cycles. As load cycle increases, the amplitude of the sensor response decreases in general.

Figure 4. Comparison between baseline response and response at (a) 100,000 cycles; (b) 160,000 cycles; (c) 200,000 cycles; (d) 290,000 cycles

The first wave packet (approx. 40-70 μ s) is the fundamental symmetric (S_0) wave packet, and the subsequent wave packets comprise of fundamental antisymmetric (A_0) wave packet, reflected waves from the sides and ends as well as scattered waves from the fatigue crack¹⁵. It is not the aim of this study to identify these wave packets. However, the information on the scattered waves is embedded in the sensor signal starting from the first wave packet. It may be advantageous to include the subsequent wave packet in the signal processing and damage quantification.

The transient energy within a stipulated time window can be represented by,

$$E = \int_{t_i}^{t_f} s^2 dt \quad (5)$$

where s is the sensor signal waveform, t_i and t_f are the start and end time of the time window respectively. A reduction in the amplitude of the sensor signal waveform implies a reduction in the transient energy. Adopting the same damage index proposed by Lim et al.¹⁵, as,

$$D.I. = \frac{E_0 - E_n}{E_0} = \frac{\int_{t_i}^{t_f} s_0^2 dt - \int_{t_i}^{t_f} s_n^2 dt}{\int_{t_i}^{t_f} s_0^2 dt} \quad (6)$$

where t_i is the start of the time window, which is the start of the first wave packet, and t_f is the end time of the time window, respectively. Here, a time window of 38-130 μ s is adopted and the result is shown in Figure 5. Similar to the EMI technique, there is an increase in the damage index during the first 40,000 cycles, which is attributed to the “settling-in” period, before it stabilise over the next 60,000 cycles. However, due to the sensitivity to noise, this damage index could fluctuate even though no damage is present. To account for this fluctuation, a repeatability study was carried out at 40,000 cycles. Sixteen readings were acquired and the mean and standard deviation are determined as 0.18 and 0.044 respectively. Assuming a normal distribution and a 95% confidence limit, the upper bound is determined as 0.25. In other words, if the data point falls below the upper bound (indicated by the dashed line), we are 95% confident that no crack has occurred. Therefore, this upper bound defines the “undamaged-zone”. Here, the first point beyond the undamaged zone is at 140,000 cycles. This implies that crack could have initiated between 120,000 cycles and 140,000 cycles. Using this crack detection algorithm, crack is assumed to have occurred at 120,000 cycles.

Figure 5. Damage index vs number of cycles

Beyond 140,000 cycles, there is still some fluctuation of data before it eventually increases with the load cycles. This observation reflects the fatigue crack growth correctly since the rate of change increases with load cycles, as defined by Paris equation.

5. DISCUSSIONS

Since the “settling-in” period occurs well before the onset of a fatigue crack, this information can be removed and data normalized for better comparison. Figure 6 shows a comparison between the EMI technique and the Lamb wave technique.

Figure 6. Normalized damage quantifier vs fatigue life

Adopting the stress intensity factor proposed by Newman²², the theoretical crack growth curve can be determined when the Paris equation²³ is used to define the crack growth rate. The parameters used in the equations are given in Table 1. The Paris coefficient, C , have to be modified using the Walker equation²⁴ to account for the stress ratio of 0.1. After modification, the Paris coefficient, C , is determined as 1.495×10^{-10} m/cycle. Crack is assumed to have initiated when the surface crack ($2c$) is 1mm. By matching the end of theoretical fatigue life with the experimental value (293,802 cycles), the surface crack of 1mm could have been initiated at 99,540 cycles. The theoretical crack growth curve is then normalized and plotted in Figure 6 for easy comparison.

Table 1. Parameters for calculation of fatigue crack growth curve.

Parameter	Value	Reference
C ($R=0$)	0.660×10^{-8} in/cycle [1.356×10^{-10} m/cycle]	Boyer ²⁵
m	2.25	Boyer ²⁵
a_0	0.34mm	Exp. [Figure 7]
c_0	0.50mm	---

Figure 7. Experimental value for a/c

The EMI technique provides an earlier indication that a crack has developed. The crack detected at 100,000 cycles is also closer to the theoretical load cycle of 106,200 cycles. This observation further enhances the fact that EMI is sensitive to incipient damage, even if it is not visible to the naked eye. However, beyond 100,000 cycles, it is obvious that the EMI technique does not provide the same trend as the theoretical crack growth curve. In fact the rate of increase is almost linear instead of an increasing rate of increase.

In comparison, the Lamb wave technique is not as sensitive in detecting the crack formation, but has a better agreement with the trend of the theoretical crack growth curve. Figure 8 shows the correlation between the theoretical surface crack length and the damage indicator. The shift in frequency is taken with reference from the undamaged state (i.e. 0.05kHz) and the change in damage index is taken with reference from the mean value (i.e. 0.18). The Lamb wave technique gives a better R^2 -value for either a linear curve fitting or a 3rd order polynomial curve fitting. This also helps to illustrate the fact that Lamb wave technique has a better correlation with the surface crack length. Therefore, based on the damage indicator used in this study, the Lamb wave technique may be a better candidate for fatigue crack growth monitoring.

Figure 8. Surface crack length vs damage indicator

6. CONCLUSIONS

In this paper, a comparative investigation is conducted on the effectiveness and efficiency of the EMI technique and the Lamb wave technique for fatigue crack identification and monitoring of welded steel connections using piezoelectric transducers. With the EMI technique, resonance peaks shift to the left in general. This implies a change in the mechanical stiffness of the system. The change increases gradually as the number of cycles increases. For Lamb wave technique, the amplitude of the sensor waveform decreases with the load cycles.

A laboratory-sized non-load carrying fillet weld specimen is used in this study. The specimen is subjected to cyclic tensile load, and the data for both techniques are acquired at stipulated intervals. Observation shows that the EMI technique provides earlier indication of a crack which is closer to the theoretical value. Thus, the EMI technique is a better candidate for fatigue crack detection. However, based on the current definition of the damage indicator, the EMI technique does not seem to reflect the surface crack length well enough. In comparison, the Lamb wave technique has a better correlation with the surface crack length based on the current damage indicator. It is therefore preferred over the EMI technique for monitoring of fatigue crack growth.

This study provides a better understanding to the limitations of the current available EMI and Lamb wave techniques. With the observations and findings from this study, an investigation on the feasibility of a hybrid EMI/Lamb wave technique may be conducted for effective integrated damage detection and monitoring of fatigue cracks.

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