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Analysis of nonuniform field emission from a sharp tip emitter of Lorentzian or hyperboloid shape

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For a sharp tip emitter, due to the non-uniform emission feature and the electron beam expansion in the vacuum, it is difficult to precisely determine the average field enhancement factor β_c as well as the effective emission area S_{eff} for a single field emitter. In this paper, we conduct a numerical experiment to simulate the electron field emission from a sharp tip emitter (Lorentzian or hyperboloid shape). By collecting the emission current I_{tot} at the finite anode area S_{tot} , we establish the criteria in using Fowler-Nordheim plot to estimate both β_c and S_{eff} , which agree well with our initial emission condition. It is found that the values of β_c and S_{eff} depend on the emitter's properties as well as the size of the anode area S_{tot} . In order to determine the precise value of β_c , S_{tot} must be large enough to collect all the emitted electrons from the sharp tip (e.g., I_{tot} reaches maximum). As an example, a Lorentzian type emitter with an aspect ratio of 10 (height over width), the effective enhancement factor is about $\beta_c = 33$ as compared to the maximal enhancement of 35 at the apex. At similar maximal enhancement factor at the apex ($=360$), both types of emitters will give different average field enhancement dependent on the collecting area. The extension of this simple model to a statistical more complicated model to simulate field emission from a cathode consisting of many field emitters is also briefly discussed. This paper should be useful to analyze and characterize field emission data together with experimental measurement. © 2013 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4798926>]

I. INTRODUCTION

The field emission (FE) process is a type of electron emission, which involves the quantum tunneling of electrons from a surface to vacuum subjected to a strong applied electrical field to lower the surface potential barrier. FE based cathode has been developed to be an alternative choice (compared to thermionic cathode) to produce electron beam required for many applications including coherent radiation sources¹⁻³ or as point sources of electrons for applications in high resolution electron microscopy.^{4,5}

The FE process is commonly described by the well-known Fowler-Nordheim (FN) law⁶ given by

$$J = A\beta^2 E_0^2 \exp\left(-\frac{B}{\beta E_0}\right), \quad (1)$$

where $A = 1.54 \times 10^{-6} \exp(10.4/\sqrt{\phi})/\phi$, $B = 6.44 \times 10^9 \phi^{3/2}$ and ϕ is the material work function in eV, β is the surface enhancement factor of the sharp tip and is a function of the emitter geometry, and $E_0 = V_g/D$ is the applied electric field with gate voltage V_g and gap spacing D . This equation implies that the amount of emitted current density at any position on a given emitting surface is only dependent on the local electric field given by $E_s = \beta E_0$.

To understand the property of field emission, the FN law has been widely used to analyze the experimentally measured field emission data by plotting the FN plot in the form of

$$\ln \frac{J_{ave}}{E_0^2} = \ln(A\beta^2) - \frac{B}{\beta E_0}, \quad (2)$$

where $J_{ave} = I_{tot}/S$ is the average emitting current density, I_{tot} is the total emitted current, and S is the emission area at the cathode. By measuring the total current I_{tot} for a given area S on the anode as a function of E_0 , followed by plotting $\ln(J_{ave}/E_0^2)$ vs $1/E_0$, we will be able to determine the gradient value of $-B(\phi)/\beta$, which is a function of ϕ and β . Specifically for a perfectly flat emitting surface with $\beta = 1$, it is straight forward that both the areas of the emitting surface (at cathode) and collecting surface (at anode) are equal, so that the slope data will directly give the value of ϕ .

If the cathode is consisting of many sharp emitters (like a field emitter array) or thin films of many sharp features, it will be difficult to use FN plot to characterize the electron field emission data as the analysis will involve all three uncertain parameters, namely emitting area (S), work function (ϕ), and enhancement factor (β). In this case, one has to always control or determine 2 out of the 3 parameters from other methods before the analysis. For example, we may solve electrostatic equation to obtain β of a sharp tip, and assume certain emission area based on the sharpness of the tip, and thus, be able to determine ϕ from the FN plot. We can also use first principal calculation to determine ϕ first and to determine β from FN plot, or we can use the above computational methods to compute ϕ and β , and to use FN plot to determine S . The problem becomes more complicated if the emitters are not uniform that we will need a careful statistical model to express the 3 parameters in their average

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values. The complexity of this issue may explain why there is a large amount of reported β from using FN plot to characterize the experimental field emission data over a wide range of sharp emitters. Some of the large field enhancement factor with $\beta > 1000$ is even physically not possible according to electrostatic models.

Even for the simplest case of a single sharp emitter, where the field enhancement β is not uniform over the protrusive surface, there is a need to define an average field enhancement factor β_{ave} . From the experiment data, in order to determine β_{ave} , we need to first measure the total emission current I_{tot} at the anode of a finite area S_{tot} . After that, we can calculate the average emission current density $J_{ave} = I_{tot}/S_{tot}$ at the corresponding applied electric field E_0 , and use the FN plot [see Eq. (2)] followed by fitting the gradient $-B/\beta_{ave}$ to obtain β_{ave} for a given ϕ . Note that according to the FN law, the real average emission current density should be expressed as I_{tot}/S_{sur} , where S_{sur} is the actual emission area on the cathode. However, in the real experiment, S_{sur} is not directly measurable for a sharp tip emitter, and it is common to use the collecting area S_{tot} for the analysis.

It is clear that the magnitude of J_{ave} will strongly depend on the anode area S_{tot} , which can actually be chosen arbitrarily in the experiment. For a sharp tip emitter, due to the nonuniform emission feature as well as the beam expansion in the vacuum, S_{tot} could be much larger than S_{sur} . As a result, the gradient and the interpolation of the linear curve ($\ln J_{ave}/E_0^2$ vs $1/E_0$) could be affected, and the subsequent fitting values might be inaccurate. Thus, the consistency of the results will need a careful review on the values of S_{tot} to be used, which is expected to be dependent on the geometrical shape of the emitters. Further questions like if it is correct to use S_{tot} to replace S_{sur} in calculating J_{ave} , and what is the relation between β_{ave} and S_{tot} , will also require some deep investigation.

In this paper, we will perform a *numerical* experiment to obtain the criteria of using FN law to characterize the field emission from a sharp emitter. We consider a sharp emitter with well-defined shape and known work function, and the electrons are emitted according to the FN law from the cathode to the anode. By collecting the current on the anode, we will perform the analysis based on FN law and investigate the variation of β_{ave} with respect to various geometrical settings, such as gap spacing D and anode area S_{tot} . The detailed models for 2 types of emitters used in this paper, namely, Lorentzian shape and hyperboloid shape are presented, respectively, in Secs. II and III. Further analysis and discussions are shown in Sec. IV. Finally, concluding remarks are presented in Sec. V. For completeness, a list of assumptions and limitations are provided in a supplementary document.⁷

II. LORENTZIAN SHAPE EMITTER

A 3D schematic of the Lorentzian shape emitter is illustrated in Fig. 1(a), which is assumed to be infinitely long in the z -direction. A rectangular anode with finite width of $2L$ is proposed to collect the emission current. It is a classical model commonly used to simulate the actual wedge shape emitters formed by GaAs,⁸ GaN,⁹ silicon,¹⁰ or organic material,¹¹ and can provide simple but good estimation on the

FIG. 1. (a) The 3D schematic of a Lorentzian shape emitter. (b) The cross-section schematic of the x - y plane.

field distribution and surface enhancement profile.¹² Nevertheless, it is designed to calculate the fringe fields in the immediate vicinity of the emitter surface, and completely ignore any defect, irregularity, and space charge effect. Thus, the fluctuations in current at constant voltage over time due to charge sputtering and the migration and generation of nanoprotusions will not be included.¹³

The cross-section of x - y plane is shown in Fig. 1(b), where h is the height of the emitter in the y -direction and w is the full-width-at-half-maximum in the x -direction. The apex of the emitter is located at $(x=0, y=h)$, and the planar cathode and anode is, respectively, at $y=0$ and $y=D$. In the figure, L is the corresponding half anode width to collect the emission line current I_z (A/m), and $S_L = \int ds$ is the corresponding *maximum* arc length on the emitter surface for which beyond the electrons emitted will be outside the collecting area. The collecting area on the anode and emitting area on the cathode are defined, respectively, as $S_{tot} = 2L \cdot z$ and $S_{sur} = 2S_L \cdot z$. The total collected current on the anode is $I_{tot} = I_z \cdot z$.

The surface enhancement profile as well as the vacuum field distribution is expressed in Eqs. (3) and (4), respectively,¹² given by

$$\beta = \frac{\sqrt{X_0^2 + (Y_0 + \epsilon)^2}}{\{[1 - (Y_0 + \epsilon)^2 + X_0^2]^2 + [2X_0(Y_0 + \epsilon)]^2\}^{1/4}}, \quad (3)$$

$$E_x + iE_y = \frac{-iE_0[x - i(y + b)]}{\{[a^2 - (y + b)^2 + x^2] - 2ix(y + b)\}^{1/2}}, \quad (4)$$

where $a \approx h + \frac{w}{3.464}$, $b \approx \frac{w}{3.464}(1 + \frac{1}{3.464h/w})$, $\epsilon = b/a \approx w/3.464h$, and (X_0, Y_0) are the normalized coordinates of the emitter surface (see Eqs. (5a) and (5b) in Ref. 12). Note that the above equations are only valid for $h/w > 1$.¹⁴

With the known surface enhancement profile, we could easily calculate the emission current density $J(X_0, Y_0)$ at every point of the emitter surface (X_0, Y_0) by using Eq. (1) at a given applied electric field E_0 . Once an electron emitted into the vacuum, we can simply track the electron trajectory (x, y) by applying Eq. (4) and Newton's law as in our previous work¹⁵ until it reaches anode at $y = D$. The electrons are considered collected if they are able to fall inside the finite anode area of $2L$ in length bounded between $(x = -L, y = D)$ and $(x = L, y = D)$. The total emission line current I_z that can be collected by the anode is given by

$$I_z = 2 \int_0^{S_L} J \cdot ds. \quad (5)$$

Note that throughout the paper, we keep the work function at $\phi = 4$ eV and $h = 100$ nm $\ll D$.

Since it is impossible to directly measure the actual emission area S_{sur} on the emitter surface, we define an experimentally measured average current (based on the collecting area on the anode), which is

$$J_{ave} = \frac{I_{tot}}{S_{tot}} = \frac{I_z}{2L} = \frac{\int_0^{S_L} J \cdot ds}{L}. \quad (6)$$

The average enhancement is then obtained by plotting $\ln(J_{ave}/E_0^2)$ with respect to $1/E_0$ and fitting the gradient. Due to the beam expansion, L (or S_{tot}) might be much larger than the real emission length S_L (or S_{sur}). As a result, J_{ave} could be much smaller than the real average emission current density emitted from the cathode given by

$$J_{sur} = \frac{I_{tot}}{S_{sur}} = \frac{I_z}{2 \int ds} = \frac{\int_0^{S_L} J \cdot ds}{\int ds}. \quad (7)$$

Thus, it is first necessary to check the validity of replacing S_{sur} with S_{tot} by comparing the β_{ave} results obtained from J_{ave} and J_{sur} as a function of L for a case that $h/w = 10$ and $D = 10$ $\mu\text{m} \gg h = 0.1$ μm .

Apparently from Fig. 2(a), L is much larger than S_L (the beam expansion is significant), and S_L increases as L becomes larger. However, in Fig. 2(b), the calculated β_{ave} values based on J_{ave} (solid line) and J_{sur} (dashed line) agree very well, which imply that it is valid to use S_{tot} and its correspond J_{ave} to estimate the property of the emitter such as β_{ave} . We further confirm that the gradient of the FN plot is rather independent of the applied field E_0 . The error is less than 4% within the studied range of E_0 , and this clearly

FIG. 2. Lorentzian shape emitter at $h/w = 10$: (a) The relation between the real emission area S_L and the anode area L . (b) The average enhancement β_{ave} obtained by using J_{ave} (solid line) and J_{sur} (dashed line).

indicates that the linear relation of $\ln J/E_0^2$ and $1/E_0$ is strictly held in our numerical experiment.

Theoretically for a fixed set of h , w , and D , the *real* sharpness or average enhancement of the emitter defined as β_c should not vary with the anode property L . Thus, it is important to know the critical L (to be controlled by the experimental setting) where its corresponding β_{ave} determined by the FN plot shown in Fig. 2(b) is the *real* sharpness β_c . In general, this critical value of L mainly depends on the gap spacing D . For a larger D , the electron beam will expand further and we need larger L to collect all the electrons emitted from a given emitting surface S_L in order to collect the same amount of total current. Fig. 3(a) illustrates the emission current I_z as a function of L at $h/w = 10$ and $E_0 = 10^8$ V/m. As expected, with increasing D (from left to right = 1 to 10 μm), larger L is required to collect the same amount of current I_z . Since most of the current concentrates near the vicinity of the tip ($\beta \gg 1$) and the emission from the tail ($\beta \approx 1$) can be omitted, the maximum values of I_z are the same regardless the gap spacing D as long as L is large enough. A critical value L_c (labeled in the figure) is defined at which the emission current I_z reaches maximum (all electrons have been collected). For example, we have $L_c \approx 0.13$, 0.38, and 0.57 μm for $D = 1$, 5, and 10 μm , respectively.

The corresponding β_{ave} result at fixed $h/w = 10$ for different D is shown in Fig. 3(b). It is clear that as L approaches to zero, the collected current I_z is mainly from the apex of the emitter, thus, we have $\beta_{ave} \approx \beta_{max} = 34.654$, which is the localized field enhancement factor at the emitter apex ($x = 0, y = h$) determined by Eq. (3). From this figure, we

FIG. 3. Lorentzian shape emitter: (a) The collected current I_z for various gap spacings D at fixed $h/w = 10$ and $E_0 = 10^8$ V/m, where L_c is the critical anode length at which I_z reaches maximum. (b) The corresponding β_{ave} result, where β_c is the true average enhancement located at L_c . (c) The comparison of β_{ave} obtained by fitting the gradient (solid line) and interpolation (dashed line).

could easily locate $\beta_{ave} = \beta_c \approx 33.37$ at the critical length $L_c \approx 0.13, 0.38$, and $0.57 \mu\text{m}$ for $D = 1, 5$, and $10 \mu\text{m}$, respectively. This value is the *real* average enhancement β_c for $h/w = 10$ and it is independent of the gap spacing. Note that these critical L_c are the *minimum* required anode sizes that one should use in the experiment in order to determine the *real* field enhancement β_c . Thus, as long as the value of L is greater or equal than L_c , the estimated $\beta_{ave} \approx \beta_c$ will be valid, and we shall elaborate this further in Sec. IV.

In Fig. 3(c), we compare β_{ave} results obtained by fitting the gradient $-B/\beta_{ave}$ with the result obtained by fitting the interpolation $\ln A \beta_{ave}^2$ at $D = 5 \mu\text{m}$. The comparison shows that the interpolation fitting result (dashed line) is much smaller than the gradient fitting result (solid line). This is simply because the empirical average current density J_{ave} is much smaller than the real emission current density J_{sur} due to the beam expansion. As a result, the linear curve plotted by $\ln J_{ave}/E_0^2$ and $1/E_0$ will significantly shift downwards and the interpolation data are no longer correct. Hence, we conclude that for a sharp tip emitter, the enhancement value can only be obtained by fitting the gradient.

In Fig. 4(a), we show the calculated results at fixed $D = 5 \mu\text{m}$ and $E_0 = 10^8$ V/m for different sharpnesses $h/w = 5, 10$, and 15 (bottom to top) with corresponding $\beta_{max} = 17, 35$, and 52 at the emitter apex. Due to larger field enhancement at high h/w , we have more emitted current and thus higher I_z . Similarly to Fig. 3(a), I_z will saturate at large anode area L that gives us the critical L_c at about $0.34, 0.38$, and

FIG. 4. Lorentzian shape emitter: (a) The collected current I_z for various h/w at fixed $D = 5 \mu\text{m}$ and $E_0 = 10^8$ V/m, where L_c is the critical anode length at which I_z reaches maximum. (b) The corresponding β_{ave} result, where β_c is the true average enhancement located at L_c . (c) The comparison of β_{ave} obtained by fitting the gradient (solid line) and interpolation (dashed line).

$0.39 \mu\text{m}$ for $h/w = 5, 10$, and 15 . Fig. 4(b) shows the corresponding β_{ave} by gradient fitting, and the *real* field enhancements are about $\beta_c = 17, 33$, and 49 for $h/w = 5, 10$, and 15 , respectively. In Fig. 4(c), the β_{ave} results using the gradient fitting (solid line) and interpolation fitting (dashed line) at $h/w = 5$ are compared, which implies that the beam expansion is still significant even for a relatively flat emitter ($h/w = 5$).

After obtaining β_c , we could directly calculate the effective emission width defined as $S_{eff} = I_z/J_c$, where J_c is calculated by substituting β_c into Eq. (1). For the Lorentzian shape emitter, the effective emission width S_{eff} depends on the emitter sharpness characterized by h/w and independent of the gap spacing D as long as $D \gg h$. For example, we have $S_{eff} \approx 0.149, 0.0515$, and 0.028 nm for $h/w = 5, 10$, and 15 , respectively.

III. HYPERBOLOID SHAPE EMITTER

In this section, we will repeat the same approach for the other type of emitter known as hyperboloid emitter, which has a cylindrically symmetry about the z -axis as illustrated in Fig. 5(a). A circular anode with finite radius R is proposed to collect the emission current. Together with the prolate-spheroidal system, this model is able to provide a simple analytical expression of electrical field distribution for Spindt-type and ellipsoid shape nanostructures.¹³ Based on

FIG. 5. (a) The 3D schematic of a hyperboloid shape emitter. (b) The cross-section schematic along the central axis.

the STM and TEM images,^{16,17} it shows a much better idealization of the actual emitter structure in contrast to the earlier spherical model.¹⁸ Therefore, it is widely used to simulate the nanorod or nanotube tips such as crystal fiber,¹⁹ carbon nanotube,²⁰ and metal/metal-oxide nanotip.^{21,22} However, in this model, the microtip is treated as a smooth hyperboloid surface,²³ and hence it may not be accurate enough to estimate an extremely atomic-sharp tip, e.g., TiO₂ nanotip in Refs. 24 and 25, which requires to handle the singularity at the emitter apex.

The cross-section along the central axis is shown in Fig. 5(b), where the apex of the emitter and the planar anode are located, respectively, at $r=0, z=D$ and $z=0$. Here, $D = a \cos \theta$ is the distance between the emitter apex and anode, a is the half-foci distance and θ is the tip half angle. R is the finite anode radius to collect the emission current I_{tot} , and (r_m, z_m) is the location on the emitting surface for which beyond the electrons emitted will be outside the collecting area $r=R$ at the anode. Thus, the collecting area on the flat anode and the actual emitting area on the cathode are, respectively, $S_{tot} = \pi R^2$ and $S_{sur} = 2\pi \int_D^{z_m} r(z) \cdot dz$.

The field enhancement profile on the emitting surface, and vacuum potential distribution are given by²³

$$\beta^{-1} = Q_0(\cos \theta) \sqrt{\xi^2 - \cos^2 \theta} \tan \theta, \quad (8)$$

$$V(\eta) = V_0 \frac{\ln[(1+\eta)/(1-\eta)(1-\eta_1)/(1+\eta_1)]}{\ln[(1+\eta_2)/(1-\eta_2)(1-\eta_1)/(1+\eta_1)]}, \quad (9)$$

where $Q_0(\cos \theta) = (1/2) \ln[(1+\cos \theta)/(1-\cos \theta)]$ is the Legendre polynomial of second kind, $\eta_1 = \cos \theta$ is the cathode surface, and $\eta_2 = 0$ is the planar anode. By combining Eqs. (1) and (8), we can calculate the emission current density $J(r_0, z_0)$ from any point (r_0, z_0) on the emitter surface. By taking the derivative of Eq. (9), we obtain the vacuum electric field, and use it together with equation of motion to calculate the electron trajectory (see Fig. 5 in Ref. 15) and to determine whether it can be collected by the anode area S_{tot} . As a result, the overall emission current I_{tot} collected by the anode is expressed as

$$I_{tot} = 2\pi \int_D^{z_m} J[r(z), z] \cdot r(z) \cdot dz. \quad (10)$$

Due to the beam expansion, the anode area S_{tot} might be much larger than the real emission area S_{sur} , we need to compare the average current density emitted from the surface J_{sur} and its counter part collected on the anode J_{ave} , which are given by

$$J_{sur} = \frac{I_{tot}}{S_{sur}} = \frac{\int_D^{z_m} J[r(z), z] \cdot r(z) \cdot dz}{\int_D^{z_m} r(z) \cdot dz}, \quad (11)$$

$$J_{ave} = \frac{I_{tot}}{S_{tot}} = \frac{2 \int_D^{z_m} J[r(z), z] \cdot r(z) \cdot dz}{R^2}. \quad (12)$$

To check the validity of using J_{ave} , instead of J_{sur} , to fit the average enhancement β_{ave} , we compare β_{ave} obtained by J_{ave} (solid line) and J_{sur} (dashed line) at $D=10 \mu\text{m}$ and $\cos \theta = D/a = 0.99523$. Fig. 6(a) shows the relationship between S_{tot} and S_{sur} , which illustrates significant beam expansion ($S_{sur} \ll S_{tot}$), while Fig. 6(b) shows good agreement and confirms that it is valid to use J_{ave} to find the field enhancement factor. Note that the maximum enhancement is $\beta_{max} = 34.6654$ when S_{tot} approaching zero (as expected).

Fig. 7(a) illustrates the total emission current I_{tot} with respect to the anode area S_{tot} for various gap spacings $D = 1, 5$, and $10 \mu\text{m}$ (bottom to top) at fixed sharpness $\cos \theta = 0.99523$ and $E_0 = 10^8 \text{ V/m}$. Similarly, we can define a critical anode area $S_c = \pi R_c^2$ [labeled in the figure] where the total current I_{tot} reaches maximum and this will be the minimum anode area to be used in the experiment. For example, we have $S_c [\text{m}^2] = 2.01 \times 10^{-12}, 4.18 \times 10^{-11}$, and 2.55×10^{-10} for $D = 1, 5$, and $10 \mu\text{m}$, respectively. The corresponding β_{ave} is illustrated in Fig. 7(b) by gradient fitting. At small S_{tot} , we have $\beta_{ave} \approx \beta_{max} = 34.6654$ which equals to the maximum enhancement on the emitter apex. The real average enhancement β_c could be determined at the corresponding S_c , which gives the same $\beta_c \approx 32.05$ [as labeled in the figure]. Note that β_c only depends on the geometrical sharpness

FIG. 6. Hyperboloid shape emitter at $\cos \theta = 0.99523$: (a) The relation between the real emission area S_{sur} and the anode area S_{tot} . (b) The average enhancement β_{ave} obtained by using J_{ave} (solid line) and J_{sur} (dashed line).

characterized by $\cos \theta$, and independent on the gap spacing. Fig. 7(c) compares the β_{ave} results by gradient fitting (solid line) and interpolation fitting (dashed line).

Similar to the calculation done for the Lorentzian shape, we see that I_{tot} will saturate at large S_{tot} because most electrons are emitted from the region near the emitter apex. However, for the hyperboloid shape [see Fig. 7(a)], the maximum value of I_{tot} significantly increases with larger gap spacing from $D = 1$ to 5 and 10 μm . In comparison, the Lorentzian shape always has a *same* saturation current which is independent on D [see Fig. 3(a)]. The difference is due to the respective mathematical models used to describe the sharpness of the emitter, which may or may not be sensitive to D . For Lorentzian shape, the enhancement profile is determined by the ratio of h/w , and it is independent of D [see Eq. (3)]. Once h/w is fixed for a given h , the sharpness of emitter is determined, and we can study the effects of gap spacing simply by changing D without changing the sharpness of the emitter. Thus, we will have the same maximum current at different D . For the hyperboloid shape, however, the field enhancement factor given by Eq. (8) is governed by both gap spacing D and the half-foci length a (through $\cos \theta = D/a$ relation). In order to maintain the same sharpness at larger D , we need to increase a accordingly, which will increase the emission current due to larger emitting area. As a result, the emission current will increase with D at fixed sharpness for hyperboloid emitter and explains the difference between Figs. 3(a) and 7(a).

FIG. 7. Hyperboloid shape emitter: (a) The collected current I_{tot} for various gap spacings at $\cos \theta = 0.99523$ and $E_0 = 10^8 \text{ V/m}$, where S_c is the critical anode area at which I_{tot} reaches maximum. (b) The corresponding β_{ave} result, where β_c is the true average enhancement located at S_c . (c) The comparison of β_{ave} obtained by fitting the gradient (solid line) and interpolation (dashed line).

Fig. 8(a) depicts the emission current I_{tot} as a function of anode area S_{tot} at $E_0 = 10^8 \text{ V/m}$ and $D = 5 \mu\text{m}$ for different sharpnesses (bottom to top) given by $\cos \theta = 0.9889$, 0.99523, and 0.99704, where their corresponding maximum field enhancements are $\beta_{max} = 17.2873$, 34.6654, and 51.7668, respectively. As the emitter becomes sharper ($\cos \theta$ increases), the emission current will increase due to larger field enhancement [see Eq. (8)]. The critical emission area S_c [m²] are found to be 3.63×10^{-11} , 4.18×10^{-11} , and 4.78×10^{-11} . The calculated β_{ave} is shown in Fig. 8(b), and we could determine the *real* average enhancement β_c at $S_{tot} = S_c$, which gives $\beta_c = 16.62$, 32.05, and 46.21, respectively. Figure 8(c) compares the β_{ave} obtained from gradient fitting (solid line) and interpolation fitting (dashed line) at $\cos \theta = 0.9889$, which again confirms that the later approach is not accurate.

Lastly, we calculate the effective emission area defined as $S_{eff} = I_{tot}/J_c$, where J_c can be obtained by substituting β_c into Eq. (1). For $\cos \theta = 0.9889$, 0.99523, and 0.99704 at $D = 5 \mu\text{m}$, we have $S_{eff} [\text{m}^2] = 9.60 \times 10^{-15}$, 3.78×10^{-15} , and 2.30×10^{-15} , respectively. However, unlike the Lorentzian shape where S_{eff} is independent on the gap spacing D , due to the change in the geometrical size (the half-foci length a varies with respect to D to maintain the enhancement profile), $S_{eff} [\text{m}^2] = 1.52 \times 10^{-16}$, 3.78×10^{-15} , and 1.52×10^{-14} for different $D = 1$, 5, and 10 μm at fixed sharpness $\cos \theta = 0.99523$.

FIG. 8. Hyperboloid shape emitter: (a) The collected current I_{tot} for various emitter geometries at $D = 5 \mu\text{m}$ and $E_0 = 10^8 \text{ V/m}$, where S_c is the critical anode area at which I_{tot} reaches maximum. (b) The corresponding β_{ave} result, where β_c is the true average enhancement located at S_c . (c) The comparison of β_{ave} obtained by fitting the gradient (solid line) and interpolation (dashed line).

IV. DISCUSSION

From previous analysis in Secs. II and III, we have illustrated that the collected current I_{tot} and the resultant average enhancement β_{ave} depend on the anode area S_{tot} [see Figs. 3(a) and 7(a)]. By defining the critical anode area (L_c for the Lorentzian shape and S_c for the hyperboloid shape) at which I_{tot} reaches maximum, we conclude that it is the minimum anode area which is required for the experimental set up to collect nearly all the emitting current, so that the FN analysis for the emitter will be valid. If the anode area S_{tot} is smaller than the critical value S_c , substantial part of the emitted current is not collected and the corresponding field enhancement result will not be accurate. If $S_{tot} \geq S_c$, according to the FN plotting given by $\ln(J_{ave}/E_0^2) = \ln[(I_{tot})/(S_{tot} \cdot E_0^2)] = \ln I_{tot} - \ln S_{tot} - \ln E_0^2$, the second term (a constant) will be cancelled out in the calculation of the gradient based on the FN plot, and thus, will not influence β_c as long as the first term accounts for all the emitted current. For example, β_c remains unchanged when the anode length (or area) is larger than L_c (or S_c) as shown in Fig. 3(b) (or Fig. 7(b)). Hence this finding suggests that researchers should use a sufficiently large anode area ($\geq S_c$ calculated here) to collect all the current emitted from a sharp emitter due to beam expansion.

Based on the calculated β_c , it is worth to compare the difference between the Lorentzian and hyperboloid emitter. From the comparison, we observe that with the same apex

FIG. 9. The comparison of the β_{ave} results between the Lorentzian shape (solid line) and the hyperboloid shape (dashed line) at $\beta_{max} \approx 361$ and $D = 5 \mu\text{m}$, as a function of anode length L (Lorentzian shape) or anode area $S_{tot} = \pi R^2$ (hyperboloid shape). This maximum $\beta_{max} \approx 361$ corresponds to $h/w = 100$ for Lorentzian shape and $\cos \theta = 0.99968$ for hyperboloid shape.

enhancement β_{max} , Lorentzian shape always has a slightly larger β_c than the hyperboloid shape. At $\beta_{max} \approx 17, 35$, and 52 , we have $\beta_c \approx 17, 34$, and 49 for the Lorentzian shape, and $\beta_c \approx 16, 32$, and 46 for the hyperboloid shape. This is because the field enhancement decreases faster away from the apex for Lorentzian shape emitter as compared to the hyperboloid field emitter (see Figs. 1 and 4 in Ref. 15), so the emission current is more concentrated near apex, which will produce a higher β_c . This effect becomes more dominant for sharper tips as shown in Fig. 9, which compares the two shapes at large $\beta_{max} \approx 361$ and $D = 5 \mu\text{m}$. Clearly, we have $\beta_c = 264$ at $L_c \approx 0.7 \mu\text{m}$ for the Lorentzian shape (solid line), which is larger than the hyperboloid shape where $\beta_c = 207$ at $S_c = 2.011 \times 10^2 \mu\text{m}^2$ (dashed line).

With experimental data, this model presented here is convenient to formulate a sharp emitter as a flat surface model with an effective emission area S_{eff} and an enhancement factor β_c . First, we perform an experiment to measure the total collected current I_{tot} at the anode with a size larger than the critical area discussed above. From the obtained I_{tot} , we can use the FN plot to estimate the critical field enhancement factor β_c and the effective emission area $S_{eff} = I_{tot}/J_c$. Thus, the field emission from a sharp emitter can be treated as a flat emission model governed by

$$I_{tot} = S_{eff} \cdot A \beta_c^2 E_0^2 \exp\left(-\frac{B}{\beta_c E_0}\right), \quad (13)$$

where S_{eff} and β_c are known quantities. Finally, this simple model can be further used to calculate emission current at any arbitrary applied field E_0 without performing additional experiment.

We speculate that this simple model may be useful to create a statistical model for a field emission cathode containing of many field emitters,^{26,27} and it is interesting to estimate the overall field enhancement of an array. Assuming that the total emission current can be written as the summation of every single emitter, e.g., $I_{tot} = \sum_{n=1}^N I_n$, where I_n is the current emitted from the n th emitter, and all

the emitters are identical and uniformly distributed ($I_{tot} = N \times I_1$), the overall enhancement for an array will be the same as a single emitter.

If the emitters are not identical and nonuniformly distributed, then it will require a statistical model with some assumed distribution in terms of sharpness of the emitter and the spacing of the adjacent emitters. Here, let us check the simplest case that all the emitters are identical with $\beta_n = \beta_0$ for $n = 1$ to $N - 1$, except for the N th emitter, which has a very large enhancement $\beta_N \gg \beta_0$ and $I_N \gg I_0$. If the number of the emitters N is small such that the sum of the current emitted from all other emitters remains much smaller than the current produced by the sharpest emitter [$I_N \gg (N - 1)I_0$], we will have $I_{tot} = (N - 1)I_0 + I_N \approx I_N$, and the overall enhancement will be close to β_N . On the another hand, if N is very large such that $I_N \ll (N - 1)I_0$, $I_{tot} \approx (N - 1)I_0$, and the overall enhancement will be close to β_0 .

Last but not least, we would like to comment on other two types of common emitters that were not studied in this paper, namely, a perfectly sharp conical shape²⁸ and knife-edge shape.²⁹ For a perfectly sharp conical shape, we believe the results should be quite similar to the hyperboloid shape since they both are cylindrical symmetry about the central axis. The only difference is that the surface enhancement is less uniform for the conical shape (the emitter is sharper), and the electron beam will be more concentrated around the emitter tip. Thus, at the same apex enhancement β_{max} , the true average enhancement β_c for the sharp conical shape emitter should be slightly larger than the hyperboloid shape. For the knife-edge emitter, it is infinitely long in the z -direction similar to the Lorentzian shape. Thus, we may speculate that the results will be closer to the Lorentzian shape. However, the knife-edge geometry has two sharp tips (see point B and C in Fig. 1(a) of Ref. 29), and they have very large maximum enhancement value approaching infinity [see Eqs. (1) and (4) in Ref. 29], which will require further investigation.

V. SUMMARY

In conclusion, we have performed a numerical experiment assuming electrons are emitted from a well-defined field emitter, and use the numerical data to mimic experimental measurements to find the criteria for which the analysis of FN law remains valid to estimate the field enhancement factor and effective emission area. Two types of emitters are studied in this paper, namely Lorentzian shape and hyperboloid shape. From the analysis, we establish a criterion to determine the critical anode area L_c/S_c for which all the emitting current are collected and it also serves as the minimum anode area that one should use in the real experiment so that the analysis of FN plot is correct. Subsequently, the enhancement factor β_c can be obtained at the appropriate L_c/S_c at specific geometrical condition and gap spacing. Lastly, we may calculate the effective emission

area S_{eff} base on the β_c values so that a sharp emitter can be treated as a flat emitting patch, which may be useful to extend to study a field emission cathode consisting of many field emitters. This paper will serve as a baseline on using FN law to characterize the field emission from a single field emitter.

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