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2012

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Inverse analyses of disk infiltrometer tests

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ABSTRACT: The disk infiltrometer is a simple device used to measure hydraulic properties of unsaturated soils at low suctions. Saturated coefficient of permeability can be estimated from the coefficients of permeability measured at low suctions. However, there are several methods to calculate the hydraulic properties from disk infiltrometer tests. In this paper, inverse analyses of disk infiltrometer tests on compacted residual soil samples were performed. The estimated unsaturated hydraulic parameters were compared with those obtained experimentally. The results show that it is possible to obtain the soil-water characteristic curve accurately from the inverse analyses of disk infiltrometer tests.

KEYWORDS: unsaturated; residual soil; infiltrometer; inverse analysis; hydraulic properties; soil-water characteristic curve

1 INTRODUCTION

Disk infiltrometer is used for in-situ measurement of unsaturated hydraulic properties of soils near saturation (Ankeny et al. 1991; Perroux and White 1988). Disk infiltrometer belongs to a group of tension infiltrometer methods which include single disk with single tension (e.g. Perroux and White 1988), single tension with disks having different radii (e.g. Smetten and Clothier 1989), and single disk with multiple tensions (e.g. Angulo-Jaramillo et al. 2000). The disk infiltrometer used in this study is a home-made single disk with single tension infiltrometer (Fig. 1). The disk infiltrometer consists of two acrylic tubes (water reservoir and bubble tube) with internal diameter of 25 mm. The height of water reservoir is 0.5 m and that of bubble tube is 1 m. The water reservoir and bubble tube are connected by a flexible tube from the top of bubble tube to the water reservoir where a porous disk is fixed at the base. The negative pressure in the bubble tube can be adjusted by moving the capillary tube in the bubble tube.

There are many methods of analysing the data from disk infiltrometer test to determine the unsaturated hydraulic properties of soils. These methods are based on either steady state flow (e.g. Ankeny et al. 1991; Smetten and Clothier 1989; Wooding 1968) or transient flow (e.g. Haverkamp et al. 1994; Smetten et al. 1994; Warrick 1992; Zhang 1997) of the disk infiltrometer test. Using transient flow is attractive due to the shorter time duration and there-

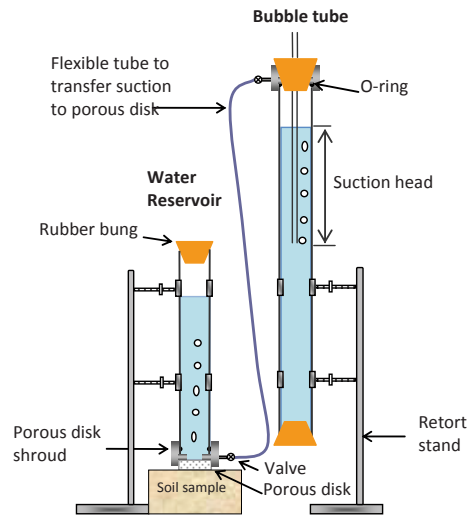


Figure 1. Schematic diagram of disk infiltrometer test set-up.

fore the test can be performed using a smaller volume of soil making the assumptions of uniform initial water content and homogenous soil valid. Semi-empirical approaches such as the method proposed by Zhang (1997) in analysing transient flow data from disk infiltrometer test is attractive as it involves only a very simple analysis to obtain the permeability at the applied suction head. However as pointed

out by Dohnal et al. (2010), Zhang's method requires the soil-water characteristic curve (SWCC) to accurately determine the permeability of unsaturated soil using the disk infiltrometer. In this study, inverse analyses of the transient flow data from the disk infiltrometer tests were used to determine the hydraulic properties of unsaturated soils. Inverse problems are well known to be typically ill-posed and ill-conditioned as uniqueness of parameters is not guaranteed. The sensitivity of the initial values of the parameters in obtaining the final set of parameters was examined with the aid of experimentally determined data.

2 THEORY OF DISK INFILTROMETER

Water flowing through disk infiltrometer can be treated as an axisymmetric infiltration process. The flow through disk infiltrometer can be grouped into steady state flow and transient flow. Richard's equation shown below is used to describe the flow through disk infiltrometer.

$$\frac{\partial \theta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[rk(h) \frac{\partial h}{\partial r} \right] + \frac{\partial}{\partial z} \left[k(h) \frac{\partial h}{\partial z} \right] - \frac{\partial k(h)}{\partial z} \quad (1)$$

where θ is the volumetric water content, h is the pressure head, z is the depth, r is a radial coordinate, and $k(h)$ is the unsaturated permeability function. For the disk infiltrometer, the following initial and boundary conditions are applicable:

$$h(r, z, 0) = h_i \quad (2)$$

$$h(r, 0, t) = h_0 \quad \text{for } 0 < r < r_0 \quad (3)$$

$$-\frac{\partial h}{\partial z} + 1 = 0 \quad \text{for } z = 0 \text{ and } r > r_0 \quad (4)$$

$$\frac{\partial h}{\partial r} = 0 \quad \text{as } r \rightarrow \infty \quad (5)$$

$$\frac{\partial h}{\partial z} = 0 \quad \text{as } z \rightarrow \infty \quad (6)$$

where h_i is the initial pressure head in the soil, r_0 is the radius of the disk and $h_0 \leq 0$ is the suction head of the infiltrometer during the infiltration process.

For transient flow, the problem can be treated as an axisymmetric problem with infiltration I from a circular source at the soil surface given by the following expression (Smetten et al. 1994):

$$I_{3D} - I_{1D} = \frac{\xi S^2}{r(\theta_0 - \theta_n)} t \quad (7)$$

where the subscripts 3D and 1D refer to axisymmetric three-dimensional and one-dimensional

processes, respectively, ξ is a constant and S is soil sorptivity. An appropriate value for ξ is 0.75 (Smetten et al. 1994). For short to medium time, Haverkamp et al. (1994) developed the following physically based infiltration equation for disk infiltrometers from Equation 7:

$$I_{3D} = S\sqrt{t} + \left[\frac{2-\beta}{3} k + \frac{\xi S^2}{r(\theta_0 - \theta_i)} \right] t \quad (8)$$

where β is a constant between 0 and 1. Equation 8 can be simplified as follows:

$$I = C_1 \sqrt{t} + C_2 t \quad (9)$$

where

$$C_1 = S \quad (10)$$

$$C_2 = \frac{2-\beta}{3} k + \frac{\xi S^2}{r(\theta_0 - \theta_i)} \quad (11)$$

Equation 7 is similar in form to Philip (1957) expression for one-dimensional transient infiltration process. Warrick (1992) and Zhang (1997) proposed different expressions for C_1 and C_2 by fitting the non-linear cumulative infiltration curve versus time as follow:

$$C_1(h_0) = A_1 S(h_0) \quad (12)$$

$$C_2(h_0) = A_2 k(h_0) \quad (13)$$

where A_1 and A_2 are dimensionless coefficients. Soil sorptivity $S(h_0)$ and permeability $K(h_0)$ can therefore be calculated as

$$S(h_0) = \frac{C_1(h_0)}{A_1} \quad (14)$$

$$k(h_0) = \frac{C_2(h_0)}{A_2} \quad (15)$$

Zhang (1997) proposed empirical equations for A_1 and A_2 for different soil-water characteristic curve (SWCC) equations to calculate $S(h_0)$ and $K(h_0)$. The van Genuchten SWCC equation is given as:

$$\frac{\theta_w - \theta_r}{\theta_s - \theta_r} = \left[\frac{1}{1 + (a h)^n} \right]^m \quad (16)$$

where θ_w is volumetric water content, θ_s is the saturated volumetric water content, θ_r is the residual volumetric water content, a , n and m are coefficients and m is given as:

$$m = 1 - \frac{1}{n}, \quad n > 1 \quad (17)$$

For van Genuchten (1980) SWCC equation, A_1 and A_2 can be written as follow (Zhang 1997):

$$A_1 = \frac{1.4b^{0.5}(\theta_0 - \theta_i)^{0.25} \exp[3(n-1.9)\alpha h_0]}{(\alpha r_0)^{0.15}} \quad (18)$$

$$A_2 = \frac{11.65(n^{0.1} - 1) \exp[2.92(n-1.9)\alpha h_0]}{(\alpha r_0)^{0.91}} \text{ for } n \geq 1.9 \quad (19a)$$

$$A_2 = \frac{11.65(n^{0.1} - 1) \exp[7.5(n-1.9)\alpha h_0]}{(\alpha r_0)^{0.91}} \text{ for } n < 1.9 \quad (19b)$$

where b is 0.55 according to Warrick and Broadbridge (1992).

To determine $S(h_0)$ and $K(h_0)$ from disk infiltrometer test, the cumulative infiltration I is plotted against \sqrt{t} and an order 2 polynomial equation is fitted to obtain $C_1(h_0)$ and $C_2(h_0)$. It is therefore expected that if SWCC of the soil is not established, there will be errors in estimating $S(h_0)$ and $K(h_0)$.

Šimůnek (1999), Šimůnek et al. (1998a), and Šimůnek et al. (1998b) suggested that using the infiltration data at several consecutive supply tension, initial water content and final water content, unsaturated permeability and SWCC of soils can be estimated accurately by inverse analysis. However inverse analysis has been deemed to be too complex for practical use and is beset with the well known problem of ill-posed and ill-conditioned of inverse analysis (Si and Bodhinayake 2005). In this study, the accuracy of inverse analyses of disk infiltrometer tests to establish the SWCC was investigated using HYDRUS program (Šimůnek et al. 2008).

3 METHODOLOGY

To model the infiltration process during a disk infiltrometer process, the SWCC and permeability function of the soil are needed. The initial and boundary conditions as given in Equations 2 to 6 are also required. In the inverse problem, SWCC and permeability function of the soil are obtained by matching the numerical model solutions with observation of some measurements during the disk infiltrometer tests. These measurements can be cumulative infiltration, infiltration rate, water content or pressure head at some points. In the disk infiltrometer test, cumulative infiltration with time is always measured. However, water content and/or pressure head are usually not measured. Furthermore, the initial pressure head or water content of the soil is generally unknown at the beginning of the test and the boundary conditions may not be well defined. In this study, the problem was constrained by performing disk infiltrometer disks on compacted soil samples

Table 1. Compaction and basic properties of the residual soil

Soil sample	DOP	WOP
Dry density (Mg/m ³)	1.75	1.75
Initial water content (%)	13.90	18.80
Initial matric suction (kPa)	350	150
Saturated permeability, k_s (m/s)	5.8E-09	5.2E-10
Optimum water content (%)	16	
Maximum dry density (Mg/m ³)	1.77	
Specific gravity, G_s	2.68	
Liquid limit (%)	39	
Plastic limit (%)	24	
Clay (%)	25	
Silt (%)	40	
Sand (%)	35	
Unified Soil Classification System (USCS)	ML	
United States Department of Agriculture (USDA) texture	Loam	

that are 10 cm in diameter and height, i.e. the boundary condition is well established. Initial matric suction of the compacted soil sample was measured using the device described in Leong et al. (2009).

A sedimentary residual soil from the Jurong Formation (Leong et al. 2003) was selected for the study. Compaction and basic soil properties of the soil according to ASTM D698 (1998) are shown in Table 1. Soils were compacted for the experiments and initial matric suction of the soil samples were measured. Subsequently, disk infiltrometer tests were conducted on the soil samples using various pressure heads. After the disk infiltrometer tests, the samples were trimmed to cylindrical specimens of 50 mm diameter by 50 mm height and 50 mm diameter by 25 mm height for saturated permeability test using a flexible wall permeameter and pressure plate test to obtain the SWCC, respectively.

4 TEST RESULTS AND INVERSE ANALYSES

Soil samples were compacted at dry-of-optimum (DOP) and wet-of-optimum (WOP) conditions. The compaction conditions, initial matric suctions and saturated permeability k_s of the soil samples are summarised in Table 1. The SWCC of the DOP and WOP soil samples are shown in Figure 2. The disk infiltrometer test results are summarized in Figure 3 where cumulative infiltration is plotted against square root of time (\sqrt{t}). For the WOP compacted soil sample, only one disk infiltrometer test was performed at pressure head $h_0 = -2$ cm as the flow rates for more negative values of h_0 were too slow. Besides cumulative infiltration, no other parameters were measured during the disk infiltrometer test. Because of the finite size of the compacted soil sam-

ple, the infiltration process during the disk infiltrometer test was only axisymmetric initially and subsequently the process became mainly one-dimensional. Therefore, inverse analyses of a one-dimensional infiltration process were conducted using HYDRUS-1D where greater weightage is given to the matching cumulative infiltration in the latter half of the disk infiltrometer test. The cumulative infiltration is adjusted for the 10 cm diameter of the compacted soil sample.

In the inverse analysis, van Genuchten SWCC equation was used. Of the four parameters (θ_r , θ_s , α and n) in van Genuchten SWCC equation, only θ_s was known from the initial condition of the sample and the other parameters have to be determined. HYDRUS-1D uses the following equation for the permeability function:

$$k(h)=k_s\left(\frac{\theta_w-\theta_r}{\theta_s-\theta_r}\right)^{\ell}\left\{1-\left[1-\left(\frac{\theta_w-\theta_r}{\theta_s-\theta_r}\right)^{\frac{1}{m}}\right]^m\right\}^2\tag{20}$$

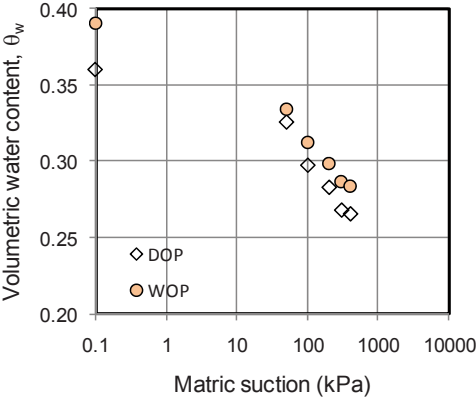


Figure 2. Soil-water characteristic curves of DOP and WOP samples.

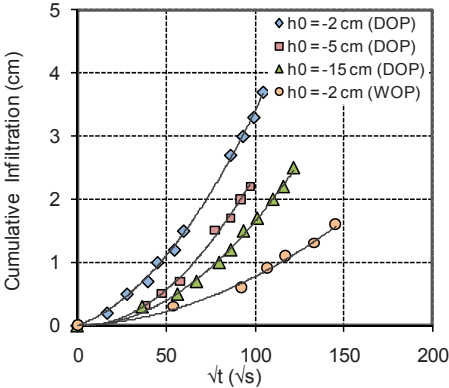


Figure 3. Disk infiltrometer test results.

Table 2. van Genuchten parameters for different soil textures from Carsel and Parrish (1988)

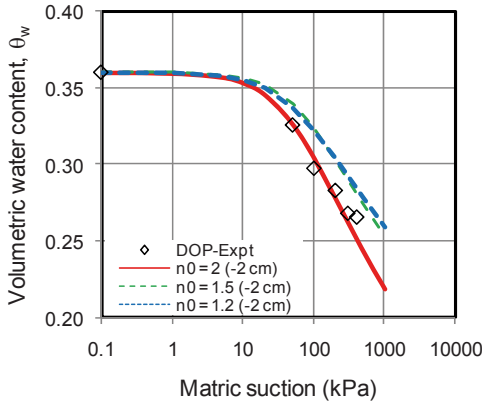
USDA Texture	α (cm ⁻¹)	N
Sand	0.145	2.68
Loamy sand	0.124	2.28
Sandy loam	0.075	1.89
Loam	0.036	1.56
Silt	0.016	1.37
Silt loam	0.020	1.41
Sandy clay loam	0.059	1.48
Clay loam	0.019	1.31
Silty clay loam	0.010	1.23
Sandy clay	0.027	1.23
Silty clay	0.005	1.09
Clay	0.008	1.09

Table 3. Final values of θ_r , α , n and ℓ from inverse analyses

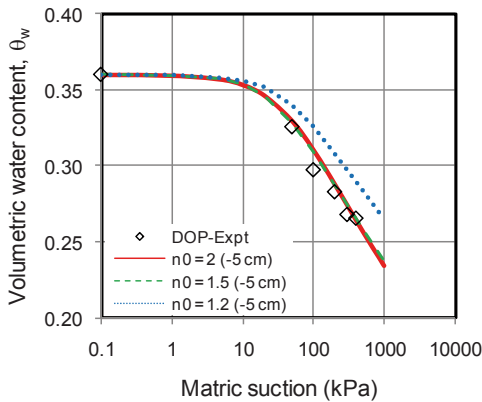
DOP sample					
h_0 (cm)	n_0	Final			
		θ_r	α (cm ⁻¹)	n	ℓ
-2	1.2	0.1000	0.0020	1.160	6.873
	1.5	0.0199	0.0020	1.152	5.365
	2.0	0.0519	0.0019	1.206	1.785
-5	1.2	0.0659	0.0018	1.134	0.818
	1.5	0.0606	0.0023	1.167	5.236
	2.0	0.1000	0.0013	1.198	0.154
-15	1.2	0.0611	0.0018	1.141	4.865
	1.5	0.1000	0.0016	1.162	6.624
	2.0	0.1000	0.0010	1.199	4.680
WOP sample					
h_0 (cm)	n_0	Final			
		θ_r	α (cm ⁻¹)	n	ℓ
-2	1.2	0.0003	0.0009	1.215	-0.009
	1.5	0.1000	0.0049	1.392	5.793
	2.0	0.0003	0.0009	1.215	-0.009

where ℓ is a pore-connectivity parameter. In this study, k_s was measured. Hence in the inverse analysis, four parameters (θ_r , α , n and ℓ) are determined. Mualem (1976) suggested that ℓ is about 0.5 for most soils. But fixing ℓ to be 0.5 will be too restrictive in the inverse analyses as the range of ℓ is between -5 and 16 (van Genuchten et al. 1991). The range of -5 to 16 was set for ℓ . Values of α and n for various USDA soil textures given by Carsel and Parrish (1988) are shown in Table 2. Table 2 shows α varies from 0.005 cm⁻¹ to 0.145 cm⁻¹ and n varies from 1.09 to 2.68. On closer examination, α has values in the order of 0.001, 0.01 and 0.1 whereas n has values in three regions 1.2, 1.5 and 2. Numerous runs were made with combinations of different initial values of α (0.001, 0.01 and 0.1) and n (1.2, 1.5 and 2). The initial value of θ_r was fixed at 0.0001 and θ_r was allowed to vary between 0 and 0.1. It was noticed that the final values of θ_r , α , n and ℓ were most sensitive to the initial values of n (n_0) and only the numerical results with different initial n values with $\theta_r = 0.0001$, $\alpha = 0.001$ and $\ell = 0.5$ are presented in this paper. The final values of θ_r , α , n and ℓ are summarised in Table 3. The non-uniqueness of the final values of θ_r , α , n and ℓ is clearly evident. Comparisons of the inversely determined SWCCs with the experimental SWCCs are given in Figure 4. The results showed that there is at least one good

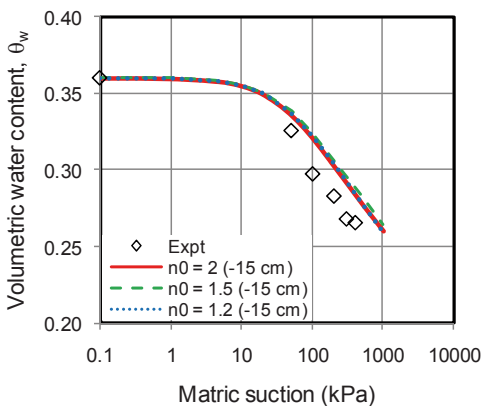
SWCC match for the compacted samples. To examine if the best matched SWCC could be determined priori, the goodness-of-fit parameters R^2 and value of the objective function (Φ) are examined in Table 4. Comparing Figure 4 and Table 4, the highest value of R^2 for each h_0 correctly predicts the best



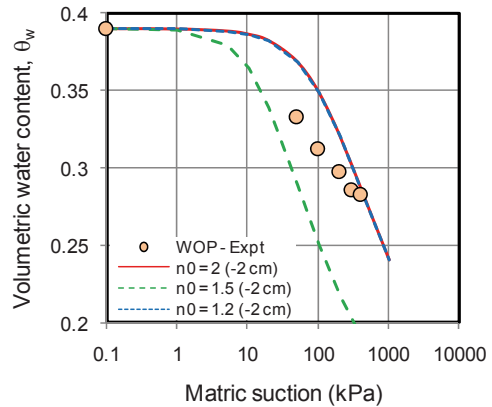
(a) DOP sample, $h_0 = -2$ cm.



(b) DOP sample, $h_0 = -5$ cm.



(c) DOP sample, $h_0 = -15$ cm.



(d) WOP sample, $h_0 = -2$ cm.

Figure 4. Comparison of experimental SWCC with SWCC from inverse analyses.

Table 4. Values of R^2 and Φ from inverse analyses

DOP sample			
h_0 (cm)	Initial n	Final	
		R^2	Φ
-2	1.2	0.98637	4.207
	1.5	0.98640	5.433
	2.0	0.98643*	4.223
-5	1.2	0.97679	1.396
	1.5	0.97676	1.392
	2.0	0.97679	1.389
-15	1.2	0.96629	5.802
	1.5	0.96615	5.787
	2.0	0.96635	5.583
WOP sample			
h_0 (cm)	Initial n	Final	
		R^2	Φ
-2	1.2	0.96190*	2.516
	1.5	0.96186	2.623
	2.0	0.96190	2.516

* Best matched SWCC for sample

matched SWCC. When there is a tie, the lowest value of Φ can be used to determine the best matched SWCC. The best matched SWCC for the DOP sample was for $h_0 = -2$ cm and $n_0 = 2.0$ and for the DOP sample, the best matched SWCC was for $n_0 = 1.2$ or 2.0. A better match of the SWCC was obtained for the DOP sample as compared to the WOP sample and this better match can be attributed to the higher initial matric suction of the DOP sample and thus higher infiltration during the disk infiltrometer test.

5 CONCLUSION

Disk infiltrometer tests were conducted for dry-of-optimum and wet-of-optimum compacted residual soil samples. The saturated coefficient of permeability, soil-water characteristic curve and the initial matric suction of the compacted samples were measured. Inverse analyses were conducted using

HYDRUS-1D. The inverse analyses showed that though the determined parameters for the soil-water characteristic curve and permeability function were not unique, it was still possible to pick the best matched soil-water characteristic curve using the R^2 value and the value of objective function.

ACKNOWLEDGEMENTS

The assistance of Mr Phua Kok Soon of the Construction Laboratory, School of Civil and Environmental Engineering, Nanyang Technological University, in fabricating the disk tensiometer is gratefully acknowledged.

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