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2012

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Estimation of P-wave velocities for unsaturated geo-materials

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ABSTRACT: Using P-wave velocity as a proxy for degree of saturation is useful in many applications. The effect of saturation on P-wave velocity of geo-materials has been frequently modeled using Biot-Gassmann equation. However Biot-Gassmann equation is not able to fully account for the effect of partial saturation. In this study, the effects of degree of saturation on P-wave velocity of rocks are investigated. Modifications to the Biot-Gassmann equation are then suggested to improve the estimation of P-wave velocities for unsaturated rocks. The modifications are evaluated with several data sets and were found to provide an improved estimate of P-wave velocity for unsaturated rocks to within $\pm 10\%$.

KEYWORDS: P-wave; S-wave; velocity; degree of saturation; unsaturated.

1 INTRODUCTION

Primary (P) and secondary (S) waves have long been used in subsurface exploration due to their ability to reflect properties like material types, structure, porosity and degree of fluid saturation. For unsaturated geo-materials, P-wave velocity (V_p) can be used as a proxy for degree of saturation if a good correlation between V_p and degree of saturation is obtained. To date, most correlations are either purely empirical or based on a modification of the famous Biot-Gassmann equation. An empirical correlation like that described in Han et al. (1986) is limited to a narrow range of geo-materials. On the other hand, some suggested modifications of the Biot-Gassmann equation are too complicated to be easily employed. For example, Naesgaard et al. (2007) and Yang (2005) suggested the use of Skempton pore-pressure parameter B to estimate V_p at various degrees of saturation. However, evaluations by Naesgaard et al. (2007) have shown that V_p is independent of Skempton pore-pressure parameter B at non-homogeneous partial saturation as the P-wave will route through the stiffer saturated zones leading to an inaccurate representation of the bulk modulus. Conte et al. (2009) introduced the soil-water characteristic curve (SWCC) into the Biot-Gassmann equation. However, SWCC varies greatly depending on the material type and structure of the geo-material. Extensive laboratory measurement will also need to be under-

taken to derive the SWCC for the specific type of geo-material hence making attempts to use this equation tedious and time consuming.

Biot-Gassmann equation provides a good basis for development of V_p equation for unsaturated geo-materials. Such an equation will not only be beneficial in site investigation and mineral prospecting (e.g. coal and oil) but also in laboratory tests for unsaturated geo-materials where V_p can be used as a proxy for degree of saturation. In this paper, a brief review of the Biot-Gassmann equation is first given followed by development of a V_p equation to account for degree of saturation. The derivation of the various parameters of the V_p equation are illustrated for rocks followed by evaluation of the V_p equation using independent data sets.

2 REVIEW OF BIOT-GASSMANN EQUATION

From elastic theory, P-wave velocity (V_p) and S-wave velocity (V_s) of a material are:

$$V_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} \quad (1)$$

$$V_s = \sqrt{\frac{G}{\rho}} \quad (2)$$

where K is the bulk modulus of the material, G is the shear modulus of the material and ρ is the bulk density of the material. For an unsaturated three-phase (air, liquid, solid) geo-material, ρ is given by the following equation:

$$\rho = (1 - n)\rho_s + S_r n\rho_w + (1 - S_r)n\rho_a \quad (3)$$

where n is the porosity, S_r is the degree of saturation, ρ_s is the density of the mineral solid, ρ_w is the density of the liquid phase (liquid is usually water in most applications i.e. $\rho_w = 1000 \text{ kg/m}^3$) and $\rho_a (=1.2 \text{ kg/m}^3)$ is the density of the gaseous phase.

The shear modulus of the geo-material G can be easily obtained using Equation 2 if V_s is measured and bulk density is obtained using Equation 3. Therefore to calculate V_p using Equation 1, bulk modulus of the geo-material K is needed. Bulk modulus K is a function of various variables like porosity, composition, mineralogy, matrix stiffness and the bulk moduli of the different mineral grains (Santamarina, 2001). In addition, V_p is highly dependent on the degree of saturation.

In Biot-Gassmann equation, K consists of bulk modulus of the matrix, K_m , and a pore-space modulus M as shown below:

$$K = K_m + \alpha^2 M \quad (4)$$

$$\frac{1}{M} = \frac{(\alpha - n)}{K_g} + \frac{n}{K_f} \quad (5)$$

$$\alpha = 1 - \frac{K_m}{K_g} \quad (6)$$

where K_g is the bulk modulus of the mineral grains, K_f is the bulk modulus of the interstitial fluids, and α is the Biot-Willis coefficient. In the derivation of K , Gassmann (1951) assumed that: (i) the medium is homogeneous and isotropic; (ii) the pores are interconnected (i.e. no occluded pores) and pressure is in equilibrium; (iii) the pore fluid is frictionless (i.e. low-viscosity fluid); (iv) relative motion between fluid and solid during propagation of the wave is negligible (applicable for low frequencies wave only); and (v) the pore fluid does not interact with the

solid matrix mineral (i.e. shear modulus remains constant).

Many of these assumptions are not valid in reality. For example, most geo-materials are anisotropic due to preferential alignment of minerals or cracks as seen in Winkler et al. (1995). Also, geo-materials that have undergone repeated drying and wetting processes would not be able to produce homogeneous saturation. Moreover, Mavko (1998) have shown that the Biot-Gassmann equation works best at low seismic frequency preferably no more than 100 Hz. However, this limitation is frequently violated as seismic surveys are usually performed using frequencies in the kHz range. Finally, the Biot-Gassmann equation is only applicable for a two-phase medium. Unsaturated geo-materials are three phase materials which consist of solid, liquid and air phases. Violations of assumptions will cause theoretical predictions of V_p to deviate from experimental results.

3 DEVELOPMENT OF V_p EQUATION

Starting from Biot-Gassmann equation and assuming K_g is much greater than K_m (i.e. $\alpha \approx 1$), V_p equations for a geo-material at zero degree of saturation and at full saturation are, respectively:

$$V_p = \sqrt{\frac{K_m + \frac{4}{3}G + \frac{K_a}{n}}{\rho}} \quad (7a)$$

$$V_p = \sqrt{\frac{K_m + \frac{4}{3}G + \frac{K_w}{n}}{\rho}} \quad (7b)$$

where K_a is bulk modulus of air (=142 kPa) and K_w is bulk modulus of water (=2.18 GPa). However, $\alpha \approx 1$ is too restrictive and any violation of the assumptions of Biot-Gassmann equation may invalidate the expression for α . In view of this, a factor I_f is introduced and Equation 7 can be rewritten in the following general form:

$$V_p = \sqrt{\frac{K_m + \frac{4}{3}G + I_f \frac{K_f}{n}}{\rho}} \quad (8)$$

where K_f is the bulk modulus of the fluid(s) in the pore space.

Values of K_m and G can be obtained if V_p and V_s of the geo-material at zero degree of saturation are

available or if V_s of the geo-material at zero degree of saturation and Poisson's ratio ν are available. If V_p and V_s of the geo-material at zero degree of saturation are available, G can be obtained using Equation 2 and K_m can be obtained as

$$K_m = V_{p,dry}^2 \rho - \frac{4}{3} G \quad (9)$$

If V_s of the geo-material at zero degree of saturation and Poisson's ratio ν are available, G can be obtained using Equation 2 and K_m can be obtained as

$$K_m = \frac{2G(1+\nu)}{3(1-2\nu)} \quad (10)$$

For degree of saturation other than zero and full, K_f is the bulk modulus of the fluid constituents occupying the pore space. To estimate K_f , Reuss and Voigt models as shown in Equations 11 and 12, respectively, are often used.

$$\frac{1}{K_f} = \frac{1-S_r}{K_a} + \frac{S_r}{K_w} \quad (11)$$

$$K_f = S_r K_w + (1-S_r) K_a \quad (12)$$

Modeling K_f according to Reuss model (Equation 11) is analogous to treating both the liquid and gaseous phases as springs connected in series while modeling K_f according to Voigt model (Equation 12) is analogous to treating both the liquid and gaseous phases as springs connected in parallel. According to Avseth et al. (2005), the Reuss model gives the lower bound and the Voigt model gives the upper bound as there is no way that nature can put together a mixture elastically softer or stiffer than the Reuss model and Voigt model, respectively.

While Equations 11 and 12 are useful upper and lower bounds for K_f , in reality the variation of K_f with S_r for a geo-material is more complicated as it depends on their matrix as well as their mineral type. Therefore it is proposed that K_f is given by the following equation:

$$K_f = (1-\beta)K_a + \beta K_w \quad (13)$$

where β is a function of degree of saturation and varies between zero and unity. One possible form for β is

$$\beta = \left\{ \frac{1}{1 + [a(1-S_r)]^b} \right\}^c \quad (14)$$

where a , b and c are coefficients dependent on material type and properties.

For the derivation of I_F and β , several sets of rock data are collated from the literature as summarized in Table 1. The data sets consist of sedimentary (limestone, sandstone, travertine), igneous (granite) and metamorphic (marble, serpentinite) rocks. The information available from these data sets include V_p at various degrees of saturation, V_s at various degrees of saturation or shear modulus G , porosity n , bulk modulus of mineral grain K_g , specific gravity G_s and bulk density ρ .

Table 1. Summary of data sets used for derivation of I_F and β

No. of Data Sets	Description	Reference
2	Fort Union Sandstone, Schuler-Cotton Valley Sandstone	Murphy (1982a)
1	Massilon Sandstone	Murphy (1982b)
1	Spirit River Sandstone	Knight et al. (1990)
1	Estailades Limestone	Cadoret et al. (1995)
6	Granite, Travertine, Marble, Serpentinite	Kahraman (2007)

3.1 Derivation of I_F

Saturation has been known to stiffen the bulk modulus of geo-materials and hence enhance the propagation of P-waves. To obtain I_F , V_p of dry ($S_r = 0$) and saturated ($S_r = 1$) rocks are examined. The values of I_F are back-calculated and plotted against porosity n in Figure 1. Figure 1 shows that generally the higher the porosity of the rock the larger is the value of I_F with the exception of two high porosity sedimentary rocks. Differences in V_p between high and low porosity sedimentary rocks were also noted by Murphy (1982a) however no porosity value on the demarcation of low and high porosity sedimentary rocks was given and no mention was made on how to account for the differences. The data in Figure 1 indicate that the increase in I_F with n forms

two parallel lines with the I_F values for metamorphic, igneous and low porosity sedimentary rocks lying on one line to the left while the I_F values for high porosity sedimentary rocks lying on the line to the right (Figure 1). Equations of the respective lines are shown in Equations 15 and 16.

$$I_F = 9.596n + 0.0272 \quad (15)$$

$$I_F = 9.596n - 2.3 \quad (16)$$

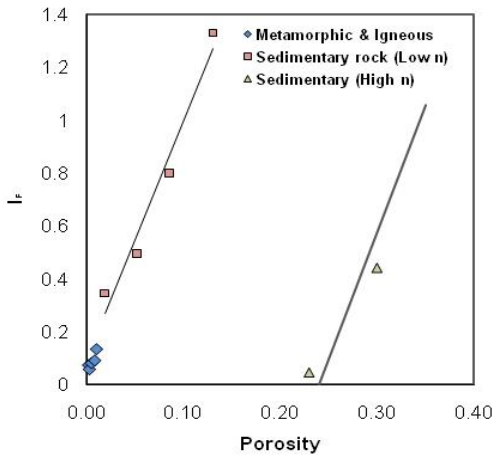


Figure 1. Correlation between I_F and porosity n .

3.2 Derivation of β

To obtain the coefficients a , b and c for β , the values of β according to Equations 8 and 13 were back-calculated for the rock data sets. The values of the coefficients a , b and c were then determined according to Equation 14. Cross-correlations of coefficients a , b and c and correlations of coefficients a , b and c with rock properties (K_g , G_s , n , ρ and combinations) were examined. The highest correlations are illustrated in Figures 2 to 4 and the best-fit equations for the correlations are shown in Equations (17) to (19), respectively.

$$c = 20.855e^{-15.385n} \quad (17)$$

$$b = 29.031c^{-0.4634} \quad (18)$$

$$a = 8.8085c^{-0.5821} \quad (19)$$

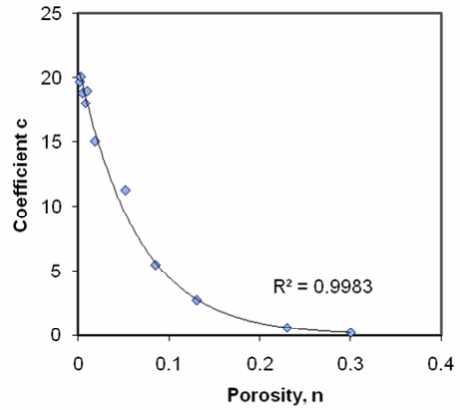


Figure 2. Correlation between coefficient c and porosity n .

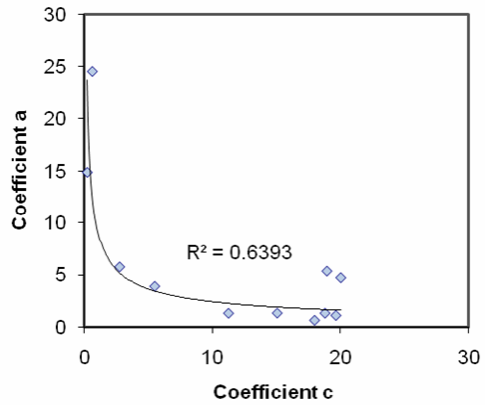


Figure 3. Correlation between coefficients c and a .

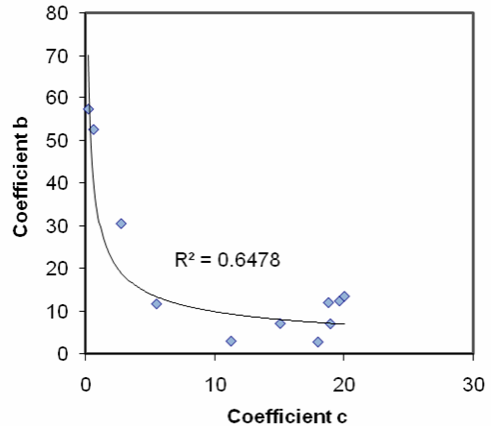


Figure 4. Correlation between coefficients c and b .

4 EVALUATION OF PROPOSED V_p EQUATION

The proposed V_p equation is evaluated with two independent data sets as summarized in Table 2. The coefficients a, b and c were obtained from Equations 16 to 18, respectively, and the value of I_F was obtained using either Equation 15 or Equation 16. Figures 5 and 6 show that the predicted values of V_p is very close to the experimental values of V_p with errors much less than 10% most of the time.

Table 2. Data for evaluation of V_p equation

Parameter	Sandstone from Chen et al. (1991)	Sierra White Granite from Murphy (1985)
Porosity, n	0.33	0.008
Mineral density ρ_s	2650 kg/m ³	2661 kg/m ³
Shear modulus, G	3.29 GPa	26.40 GPa
Mineral bulk modulus, K_g	40 GPa	56 GPa
Coefficient a	28.87	1.62
Coefficient b	74.72	7.53
Coefficient c	0.13	18.44
I_F	0.867	0.10

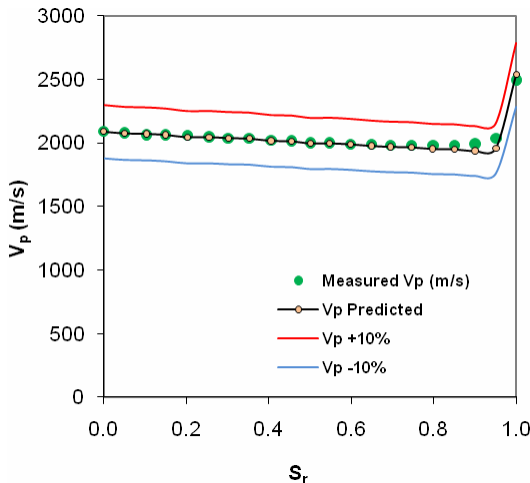


Figure 5. Predicted V_p vs S_r for Sandstone by Chen et al. (1991).

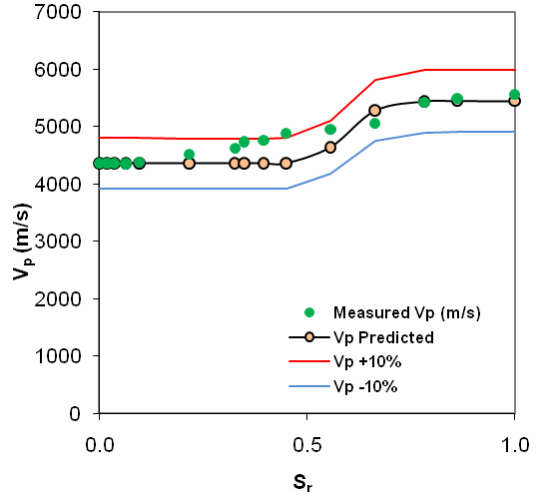


Figure 6. Predicted V_p vs S_r for Sierra White Granite by Murphy (1985).

5 CONCLUSION

An equation to estimate V_p for geo-materials at various degrees of saturation is proposed. Method to obtain the parameters in the V_p equation was illustrated for rocks. The proposed V_p equation was evaluated with two independent rock data sets. Very good agreement between estimated and experimental values of V_p was obtained, with errors much less than 10% most of the time. The robustness of the proposed V_p equation need to be evaluated with more extensive rock data sets. Further research is also needed to examine the validity of the proposed V_p equation for unsaturated soils.

ACKNOWLEDGMENTS

The financial support by DSTA research project fund, PTRC-CEE/DSTA/2010.01, is gratefully acknowledged.

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