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Enhancement of Piezoelectric Energy Harvesting with Multi-Stable Nonlinear Vibrations

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ABSTRACT

The need for long-term solutions to power various wireless sensor systems has been driving the research in the area of energy harvesting for the past decade. The present paper brings forth an investigation into the realm of piezoelectric energy harvesting (PEH) using nonlinear vibrations. A piezoelectric cantilever beam with a magnetic tip mass interacting with additional magnets around it forms a multi-stable nonlinear PEH configuration. The study indicates that the multi-stable configuration provides a widened bandwidth as compared to the conventional linear PEH devices and an increased voltage output as compared to many other PEH devices. An experimental parametric study is conducted to arrive at an optimal configuration for the performance enhancement of the harvester along with a glimpse into the enhanced magnetostatic interactions equations and various possible magnetic nonlinear configurations for the given conditions.

Keywords: energy harvesting; broadband; nonlinear vibrations; multi-stable configuration.

1. INTRODUCTION

In the quest for long-term service of various low-power sensor systems and wireless sensor networks, there has been prominent research in the field of energy harvesting. Vibration based piezoelectric energy harvesting (PEH) has gained popularity owing to the fact that the piezoelectric materials provide a high power density. Most initial harvesters consisted of a linear PEH device vibrating at resonance. Due to several drawbacks of linear PEH devices, a gradual shift towards the nonlinear PEH has been observed lately [1-4]. The non-linearity not only enhances the energy conversion capabilities of the harvesters but also increases the operating bandwidth. It has been of prime accord to enhance the operating bandwidth several times over the linear cases without much loss in the amplitude.

The use of magnets to induce nonlinearity was shown to have increased the efficiency of the PEH systems considerably [5-10]. The application of the magnetic induced nonlinearity is not limited to piezoelectric energy harvesting but can be applied to other mechanisms such as the electromagnetic and electrostatic transductions. The monostable and bistable configurations which come into existence due to the presence of magnets have been stated in the literature [5]. The dipole-dipole magnetostatic interaction equations were used by most of the authors to incorporate the magnetic coupling induced into the dynamics of the PEH system. Though the dipole-dipole formulation gives a good degree of accuracy, it restricts the application within the sphere of its corresponding assumptions. These drawbacks can be overcome by the use of the enhanced magnetostatic interaction formulation [12-14]. The applications of this formulation are stated in the forthcoming sections of this paper.

This paper primarily introduces the concept of multistable nonlinear vibrations induced with the help of magnets. A detailed experimental parametric case study for such vibrations applied to a PEH device is performed. The use of the modified magnetostatic interaction equations provides an enhanced accuracy in the potential energy and force formulations for the nonlinear magnetic effects. These equations have been used to give a physical representation of the potential energy variation for the experimental case studies.

2. ENHANCED FORMULATION FOR MAGNETOSTATIC INTERACTION EQUATIONS

The presence of magnets in the PEH system induces nonlinearity into the system and the most popular formulation to express this nonlinearity is using the dipole-dipole energy and force equations. The existence of the potential wells and their distribution can be explained with the help of these equations [5, 7]. Though the dipole-dipole equations represent the potential energy and force distribution of the magnetostatic interaction to a good degree of accuracy, it is important to

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consider the limitations this formulation poses because of the assumptions in place. The important assumptions to be considered are that the magnets are replaced by point dipoles, which are axially aligned at all times and separated with considerable distance from each other. But when the separation between the magnets is small and comparable to the size of the magnets, the shape effects play a very prominent role and the dipole-dipole equations show a very low degree of accuracy at such close separation distances [12]. The enhanced formulation developed considering the shape effects does have a few assumptions of its own, yet it allows for a better flexibility compared to the dipole-dipole formulation.

The assumptions for the enhanced formulation of cylindrical magnets consist of the direction of the magnetization vectors along the axis of symmetry of the cylinders, the magnetization vector rotation being restricted and the magnets being uniformly magnetized. From an experimental point of view, these assumptions can be easily applied to the system when the magnets are placed close to each other. The elemental mathematical equations for magnetostatic interaction forces were obtained from literature [14] and modified on a case to case basis, as there is no analytical solution available for these equations; they are simulated using MATLAB programs. Fig. 1 shows a typical representation of a system of two magnets.

Fig. 1. Typical representation of two cylindrical magnets placed 'x' m axially apart and 'r' m laterally apart.

2.1. Interaction equations for two magnets

In Fig.1 the two magnets are placed with their magnetization vectors aligned in the same direction i.e. there will be an attractive force between the two magnets. The interaction energy for such a typical system is given by

$$E_{i,1-2}(x,r) = \varepsilon \mu_o M_1 M_2 \pi R_1 R_2^2 \int_0^\infty J_0\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_1}{R_2}q\right)}{q^2} J_1(q) U(x,q) dq \quad (1)$$

$$\text{Where, } U(x,q) = \left[e^{\left(-|2d_2-x|\frac{q}{R_2}\right)} + e^{\left(-|2d_1-x|\frac{q}{R_2}\right)} - e^{\left(-|2d_1+2d_2+x|\frac{q}{R_2}\right)} - e^{\left(-|x|\frac{q}{R_2}\right)} \right]$$

$$d_i = \frac{t_i}{2}, i = 1, 2; z = d_1 + d_2 + x;$$

$$\varepsilon = \begin{pmatrix} +1 \text{ for attractive configuration} \\ -1 \text{ for repulsive configuration} \end{pmatrix}$$

$E_{i,1-2}$ is the interaction energy between the magnets 1 and 2; μ_o is the permeability of vacuum; M_1 and M_2 are the saturation magnetizations of magnets 1 and 2, respectively; R_1 and R_2 are the radii of the cylindrical magnets; J_0 and J_1 are the modified Bessel functions of the first type of order 0 and 1, respectively; r is the lateral separation between the magnetic axes, x is the axial separation between the end of the magnets, the Bessel functions of q define the shape of the magnets as described in [12] and t_i is the height of the cylindrical magnets.

The force equations based on this Energy formulation are obtained by differentiating the Energy term with z and r respectively.

$$F_z(x, r) = -\frac{\partial E_{i,l-2}(x, r)}{\partial x}, F_y(x, r) = -\frac{\partial E_{i,l-2}(x, r)}{\partial r} \quad (2)$$

$$F_z(x, r) = -\varepsilon\mu_o M_1 M_2 \pi R_1 R_2 \int_0^\infty J_0\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_1}{R_2}q\right)}{q} J_1(q) U(x, q) dq \quad (3)$$

$$F_y(x, r) = \varepsilon\mu_o M_1 M_2 \pi R_1 R_2 \int_0^\infty J_1\left(\frac{rq}{R_2}\right) \frac{J_1\left(\frac{R_1}{R_2}q\right)}{q} J_1(q) U(x, q) dq \quad (4)$$

When the lateral displacement between the magnets is restricted and only axial displacement is allowed i.e. $r = 0$. The Eqs.(3) and (4) are further simplified down to

$$F_z(x) = -\varepsilon\mu_o M_1 M_2 \pi R_1 R_2 \int_0^\infty \frac{J_1\left(\frac{R_1}{R_2}q\right)}{q} J_1(q) U(x, q) dq \quad (5)$$

$$F_y = 0 \quad (6)$$

When the magnets are assumed to be point dipoles. (Expansion of the Bessel functions in Eq. (5) about $q=0$ and applying $t \ll x$) $F_z(x)$ simplifies down to Eq. (7) which can infact be applied for the dipole-dipole approximation condition.

$$F_z(x) \approx \frac{-3\mu_o}{2\pi} m_1 m_2 \frac{1}{x^4} \quad (7)$$

where, $m_i = M_i (\pi R_i^2 t_i)$

Table. 1. Different types of magnets used and their properties

Magnet	Radius in meters	Height in meters	Saturation magnetization in A/m
Type-1 (NdFeB)	0.004	0.015	1900000
Type-2 (SmCo5)	0.0025	0.004	2000000
Type-3 (NdFeB)	0.003	0.002	1353000
Material: NdFeB – Neodymium-Iron-Boron, SmCo5 – Samarium-Cobalt.			

Numerical analysis of Eqs. (5) and (7) was performed in MATLAB using the data for magnet type-1 and magnet type-2. The corresponding curves obtained are as shown in Fig.2.

It is evident from Fig. 2 that for the interaction between magnets of type-1 and type-2, the accuracy of using the dipole formulation can yield erroneous results if the magnets are placed close to each other (of the order of twice the length of the largest magnet). Most literature dealt with distances in the close proximity of the magnets where the flux is relatively large. Hence to improve the accuracy in the modeling of the nonlinear force for the PEH device, the enhanced formulation is more suitable to the usage of the dipole-dipole formulation. Considering this fact, all the potential energy distribution curves shown in the coming sections of this paper are derived using this enhanced formulation only.

Fig. 2. Graphical representation of variation of force (F_z) with respect to the axial displacement (x) between magnet type-1 and magnet type-2 from Eqs.(5) and (7).

3. PHENOMENON OF MULTISTABILITY

The concepts of bistable behavior have been dealt at length in the review paper [9]. The bistable behavior can be described as the state where the system can exhibit two stable equilibrium positions for a given configuration. Similarly, multistability is a phenomenon where the system can exhibit many stable equilibrium positions.

When a simple linear elastic single degree of freedom (mass-spring-damper) system is subjected to an external force, it has a single equilibrium state which can be observed in its potential energy (PE) distribution curves. The system dynamics can be derived from the energy equations [15] and the total energy of such a system can be represented as shown in Eq. (8). The PE of such a system can be expressed in terms of its spring stiffness, Eq. (9) and the PE distribution can be shown as in Fig. 4 (b) & Fig. 4 (c). When the system is subjected to the effect of a magnetic force in order to induce some non-linearity, the PE of the system gets modified to Eq. (10). The magnetic interaction force acts along with the external force on the system and the magnetic interaction energy affects the PE distribution of the system.

$$E_{Total} = E_{Potential} + E_{Kinetic} + E_{Damping} + E_{External} \quad (8)$$

$$E_{Potential} = \frac{1}{2} K_{EQ} x(t)^2 \quad (9)$$

$$E_{Potential} = \frac{1}{2} K_{EQ} x(t)^2 + \sum E_i \quad (10)$$

Where, $E_{Potential}$ consists of the potential energy from the spring, $E_{Kinetic}$ consists of the Kinetic energy from the acceleration of the mass, $E_{Damping}$ consists of the Damping energy, $E_{External}$ consists of the work done by the external force, E_{Total} is the total energy of the system, E_i is the magnetic interaction energy between two magnets, K_{EQ} is the equivalent spring stiffness and $x(t)$ is the displacement caused due to the application of the external force.

3.1. Repulsive magnetic configuration.

When a pair of two repulsive magnets is introduced into the system, there are two extra equilibrium states induced into the system, out of the three equilibrium states two are stable and one is an unstable state, this can be observed from the PE distribution for a bistable case. The two troughs will hence forth be referred to as potential wells and the crest will be referred to as the potential barrier. When the system traverses through such an arrangement of potential wells and potential barriers, the behavior of the response is no longer linear but a nonlinear and chaotic behavior is likely to be observed depending on the magnitude of the external force acting on the system. If the system possesses enough energy to overcome the potential barrier it would be able to traverse through both the potential wells else it might get stuck in any of the potential wells.

If the number of magnets is increased by one, i.e. the number of interaction energy terms in the PE equation is increased to two; this arrangement would look as represented in Fig. 4 (a). Such an arrangement would result in the addition of two more equilibrium positions giving a total of five equilibrium states (three of which are stable and two unstable) as shown in Fig. 4 (b). Thus, with addition of magnets the number of potential wells and barriers are likely to increase giving a multistable configuration.

Fig. 3. Typical representation of a mass-spring-damper system.

Fig. 4. (a) Typical representation of three cylindrical magnets placed with their magnetization vectors pointing in the same direction. (b) Representation of the PE distribution for different stability conditions with magnets in repulsive configuration. (c) Representation of the PE distribution for different stability conditions with magnets in repulsive configuration. (d) Representation of the PE distribution for different neutral stability conditions with magnets in both attractive and repulsive configurations.

Fig. 5. Schematic of different equilibrium positions.

3.2. Attractive magnetic configuration.

Similar to the discussion in the above section, if a pair of two attractive magnets is introduced into the system, there will be a single stable equilibrium position and two unstable positions which may not necessarily be equilibrium states, it is exciting to observe that the potential energy is stretched to a new minimum state as shown in Fig. 4 (c). The outer points where the curvature changes can be distinguished as the potential barriers (unstable positions) and the potential well (stable position) shown in the figure is very prominent. When the system traverses through such an arrangement, the nonlinear response becomes obvious, if the system doesn't possess enough energy to overcome the deep potential well then it is likely to get stuck in that state.

On increasing the number of magnets, a similar arrangement to that of the repulsive magnets is obtained. But it is important to note that in this case the number of potential barriers is greater than the stable potential wells. This might come as an advantage or a drawback depending on various factors which will be discussed in subsequent sections. As the multistability is increased with increase in the number of magnets, the outer potential barriers start to deviate further away giving more room for the nonlinear behavior.

3.3. Neutral equilibrium configuration.

The monostable and bistable configurations have been mentioned widely in the available literature. But, there has been no clear explanation with regard to the neutral equilibrium configuration. An attempt to elucidate this phenomenon is performed in this section. Most linear systems possess a point of minimum potential energy on the PE distribution, but when the effect of nonlinearity contributes to a system there is a likelihood of a situation where in the minimum potential energy is present at a series of consecutive points as represented in Fig. (4) (d). This condition is similar to the neutral equilibrium state shown in Fig. (5). and henceforth will be referred to as neutrally stable state. It is best associated with an intermediate state where a system goes from a linear to a nonlinear phase. The neutral stability can be observed in the presence of magnets in both attractive and repulsive configurations as shown in Fig. (4) (d). The only difference being the extent to which it is present, which primarily depends on the number of magnets and their configuration and magnetization strengths. It is observed that the neutral stability aids in the enhancement of both bandwidth and magnitude which is discussed in detail in the following sections of this paper.

4. EXPERIMENTAL CASE STUDY

An experimental parametric case study was performed to investigate the behavior of a PEH device subjected to nonlinear vibrations caused by the interaction of magnets. A bimorph harvester was formed by attaching two piezoelectric transducers on the top and bottom at the root of an aluminum beam suspended at one end to form a cantilevered PEH device. The piezoelectric transducers used were Macro fiber composites (MFC) having model nos.M2807-P2 and M2814-P2 manufactured by Smart Material Corp. and having a capacitance of 12.4nF and 25.7nF respectively. The MFC transducers were bonded to the aluminum beam using an epoxy and the open circuit voltage V_{OC} for each transducer was monitored separately using a data acquisition system (NI 9229) and computer software (NI Signal express). A tip mass with an encased magnet was attached to the end of the beam, the magnets used in the tip mass were of type-2 and type-3 yielding a tip mass of 10.78g and 8.12g respectively. The end magnets (type-1) were encased in an adjustable plastic holder and aligned parallel to the axial direction of the tip mass when it was at rest. The properties of different types of magnets are listed in Table-1; the typical experimental setup and a schematic layout of the experimental setup are as shown in Fig. 6(a) and 6(b).

Fig. 6. (a) Typical experimental setup for the PEH device at rest position. (b) Schematic layout of the experiment setup.

The whole arrangement was fixed onto the movable arm of a seismic shaker (APS 445N dynamic shaker) and given a sinusoidal excitation. The acceleration was monitored using an accelerometer attached to the base of the experimental setup which was in turn connected to a data acquisition system (NI 9234) and a computer. The frequency and the magnitude of vibrations were maintained constant till a steady state response was observed. The Root Mean square (RMS) values of both acceleration and V_{OC} were monitored and noted down from time to time. At resonance in the nonlinear configuration, non periodic vibrations were observed and the data obtained was time averaged to obtain a significant approximation than record the instantaneous values.

The experimental parametric study included the excitation of the PEH device at different accelerations and changing the stiffness of the cantilever beam. The stiffness of the PEH device (K_{EQ}) is calculated based on the simplification of the system as a SDOF model [2]. In order to compare the response of various beams the plots shown in subsequent sections are done with voltage ratio as the ordinate and the frequency as the abscissa. The voltage ratio is given by Eq. (11).

$$V_{OC-Ratio} = \frac{V_{OC-RMS}}{V_{OC-Peak}} \quad (11)$$

Where, $V_{OC-Ratio}$ is the voltage ratio, V_{OC-RMS} is the open circuit RMS voltage value at a given frequency and $V_{OC-Peak}$ is the peak open circuit RMS voltage value for a linear case at the same acceleration.

4.1. Case-I: Tip magnet and the end magnets in repulsive configuration.

Once the PEH device was set on the shaker and the excitations were given, with a steady rate of increase in frequency the values of open circuit voltage were recorded from time to time; when there was no effect of end magnets, the system behaved like any other linear energy harvester. The observations for such a linear configuration are represented by the

black curves in the voltage ratio vs frequency curves, Figs. (7) and (8). When the end magnets were placed in a repulsive configuration with the tip magnet, nonlinearity gets induced into the system, the nonlinear behavior can be noted as the red and the green curves which represent forward frequency sweep and a backward frequency sweep respectively.

Literature stated that the repulsive configuration doesn't provide an optimum response under periodic loading, but that was limited to the presence of a single end magnet. Though a hardbound conclusion cannot be drawn on that statement but it can be concluded with confidence that as the stiffness decreased the broad band characteristics of a PEH get increased. The observations shown in Fig.(7) (a), (b) and (c) were obtained for a stiffer beam ($K_{EQ}=102.22\text{N/m}$) with a tip mass of type-2 and end magnets of type-1 at an RMS acceleration of 0.1g, 0.2g and 0.3g respectively, where $g=9.81\text{m/s}^2$. It is interesting to note that the behavior can be correlated to the PE distribution shown in Fig. (7) (e). The system got stuck in a single potential well at $x=5\text{mm}$ (axial separation) even at an acceleration of 0.3g as it couldn't overcome the potential barrier, the distribution at $x=8\text{mm}$ signifies a neutrally stable state between a bistable and a tristable state, and $x=10\text{mm}$ shows a neutrally stable state before the occurrence of bistability. This can also be observed in the voltage ratio curves at 0.3g, there is the presence of two significant jumps for response at $x=8\text{mm}$.

When the stiffness of the beam was reduced ($K_{EQ}=85.1\text{N/m}$) and the tip magnets were modified to type-3 keeping the end magnets of type-2 in repulsive configuration, the PE distribution gets modified as shown in Fig. (7) (f). There is a deep potential well at $x=3\text{mm}$ separation where the beam got stuck in the potential well, but the significance of this observation is the increased bandwidth observed for the intrapotential vibration. It was only at $x=6\text{mm}$ that the beam was able to traverse through all the potential wells which is visible in the voltage ratio curves for an acceleration of 0.3g for this configuration shown in Fig. (7) (d). The notable inference from this section is that the non-linearity due to repulsive configuration yields increased bandwidths though the maximum peak voltage is well below the linear configuration, if sufficient energy is available for the system to traverse the entire potential wells and barriers the bandwidth obtained from a harvester can be increased significantly.

Fig. 7. (a)-(c) Curves obtained for voltage ratio with varying frequency for an acceleration of 0.1g, 0.2g and 0.3g for a beam of stiffness equal to 102.22N/m. (d) Curves obtained for voltage ratio with varying frequency for an acceleration of 0.3g for a beam of stiffness equal to 85.1N/m. (e)-(f) PE distribution of the beams with stiffness 102.22N/m and 85.1N/m respectively for varying axial separation lengths.

4.2. Case-II: Tip magnet and the end magnets in attractive configuration.

Taking cue from the previous section, this section deals with the observations from the PEH device with the tip magnet and the end magnets placed attracting each other. Fig. (8) (a)-(d) show the response voltage ratio values for a very flexible beam ($K_{EQ}=61.2\text{N/m}$) when it was subjected to increasing excitation amplitudes of 0.1g, 0.2g, 0.3g and 0.4g. The tip magnet consisted of magnet of type-2 and the end magnets consisted of magnets of type-1. The PE distribution for such a configuration is shown in Fig. (8) (e), it is interesting to note that for this configuration when the axial separation is about $x=8\text{mm}$ the system is in a range of neutrally stable configuration and when it is at $x=5\text{mm}$ the system is just away from a neutrally stable configuration, this can be noted as the slight rise in the PE curve at the center, this effects the system response at higher acceleration values of 0.3g and 0.4g where there is a slight dip in V_{OC} response near the peak. There is a prominent double jump phenomenon observed in the response for $x=5\text{mm}$, this can be related to the significant separation between the potential barriers which is only possible in the presence multistable configurations (presence of more number of magnets) and at excitation levels of 0.4g the system was able to traverse the whole PE curve thus the response is both broadband and the voltage ratio is greater than 1.

Consequently, the stiffness of the beam was increased ($K_{EQ}=75.5\text{N/m}$ and $K_{EQ}=85.1\text{N/m}$) and experimental runs were performed. Fig. 8 (e) and (f) show the nonlinear behavior of the PEH device at $x=5\text{mm}$ and an acceleration of 0.3g for both the cases. When the experiments were run using the configuration with stiffness of 85.1N/m, the tip mass was changed to type-3 and the end magnets were of type-1 all throughout. The PE curves for both the cases are as shown in Fig. (8) (h) and (i), it is to be noted that the potential well in Fig. (8) (h) is more steep in comparison to that of Fig. (8) (i) and its effect can be directly seen in the response curves. The magnitude of the Voltage obtained and the bandwidth are both enhanced in Fig. (8) (i); the primary reason attributed to this can be due to the fact that the beam is able to traverse more distance at the same acceleration for the latter case thus covering a larger part of the PE curve. Though the effect of the central potential barrier is not so prominent in the response curves, it is likely to be observed at higher excitation levels.

The most important inferences from this section includes the fact that it is possible to achieve voltage levels higher than that of the linear cases when the system is subject to multistable conditions and the bandwidth obtained is significantly larger at a higher voltage ratio, say 0.6 times the peak. (Fig. 8 (d)).

(a)

(b)

(c)

(d)

(e)

(f)

Fig. 8. (a)-(d) Curves obtained for voltage ratio with varying frequency for an acceleration of 0.1g, 0.2g, 0.3g and 0.4g for a beam of stiffness equal to 61.2N/m. (e)-(f) Curves obtained for voltage ratio with varying frequency for an acceleration of 0.3g for beams with stiffness equal to 75.5N/m and 85.1N/m respectively. (g)-(h) PE distribution of the beams with stiffness 61.2N/m, 75.5N/m and 85.1N/m respectively for varying axial separation lengths.

5. CONCLUSION

The employment of magnets induces nonlinearity into a vibration energy harvesting system and an attempt to utilize the multistable configurations has been brought forward. This paper indicates that the use of multistable configurations not only enhances the bandwidth but also has the ability to increase the open circuit voltage of the piezoelectric energy harvester. The observations in this paper are a stepping stone to aid the design of broadband and efficient PEH devices. Out of the two most popular attractive and repulsive configurations, for a periodic excitation case, the attractive configuration has a slight edge over the repulsive configuration. The reason is that for the same conditions of excitations and axial distance from the end magnets (Fig. 7 (d) and Fig. 8 (f)) there is inter-well traversing during resonance and the magnitude of maximum voltage response is also slightly higher. Moreover, the transition stages between linear and nonlinear behaviors where the neutrally stable states come into the picture provide an efficient way of increasing the bandwidth.

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