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The focusing property of the spiral Fibonacci zone plates

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ABSTRACT

The concept of fractal, such as Cantor set, has been combined with the Fresnel zone plates (ZPs) to improve its imaging capability and introduce additional freedom for optimizing. In this letter, Fibonacci zone plates (FiZPs), which is generated by means of Fibonacci fractals, was proposed. The FiZPs can be constructed by the generating rules of the Fibonacci sequences similar with the Cantor set zone plate published. Numerical simulation based on Fresnel diffraction is performed to verify the on-axis intensity distribution of the FiZPs. According to the simulated results, the FiZPs can focus the incident beam as conventional zone plates as well but with fractal on-axis intensity profile. The theoretical results are also verified by the experiment by means of phase-only spatial light modulator (pSLM).

Furthermore, the spiral Fibonacci zone plate (sFiZPs) was designed by overlapping the spiral phase on the FiZPs. Both numerical simulations and experiments are performed and the results are coincident well with each other.

Keywords: Fibonacci fractals, Fresnel zone plates, spiral phase, fractal zone plates

1. INTRODUCTION

Fractal structures, introduced by Mandelbrot [1] two decades ago, play an important role in understanding and describing a variety of phenomena in various fields of science, such as biology, geophysics, optics [2-4] and physics [5]. The research on light interaction with fractals objects and the discovery of fractal properties of the electromagnetic fields resulted in the fractals optics [6, 7]. Especially, the diffractions of the fractals structures and objects have attracted much attention. Both theoretical analysis and experimental results showed that the fractal features could be extracted from the diffractive patterns of the fractal objects [6]. Therefore it is useful to analyze the diffraction in different type of fractals.

Many efforts have been engaged in the deterministic and random fractals based on the Frannhofer and Fresnel diffraction for both theoretical and experimental studying [8-10]. Moreover, it has been shown that some optical fields have an intrinsic fractal structure [11, 12]. Most of the research on the fractal in the frame of the positional coordinates, such as Cantor set grating [13], Fibonacci grating [14] etc.. Nevertheless, under the other frame, such as the square of the positional coordinates similar with the Fresnel zone plates, the fractal behavior can be observed as well [15].

Fresnel zone plate, which is designed to focus the incident light with diffraction but not refraction, has been found many applications in optics [16], especially in soft x-ray microscopy [17] and THz tomography [18] in which the refraction elements are invalid due to the strong absorption of materials. Therefore, it has been an important field to design the zone plate to enhance the focusing or improve the imaging quality. Recently, fractals zone plates (FraZPs) [15, 19-22] along the square of the transverse coordinates have been reported. The focusing properties of the FraZPs for the case of triadic Cantor set modulation have been analytically and experimentally studied. Both theoretical and experimental results showed that the on-axis intensity distribution of the FraZPs exhibits fractal behavior in comparison with FZPs.

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In this paper, we proposed one novel FraZPs by considering the Fibonacci fractals. First the constructed rule of the Fibonacci zone plates (FiZPs) will be depicted. According to structures, on-axis intensity distribution was studied by means of Fresnel diffraction theory. Subsequently, experiments based on phase-only spatial light modulator (pSLM) were performed to verify the theoretical results. The experimental results are coincident well with the theoretical prediction. Multiple foci were found according to the FiZPs with fractal profile. Furthermore, spiral FiZPs (sFiZPs), which overlap the spiral phase with FiZPs, were introduced. The analytical formula of the on-axis intensity distribution was also presented according to the Fresnel diffraction theory and the generating rule of Fibonacci fractal. Multiple focusing optical vortex are formed due to the sequence focusing properties of the FiZPs, which are expected to be employed for trapping and rotating microparticles located at various positions simultaneously. Experiment is also performed to verify the theoretical analysis by phase-only spatial light modulator (pSLM). The spiral phase and FiZPs are encoded together in the phase pattern to generate the multiple focusing optical vortex. The recorded images by CCD in specific location along the propagation direction are coincident with theoretical prediction well. In additional, we also studied the fractal property of the on-axis intensity distribution.

2. FIBONACCI ZONE PLATES

2.1 Fibonacci fractals

Fibonacci fractals, named with the inventor Italian mathematician Leonardo da Pisa, called Fibonacci (1180-1240), have been one kind of popular structures in several fields such as biology, quasicrystals. In biology, Fibonacci number can be used to describe the growth of an idealized rabbit population. Fibonacci grating are also studied comprehensively with semiconductor multilayers structures or dielectric multilayers as filter or quasicrystals microcavity.

The structures of Fibonacci fractals are illustrated in Fig. 1 [23]; The Fibonacci fractals contain two different separations A and B as shown in Fig. 1.

Figure 1. The schematic of the Fibonacci fractals

The Fibonacci recursion is depicted in Figure 1. We consider such a structure as periodic, with the elementary cell composed by the m th generation of the Fibonacci sequence, which is obtained from the recursion relation $F_m(A, B) = F_{m-1}(A, B) + F_{m-2}(A, B)$, ($m \geq 2$), and the initial conditions $F_0(A, B) = A$, $F_1(A, B) = B$. According to the generating rule, the given order one dimension Fibonacci fractals can be achieved.

2.2 The circular Fibonacci grating (CFiG) and Fibonacci zone plates (FiZPs)

To construct an axis-symmetric 2D device, the one-dimension structure is rotated one round with an axis. Therefore, with one-dimension Fibonacci fractals, two elements can be formed by means of rotating operation under two different coordinates, i.e. circular Fibonacci grating and FiZPs under positional coordinate and square positional coordinate respectively.

Figure 2. The circular Fibonacci grating (a) and the Fibonacci zone plates (b)

As shown in Figure 2, circular Fibonacci grating was constructed by rotating the 1D Fibonacci structures with an axis in positional coordinates and the Fibonacci zone plate (FiZP) was generated under square positional coordinates with the same rule. Here the ratio of length between part B and part A equals the golden mean $\gamma = \frac{\sqrt{5}+1}{2}$, and the recursion number is 10.

2.3 The far field diffraction of the CFiG and FiZPs

As described in introduction, the diffraction property of the FiZPs is the main concern in this paper. First, we will investigate the far field diffraction of the FiZPs. The far-field diffraction pattern of CFiG is also presented for comparison.

Theoretically, the far field diffraction of the optical devices can be achieved by the Fraunhofer diffraction, i.e. Fourier transform of the transmittance function of the devices, which can be formulated as below [19]:

$$I(r) = |FT\{T(r)\}|^2 \quad (1)$$

Here, $T(r)$ denotes the transmittance function of the optical elements and $FT\{\bullet\}$ represents the Fourier transform. r is the radial position of observed point. The transmittance function of CFiG and FiZPs can be formulated as:

$$T(r) = F_1 + F_2 + \sum_{i=1}^n F_i(A, B), CFiG \quad (2)$$

$$T(r^2) = F_1 + F_2 + \sum_{i=1}^n F_i(A, B), FiZPs \quad (3)$$

Figure 3 shows the far-field patterns of the CFiG and FiZPs. It is obvious that the CFiG can form multiple rings in the far field and the FiZPs focus the incident light. To achieve the numerical simulation, we set the transmittance of A part as zero and that of B part as unit.

Figure 3. The far-field diffraction pattern of CFiG (a) and FiZPs (b)

2.4 The on-axis intensity distribution of the FiZPs

In general, on-axis intensity distribution of optical devices, which is within the Fresnel diffraction range, is the major aspect to characterize its focusing and imaging behavior. Since the diffraction pattern of CFiG is multiple rings, we will only consider the on-axis intensity distribution of the FiZPs in this paper. After considering the symmetry of FiZPs, the on-axis irradiance of FiZPs can be generalized in the form:

$$I(r) = \left(\frac{2\pi}{\lambda r} \right)^2 \left| \int_0^a T(r_0^2) \exp\left(-i \frac{\pi}{\lambda r} r_0^2\right) r_0 dr_0 \right|^2 \quad (4)$$

Where a is the maximum extent of the pupil function and λ is the wavelength of the light. The same new variable was adopted as Ref. [19] to simplify the numerical calculation, as below:

$$\zeta = (r_0 / a)^2 - 0.5 \quad (5)$$

Thus we defined $q(\zeta) = T(r_0)$. To simplify the formula further, normalized axial coordinate was used as $u = \frac{a^2}{2\lambda r}$.

Therefore, the irradiance along the optical axis can be expressed as:

$$I_0(u) = 4\pi^2 u^2 \left| \int_{-0.5}^{0.5} q(\zeta) \exp(-i2\pi u \zeta) d\zeta \right|^2 \quad (6)$$

From Eq. (6), it is obvious that the axial irradiance is expressed in terms of the Fourier transform of the transmittance function. Due to the fractals profile of the diffraction screen, the irradiance along the optical axis would present the intrinsic property of the fractals, i. e. the self-similarity as well.

Figure 4 shows the on-axis intensity distribution of the FiZPs with constructed parameter $\gamma = \frac{\sqrt{5}+1}{2}$, the order or recursion of Fibonacci sequence is 13 and the maximum radius of belt set as 6.8 mm. The wavelength of incident light is 532 nm.

Figure 4. The on-axis intensity profile of the CFiG (a) and FiZPs (b)

From Figure 4(b), the on-axis intensity profile of FiZPs shows a series of discrete lines, which manifest the discrete focal points. The multiple focal points profile is similar with the Cantor-set fractal zone plates. In comparison with FiZPs, the contrast of the intensity profile of CFiG is very low due to the regular positional coordinates.

3. EXPERIMENTAL RESULTS

3.1 Experimental configuration

To measure the on-axis intensity of FiZPs, it is convenient to make use of phase-only spatial light modulator (pSLM) to generate FiZPs. In common optical configuration, the incident light was impinged onto the pSLM after collimating and filtering. The FiZPs pattern generated in computer was driven to the pSLM via VGA port of the personal computer. The incident will be modulated by the pSLM with designed phase delay and then the diffraction pattern can be observed by using of CCD camera located at the optical axis of the pSLM. We also adopt this general configuration. As shown in Figure 5, HoloEye model LCR2500 (pixel: 1024x768, pixel size: 19 μm) was used as the pSLM and DataRay WinCamD (pixel spacing: 13.4 μm) was used to record diffraction pattern. The incident light at 532 nm emitted from diode pumped solid state (DPSS) laser.

Figure 5. the schematic of the optical configuration to record the diffraction of the FiZPs

3.2 Experimental results

The CCD camera was loaded in one rail enabling to record the intensity distribution along the optical axis of FiZPs. Figure 6 shows the diffraction pattern of the FiZPs. The constructed parameters of FiZPs used in experiment is the same as the parameters in Figure 4.

Figure 6. The diffraction pattern of FiZPs with different propagation distance (a) $z=50.3$ cm, (b) $z=59.2$ cm, (c) $z=1.215$ cm, (d) $z=142.9$ cm.

As we can see from Figure 6, the on-axis intensity distribution of FiZPs is similar with Cantor-set zone plates with focusing capability. Both dark center and bright center appear along axis with different propagation distance, which determined by the destructive and constructive interference of the wave from different wave belts. The positions of dark centers and focusing points are coincident the calculated on-axis intensity distribution with Eq. (6).

4. SPIRAL FIBONACCI ZONE PLATES

4.1 Spiral Fibonacci zone plates (SFZPs)

Next, we will introduce another diffractive elements, i.e. the spiral Fibonacci zone plates, which is generated by overlapping spiral phase on the FiZPs. The image of the pattern is shown in Figure 7.

Figure 7. The phase pattern of SFiZPs

The transmittance of SFiZPs can be written as:

$$T(r^2) = \left[F_1 + F_2 + \sum_{i=1}^n F_i(A, B) \right] \exp(im\phi), \text{SFiZPs} \quad (7)$$

Here, m is the topological charge of spiral phase and ϕ is the azimuthal angle. Similar to FiZPs, SFiZPs' on-axis intensity can be expressed as the Fourier transformation of the transmittance function. It is expected that the on-axis intensity will be compensated with that of FiZPs, for example, the focal point in which light was focused by FiZPs would appear as a dark hole as occurring in FraZPs overlapping with spiral phase [24].

The same experimental setup can be adopted to study the diffraction of SFiZPs. Figure 8 shows the recorded images by means of CCD camera behind the pSLM with same parameters in Figure 6 except the unit topological charge spiral overlapping. In comparison with Figure 6, it is obvious that the spiral FiZPs can convert the focal point as a dark hole, which can be expected application in optical trapping or micro-particles manipulation. The distances recorded the image in Figure 8 (a) and (b) are correspondent to (a) and (d) in Figure 6, respectively. The radius of the dark hole will enlarge with increasing of topological charge of spiral phase, which shown in the inset of Figure 8 with topological charge equal to 3.

Figure 8. The intensity profile of the spiral FiZPs (a) $z=50.3$ (b) $z=142.9$ cm with unit topological charge, inset shows the intensity profile of spiral FiZPs with topological charge equal to 3.

5. CONCLUSION

We developed one new type fractal zone plates by introducing Fibonacci sequences into the generating rule of zone plate. First, theoretic analysis was performed to achieve the on-axis intensity distribution of FiZPs. Simulation results showed that FiZPs could focus incident light into multiple points similar with Cantor-set FraZPs. Meanwhile, the Fibonacci grating was considered for comparison. Subsequently, experiments based on pSLM were performed to verify the on-axis intensity profile of FiZPs. The experimental results are coincident well with theoretical prediction. Furthermore, one

compensatory device, which overlapping with spiral phase on FiZPs, was designed. SFiZPs will form a dark hole in the focusing point by FiZPs. Experimental results show clearly the dark hole and its radius will increase with large topological charge.

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