Time-frequency and time-frequency-rate representations using the cross quadratic spectrum

Saman S Abeysekera

2013

Abeysekera, S. S. (2013). Time-Frequency and Time-Frequency-Rate Representations using the Cross Quadratic Spectrum. 2013 IEEE Tencon-Spring Conference, pp.500-504.

<https://hdl.handle.net/10356/99082>

<https://doi.org/10.1109/TENCONSpring.2013.6584495>

© 2013 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. The published version is available at: [<http://dx.doi.org/10.1109/TENCONSpring.2013.6584495>].

Downloaded on 26 Apr 2023 22:49:38 SGT

Time-Frequency and Time-Frequency-Rate Representations using the Cross Quadratic Spectrum

Saman S. Abeysekera

School of Electrical and Electronic Engineering
Nanyang Technological University
50, Nanyang Avenue, SINGAPORE 639798.
esabeysekera@ntu.edu.sg

Abstract — Traditionally used time-frequency distributions such as the STFT and the WVD are not suitable for FM signal characterization at low SNRs. A new method is proposed using the cross quadratic spectrum (XQS) which provides accurate temporal characterization for both signal's instantaneous frequency and the instantaneous-frequency-rate.

Keywords: Time-frequency distributions; short-time Fourier transform (STFT); Wigner-Ville distribution (WVD); Polynomial Fourier transform (PFT).

I. INTRODUCTION

Frequency modulated (FM) signals are encountered in many engineering applications. For example, frequency-shift keying and continuous phase modulation which comprise of FM signals are used in communication systems [1]. Linear FM (LFM) and quadratic FM (QFM) signals are common in radar and sonar transmissions [2]. Sinusoidal FM signals are present in astronomical signals arising from binary pulsar motion [3]. Certain signals of biological nature such as those in bat echolocations and dolphin communications are also consist of FM signals [4]. Usually it is necessary to characterize the FM signal in these applications. One of the most widely used FM signal characterization is obtained using the instantaneous frequency (IF) of the signal. The IF can be obtained through the short-time Fourier transform (STFT) or the Wigner-Ville distribution (WVD) of the signal [5]. Another important parameter for FM signal characterization is the instantaneous frequency-rate (IFR) which is also known as the chirp rate [6]. The Radon transformed WVD has been proposed in [7] for the determination of IFR. Most methods of FM parameter characterization techniques reported in the literature are mainly classified under the use of polynomial phase signals or the use of higher order ambiguity functions [8-10]. These methods are generalization of the WVD kernel and involve non-linearities of the second-order or higher-orders. As such, their performances at low signal-to-noise ratios (SNRs) are poor. However, many signal processing environments are associated with low SNRs. In this context, the polynomial Fourier transform (PFT) being a generalization of the Fourier transform is useful for FM signal characterization at low

SNRs [16, 17]. It is noted here that a large amount of research addresses the parameter estimation capabilities of time-frequency distributions. However, the low SNR detection threshold study of the methods are rather limited. The main contributions of this paper are as follows: (i) The investigation of the detection capabilities of the existing FM signal characterization methods, e.g. the use of STFT, WVD and PFT, (ii) proposal of an improved representation for the IF and IFR using the cross polynomial (quadratic) Fourier transform.

II. SIGNAL MODEL

This paper considers signals in the form given by

$$z(t) = Ae^{j\phi(t)} + v(t) \quad (1)$$

where $v(t)$ is a white Gaussian noise signal with variance σ^2 . The signal to noise ratio is defined as, $SNR = A^2/\sigma^2$. The instantaneous frequency (IF) and the instantaneous frequency-rate (IFR) associated with the signal in (1) are respectively given by,

$$f_i(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt} \quad , \quad \beta_i(t) = \frac{1}{2\pi} \frac{d^2\phi(t)}{dt^2} \quad (2)$$

Assuming a sample rate of $1Hz$, a sampled sequence corresponding to the signal in (1) is obtained as,

$$z(n) = Ae^{j\phi(n)} + v(n) \quad , \quad (0 \leq n \leq N-1) \quad (3)$$

where N is the observed number of samples. For example, a sampled linear frequency modulated (LFM) signal results when the phase function is selected as $\phi(n) = 2\pi(f_0n + (1/2)\beta_0n^2)$. For the LFM signal the sampled IF and IFR are respectively given by $f_i(n) = f_0 + \beta_0n$, and $\beta_i(n) = \beta_0$.

III. SIGNAL CHARACTERIZATION USING STFT AND WVD

This section investigates the use of the STFT and the WVD as time-frequency distributions for IF estimation. In particular, emphasis is on the processing of signals at low SNRs. The STFT and the WVD of the signal in (3) are respectively given by,

$$Z(k) = \sum_{n=0}^{N-1} z(n) e^{-j2\pi kn/N}, \quad (0 \leq k \leq N-1) \quad (4)$$

$$W(k) = \sum_{n=0}^{N-1} z(n) z^*(N-1-n) e^{-j2\pi kn/N}, \quad (0 \leq k \leq N-1) \quad (5)$$

Using a peak search on either the $|STFT|^2$, i.e. $|Z(k)|^2$ or the WVD, $W(k)$, the IF of the signal can be obtained. For an LFM signal with $f_0 = 0.20$ Hz and $\beta_0 = 0.20 \times 10^{-5}$ Hz/s, Figure 1 shows the behavior of the mean-square error (*m.s.e.*) of the IF estimation with respect to the SNR and the signal duration, N , for the $|STFT|^2$ and the WVD, respectively. At high SNR, both methods reach the Cramer-Rao lower bound (shown as long dashed lines). At low SNRs, the advantage of the $|STFT|^2$ is clear. The SNR thresholds (shown in Figure 1 as dotted lines) for the $|STFT|^2$ and the WVD are respectively given by

$$SNR_{th-STFT} = 24/N, \quad SNR_{th-WVD} = 12/\sqrt{N}. \quad (6)$$

Lowering the SNR below these thresholds would result in detection outliers [11].

Figure 1. M.S.E for IF estimation from STFT (upper) and WVD (lower). $N=32, 64, 128, 256, 512$, and 1024 (increasing down) in each plot.

Note that both STFT and the WVD are associated with an optimal window length for IF estimation. The optimal window

length which maximally concentrate the signal energy around the IF can be determined using the stationary phase principle and have been shown to be [12] [13]

$$L_{opt-STFT} = \sqrt{1/\beta_i(t)}, \quad L_{opt-WVD} = \left(24 \left| \frac{d\beta_i(t)}{dt} \right| \right)^{1/3} \quad (7)$$

Figure 2 shows the *m.s.e.* of IF estimation versus SNR when the LFM signal is selected as $f_0 = 0.20$ Hz and $\beta_0 = 0.30 \times 10^{-4}$ Hz/s. According to (7) the optimum WVD window length for the LFM is infinity, and thus it is often claimed that the WVD is optimum in the detection of LFM signals. On the other hand, the optimum $|STFT|^2$ window length for Figures 1 and 2 respectively are 707 and 183, and choosing a processing length greater than that in (7) is evident from the figures. It is worth to note here that the superiority of the WVD over the $|STFT|^2$ is achieved at high SNRs. At lower SNR, the $|STFT|^2$ has better performance, however it is limited by the optimal window defined in (7). Although, the advantage of the WVD in LFM signal detection is often highlighted in the literature its low SNR threshold is often not considered. In the following section we investigate other means of achieving optimal signal concentration for the possibility of IF estimation at a low SNR.

Figure 2. M.S.E for IF estimation from STFT (upper) and WVD (lower). $N=32, 64, 128, 256, 512$, and 1024 (increasing down) in each plot.

IV. DE-CHIRPING AND QUADRATIC SPECTRUM

In [14], the authors have discussed the optimal LFM signal detection through the use of the WVD. Later, it has been shown that the optimal LFM detection can be equivalently and efficiently performed in the time domain using a simple 'de-chirp' procedure. The de-chirping can also be expressed through the polynomial Fourier transform (here limited to the quadratic term representation) as given by [16],

$$Q(k, \beta) = \sum_{n=0}^{N-1} z(n) e^{-j\pi\beta n^2} e^{-j2\pi kn/N} \quad (0 \leq k \leq N-1) \quad (8)$$

The transform in (8) is denoted here as the quadratic Fourier transform (QFT). Using the modulus square of the QFT one can obtain a quadratic Fourier spectrum (denoted here as QFS). The QFS can also be implemented as segment averaging in an analogous manner to the implementation of the Welch periodogram [18]. In the following we would look at the performance of the QFS for the purpose of FM signal characterization.

Figure 3 shows the QFS of an LFM signal given by

$$z(n) = A e^{j(2\pi f_0 n + \pi \beta_0 n^2)} + v(n), \quad (-N/2 \leq n \leq N/2 - 1) \quad (9)$$

The signal description is in a symmetrical form around $n = 0$, and has following parameters: Centre (at $n = 0$) IF = 0.20Hz , IFR = 0.30Hz/s , SNR = 10dB . Duration, $N = 1048$ samples, segment length, $P = 64$ samples. The 3-D QFS plot (obtained through de-chirping and segment averaging) shown on top of Figure 3 clearly shows a peak appearing at corresponding IF and the IFR. However, the resolution in the QFS in Figure 3 is poor in the sense of localizing the signal energy at single IF and single IFR points. The middle and lower plots of Figure 3 show cuts of the QFS projected on to the frequency and the frequency-rate axes respectively. These projections elaborate the poor resolution of the QFS.

The noise level of the QFS: Let the noise power level, e.g. region between 0.6 and 0.8Hz of the QFS, be ξ . It is easy to show the following noise statistics. (These results are as same as those obtained for the averaged periodogram.)

$$\bar{\xi} = \sigma^2 / P, \quad \sqrt{\bar{\xi}^2} = \sigma^2 / \sqrt{NP/2} \quad (10)$$

For the parameters used in Figure 3 the noise mean and the standard deviations are -28.0dB and -31.6dB respectively. The noise mean has been removed from the middle and lower plots of Figure 3.

V. CROSS QUADRATIC SPECTRUM

To improve the resolution of the representation, here we propose to use a cross quadratic spectrum (XQS) between forward and backward segments, symmetrically selected around the centre of the signal. The XQS is constructed in the following manner.

- Fragment the data in to forward and backward segments of duration P . The i^{th} forward and backward segments are respectively selected as,

$$\begin{aligned} z_i^f(n) &= z((i-1)P + n + N/2) \\ z_i^b(n) &= z(-(i-1)P - n - 2 + N/2) \end{aligned} \quad ; \quad (0 \leq n \leq P-1). \quad (11)$$

- Perform QFT on both segments

$$\begin{aligned} Q_i^f(k, \beta) &= \sum_{n=0}^{P-1} z_i^f(n) e^{-j\pi\beta\alpha_i(n)} e^{-2j\pi kn/P} \\ Q_i^b(k, \beta) &= \sum_{n=0}^{P-1} z_i^b(n) e^{-j\pi\beta\alpha_i(n)} e^{-2j\pi kn/P} \end{aligned} \quad ; \quad (12)$$

where $\alpha_i(n) = ((i-1)P + n + N/2)^2$.

- Construct the XQS as a segment average of the products of the forward and backward QFTs.

$$XQS(k, \beta) = \left| \sum_{i=0}^{\lfloor N/P \rfloor - 1} Q_i^f(k, \beta) Q_i^b(k, \beta) \right| \quad (13)$$

Figure 3. (upper) QFS of an LFM; (middle) Projection of QFS to the frequency axis; (lower) Projection of QFS to the frequency-rate axis.

Note that the construction in (12) allows coherent addition of all the product terms such that the signal energy is concentrated around the IF at $n = 0$. Figure 4 shows the XQS resulting for the same signal as used in Figure 3. Again, the 3-D XQS plot of Figure 4 peaks at corresponding IF and the IFR. Improved resolution of the projections in Figure 4 in comparison to Figure 3 is clear.

The noise level of the XQS: Following noise statistics can be derived for XQS.

$$\bar{\xi} = 0, \quad \sqrt{\bar{\xi}^2} = \sigma^2 / \sqrt{NP/2} \quad (14)$$

Therefore, the noise standard deviations is as same as that of QFS but the noise in XQS has zero mean, thus has the advantage of not having to estimate the noise power for mean removal for obtaining unbiased estimates.

Figure 4. (upper) XQS of the LFM; (middle) Projection of XQS to the frequency axis; (lower) Projection of XQS to the frequency-rate axis.

VI. APPLICATIONS OF XQS

Detection of Multi-component Signals:

Consider a multi-component FM signal which consists of the addition of 3 LFM signals. The centre IF and the IFR of the 3 LFM signals are given by, $\{0.20 \ 0.30\}$, $\{0.23 \ 0.40\}$ and $\{0.33 \ 0.35\}$. The respective SNRs are $\{0, -3, -6\}dB$. Top of Figure 5 shows the QFS of the signal while the bottom plot shows the XQS of the signal obtained using the same parameters as used in Figures 3 and 4. The superior ability to detect the 3 signals due to the resolution improvement of the XQS is clearly evident.

Figure 5. (upper) QFS and (lower) XQS of the 3 LFM signals.

Time-Frequency and Time-Frequency-Rate Representations:

Finally, Figure 6 shows the use of the XQS in characterization of a QFM signal. The simulated QFM signal is of $N=12,800$ samples at $SNR=-13dB$. Sampling rate is 1Hz. The IFR of the QFM signal is given by

$$\beta(n) = 2 \times 10^{-7} (n - N/2) \text{ Hz/s}; (-N/2 \leq n \leq N/2 - 1) \quad (15)$$

According to (6), for $SNR = -13dB$, the required data length to avoid outliers in using the STFT and the WVD are 256 and 16000, respectively. Thus, the WVD would not be able to characterize the above signal. On the other hand, according to (7) STFT requires a window length less than 100. Figure 6 (top) shows the contour plot of the time-frequency distribution

obtained using a 256-length STFT which shows obscure signal characterization. Here we propose to use the XQS of (13) to obtain IF and IFR representation of the signal. This is achieved by slicing the XQS at its peak position along the frequency-rate axis and the frequency axis, respectively. The middle and lower plots of Figure 6 show the IF and IFR representation thus obtained using a window length of 256. As it is noted that the shortest window duration for processing at an SNR of -13dB , is 256, we have demonstrated that XQS can be used for accurate IF and IFR characterization even for non-linear FM signals.

Figure 6. (upper) Time-Frequency distribution obtained from STFT; (middle and lower) IF and IFR representations obtained from XQS.

VII. CONCLUSIONS

It is shown that the traditionally used time-frequency distributions, e.g. the STFT and the WVD are not suitable for FM signal characterization at low SNRs. The WVD requires a significantly higher SNR for outlier avoidance while the STFT suffers from the requirement of a smaller window length for energy concentration. In this paper a new method using the cross quadratic Fourier spectrum (XQS) is proposed which provides accurate temporal characterization for both signal's instantaneous frequency and the instantaneous-frequency-rate.

REFERENCES

- [1] J. G. Proakis, *Digital Communications*, McGraw Hill, New York, 1995.
- [2] S. Peleg and B. Porat, "Linear FM parameter estimation from discrete-time observations," *IEEE Transactions Aerosp. Electron. Syst.*, vol. AES-27, pp. 607-614, July 1991.
- [3] S. M. Ransom, J. M. Cordes, and S. S. Eikenberry, "A new search technique for short orbital period binary pulsars," *Astrophysical Journal*, vol. 589, pp. 911-920, 2003.
- [4] D. R. Griffin, *Listening in the dark*. Yale Univ. Press, New York, 1958.
- [5] L. Cohen, *Time-Frequency Analysis*, Englewood Cliffs, NJ:Prentice Hall, 1995.
- [6] T. Abotzoglou, "Fast maximum likelihood joint estimation of frequency and frequency rate," *IEEE Transactions Aerosp. Electron. Syst.*, vol. AES-22, pp. 708-715, 1986.
- [7] J. C. Wood and D. T. Barry, "Radon transformation of time-frequency distributions for analysis of multicomponent signals," *IEEE International Conference, Acoustic, Speech and Signal Processing, ICASSP-92*, vol. 4, pp. 257-261, 1992.
- [8] S. Peleg and B. Friedlander, "The discrete polynomial phase transform," *IEEE Transactions on Signal Processing*, vol. 43, pp. 1901-1914, Aug. 1995.
- [9] S. Barbarossa and V. Petrone, "Analysis of polynomial phase signals by an integrated generalized ambiguity function," *IEEE Transactions on Signal Processing*, vol. 47, pp. 316-327, Feb. 1997.
- [10] P. O'Shea, "A fast algorithm for estimating the parameters of a quadratic FM signals," *IEEE Transactions on Signal Processing*, vol. 52, pp. 385-393, Feb. 2004.
- [11] S. S. Abeysekera, "Efficient frequency estimation using the pulse-pair method at various lags," *IEEE Transactions on Communications*, vol. 54, no. 9, pp. 1542-1546, September 2006.
- [12] L. R. O. Storey, "An investigation of whistling atmospherics," *Phil. Trans. Roy. Soc.*, vol. A246, pp. 113-141, 1953.
- [13] S. S. Abeysekera, *Time-Frequency analysis of Electrocardiographic signals*, Ph.D. dissertation, Univ. Queensland, Australia, 1989.
- [14] S. Kay and G. F. Boudreaux-Bartels, "On the optimality of the Wigner distribution for detection," *IEEE International Conference, Acoustic, Speech and Signal Processing, ICASSP-85*, pp. 27.2.1-4, 1985.
- [15] W. Li, "Wigner distribution method equivalent to dechirp method for detecting a chirp signal," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. ASSP-35, pp. 1210-1211, Aug. 1987.
- [16] V. Katkovnik, "A New form of Fourier transform for time-frequency estimation," *Signal Processing*, vol. 47, pp. 187-200, 1995.
- [17] G. Bi, X. Li, and C. M. S. See, "LFM signal detection using LPP-Hough transform," *Signal Processing*, vol. 91, pp. 1432-1443, 2011.
- [18] P. D. Welch, "The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms," *IEEE Trans. on Audio and Electroacoustics*, vol. AU-15, pp. 70-73, 1967.