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Asymmetric Iterative Multi-Track Detection for 2D Non-binary LDPC Coded Magnetic Recording

Guojun Han, Yong Liang Guan, Kui Cai, and Kheong Sann Chan

Abstract

The inter-track interference (ITI) is a major impairment for the next-generation magnetic recording systems. The single-track read channel model, combined with anti-ITI signal detection schemes, is a general way to recover single-track data from a readback signal corrupted by the ITI. In contrast to the general way, the multi-track read channel model can be used to simultaneously recover multi-track data from a readback signal and we do not need to mitigate the ITI. However, the performance of the multi-track detection is severely degraded by the weak signals from the sidetracks and the indistinguishable symbols. In this paper, to improve this performance, we propose an asymmetric iterative multi-track detection (A-IMD) scheme, which uses two parallel jointed six-symbol-based detectors concatenated with two parallel non-binary low-density parity-check (NB-LDPC) decoder over GF(4) to iteratively recover four tracks' data from the two channel readback signals. Simulation results demonstrate that the performance of the multi-track detection can be greatly improved by using the proposed A-IMD scheme.

Index Terms

Bit-patterned media recording (BPMR), inter-track interference (ITI), multi-track detection, non-binary low-density parity-check (NB-LDPC) codes.

I. INTRODUCTION

Bit-patterned media recording (BPMR), heat (or microwave) assisted magnetic recording (HAMR) and shingled writing (SW)/two-dimensional magnetic recording (TDMR) have been explored to further increase the areal density of magnetic recording. However, with the reduction of the track pitch, the inter-track interference (ITI) has become a major impairment for these techniques. The newly introduced ITI when combined with the conventional inter-symbol interference (ISI) forms a two-dimensional (2D) interference which severely degrades the performance of channel detection.

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The single-track read channel model, combined with anti-ITI signal detection schemes which use the estimated sidetrack information, is a general way to recover single-track data from a readback signal corrupted by ITI [2], [3]. For example, in [2], a multi-track detection technique, which first performs sidetrack detections and then uses the estimated sidetrack information to assist the recovery of center track data from channel readback signals, is applied to joint-track equalized and 2D equalized BPMR channels. In [3], through splitting a binary low-density parity-check (LDPC) codeword into three segments and recording them on three adjacent tracks, the authors propose a scheme, which uses iterative detection between three inner detectors and an outer decoder to estimate the sidetrack information for each detector, to recover all data from the three tracks. Similar schemes in [4], [5] also use the estimates of sidetracks to mitigate the ITI in shingled writing (SW) and two-dimensional magnetic recording (TDMR) systems.

In contrast, the multi-track read channel model needs not consider the ITI and can simultaneously recover multi-track data from a readback signal. For example, an iterative multi-track detection (IMD) scheme with an eight-symbol-based detector concatenated with an eight-symbol-based decoder is used to perform multi-track detection in [6]. However, due to the indistinguishable symbols and the weakness of signals from sidetracks, the performance of this multi-track detection is severely degraded. In this paper, we consider a multi-track read channel model, called multiple islands per read head model [7], whose output from the read head is a function of the magnetization from several independently written tracks of islands, and propose an asymmetric iterative multi-track detection (A-IMD) scheme for 2D non-binary LDPC (NB-LDPC) coded BPMR channels to improve the performances of the channels. The proposed scheme consists of two parallel six-symbol-based detectors, which have two common track inputs and one separate track input, concatenated with two parallel NB-LDPC decoders over GF(4). Since the symbol-based detectors use six possible input symbols while the symbol-based decoders use four possible input symbols, the symbol probability mapping between the detectors and the decoders is asymmetric. The iteration between the detectors and decoders improves the detection performance of sidetracks and finally improve the performance of the multi-track detection. Furthermore, the A-IMD scheme can also use the symmetry of the channel response matrix to reduce the complexity of the standard Bahl-Cocke-Jelinek-Raviv (BCJR) detection algorithm [8].

The rest of the paper is organized as follows. Section II introduces the “multiple islands per read head” model. Section III presents the proposed A-IMD scheme for 2D NB-LDPC coded BPMR channels. The simulation results are demonstrated and discussed in Section IV. Finally, Section V concludes the paper.

II. CHANNEL MODEL

In the “multiple islands per read head” model for patterned media recording, a track consists of multiple parallel sub-tracks, and the read head, when centered over the middle sub-track, spans a specified fraction of the outer sub-tracks. Therefore, the readback signal is a function of the magnetization from all three sub-tracks. The discrete-time readback signal $r_{k,l}$ can be written as

$$r_{k,l} = \sum_n \sum_m h_{n,m} x_{k-n,l-m} + n_{k,l}, \quad (1)$$

where $x_{k,l}$ is the k -th magnetic data bit of the l -th track, $h_{n,m}$ is the 2D discrete-time island response of the channel, and $n_{k,l}$ is additive white Gaussian noise (AWGN).

For a $m = 3, n = 3$ scenario, the read head yields a channel response matrix of the form

$$H_I = \begin{pmatrix} h_{-1,1} & h_{0,1} & h_{1,1} \\ h_{-1,2} & h_{0,2} & h_{1,2} \\ h_{-1,3} & h_{0,3} & h_{1,3} \end{pmatrix}. \quad (2)$$

Let -1 represent data bit 0 and +1 represent data bit 1 on the magnetic recording media. If an arbitrary pattern of magnetization can be written on the three islands, there will be eight possible recorded symbols. However, due to the limitation of read head-medium geometry and the reading mechanization, these symbols cannot be easily distinguished and recovered from the readback signals by using the symbol-based BCJR detector [8]. For example, if the read head spans over all three islands, the assumed symmetries of the head-medium geometries yield a channel response matrix of the form

$$H_{II} = \begin{pmatrix} \beta & b & \beta \\ \beta & b & \beta \\ \beta & b & \beta \end{pmatrix}, \quad (3)$$

and if the read head partially spans the outer sub-tracks, the assumed symmetries of the head-medium geometries yield a channel response matrix of the form

$$H_{III} = \begin{pmatrix} \alpha & a & \alpha \\ \beta & b & \beta \\ \alpha & a & \alpha \end{pmatrix}, \quad (4)$$

where a, b, α and β are the relative contribution of islands to the readback signals. Suppose the previous symbol $(x_{k-1,1}, x_{k-1,2}, x_{k-1,3})$ and current symbol $(x_{k,1}, x_{k,2}, x_{k,3})$ are fixed. For the channel response matrix (3), according to the noiseless readback signal $y_{II}(k)$,

$$y_{II}(k) = \beta(x_{k-1,1} + x_{k-1,2} + x_{k-1,3} + x_{k+1,1} + x_{k+1,2} + x_{k+1,3}) + b(x_{k,1} + x_{k,2} + x_{k,3}) \quad (5)$$

it can be see that the readback signal is depend on the sum of $x_{k+1,1}$, $x_{k+1,2}$ and $x_{k+1,3}$ of the next symbol. Therefore, for this type of readback model, there only exist four distinguishable symbols and the corresponding information rate is $2/3$ bits per island. For the channel response matrix (4), according to the noiseless readback signal $y_{III}(k)$,

$$y_{III}(k) = a(x_{k,1} + x_{k,3}) + \beta(x_{k-1,2} + x_{k+1,2}) + b x_{k,2} + \alpha(x_{k-1,1} + x_{k-1,3} + x_{k+1,1} + x_{k+1,3}) \quad (6)$$

it can be see that the readback signal does not change when we exchange $x_{k+1,1}$ and $x_{k+1,3}$ of the next symbol. Therefore, for this type of readback model, it can not distinguish the symbols $(-1, -1, 1)$ and $(1, -1, -1)$, $(-1, 1, 1)$ and $(1, 1, -1)$ from each other and the corresponding information rate is $\log_2(6) = 0.8617$ bits per island.

Fig. 1. Information bit encoding and writing process.

III. ASYMMETRIC ITERATIVE MULTI-TRACK DETECTION

For the channel response matrix (3), since the read head spans over all three islands, the readback signal is vulnerable to the interference of the other tracks, while for the channel response matrix (4), this interference can be avoided since the read head only partially spans the two adjacent tracks. Therefore, for the channel response matrix (4), if we use the IMD scheme to perform channel detection, we do not need to consider the ITI. However, due to the indistinguishable symbols caused by the symmetry of channel response matrix and the weakness of signals from sidetracks, this detection performance is severely degraded. A small shift off the center track read can eliminate some symmetries of the response matrix. However, if the shift off is too small, it still has difficulty to distinguish all eight possible symbols, and a large shift off will introduce new ITI. Moreover, the off-track shift read can further reduce the readback signal strength of one sidetrack. In the following, both the 2D NB-LDPC coding and the A-IMD scheme are together used to improve the detection performance of this channel.

A. 2D NB-LDPC Coded BPMR Channels

Binary LDPC codes [9] have been shown to approach the Shannon limit performance when decoded with iterative belief-propagation (BP) for very long block lengths [10], [11]. But for short to moderate block lengths, NB-LDPC codes defined over $\text{GF}(q)$ ($q > 2$) have shown significant improvement in performance over their binary counterparts [12]. Here we use NB-LDPC codes over $\text{GF}(4)$ to encode information bits and further to eliminate the indistinguishable symbols mentioned in Section II. As shown in Fig. 1, on the encoder side, an information bit sequence is first converted to a non-binary symbol sequence, then the symbol sequence is encoded by a NB-LDPC encoder over $\text{GF}(4)$ and each two encoded frames are recorded on the four adjacent tracks, where the high bits of symbols in the encoded frame 1 and 2 are recorded on the tracks 1 and 3, respectively, while the low bits of symbols in the encoded frame 1 and 2 are recorded on the tracks 2 and 4, respectively. In Fig. 1, the modulators are used to generate writing signals. Since each bit of encoded symbols is recorded on the adjacent tracks rather than on the same track, this forms a 2D NB-LDPC coded BPMR channel.

The 2D symbols recorded on the islands and the associated outputs are shown in Fig. 2. Two bits of each symbol are separately recorded on two adjacent tracks and four tracks form a group. The readback signal from tracks 1, 2 and 3 is denoted as read head output 1, while the readback signal from tracks 2, 3 and 4 is denoted as read head output 2. Tracks 2 and 3 are common tracks for read head output 1 and read head output 2.

Fig. 2. 2D recorded symbols on the islands and the associated outputs.

B. Asymmetric Iterative Multi-Track Detection Scheme

The A-IMD scheme is shown in Fig. 3, it uses the outputs from the upper three tracks and the lower three tracks of a group to perform joint iterative detection. The scheme can be divided into two identical parts: the detection of the upper tracks and the detection of the lower tracks. Each part consists of a reduced-BCJR detector, a symbol probability mapping from detector to decoder, a symbol probability mapping from decoder to detector and a NB-LDPC decoder. Since the upper and lower symbol-based detectors use six possible input symbols and work hand in hand, while the upper and lower symbol-based decoders use four possible input symbols and work separately, the symbol probability mapping between the detectors and the decoders is asymmetric. The symbols for multi-track detection and the symbols for 2D NB-LDPC decoding are shown in Fig. 2. To perform the symbol mapping between the detector and the decoder, the extrinsic information is exchanged between the upper detection and the lower detection. The new scheme can be implemented in parallel or serial. For simplicity, here we only consider the serial detection method.

C. Reduced-BCJR Detection

In the proposed A-IMD scheme, we use the symbol-based BCJR algorithm for channel detection [8]. Due to the symmetry of the channel response matrix, the number of states and the number of input symbols of the channel detection trellis is greatly reduced in [2], [13]. Suppose $x_{k+1,1}$, $x_{k+1,2}$ and $x_{k+1,3}$ are the three binary inputs for channel detection trellis. Since $x_{k+1,1}$ and $x_{k+1,3}$ have the identical contribution to the channel readback signal, we can use binary input $x_{k+1,2}$ and non-binary input w_{k+1} to represent the inputs, where $w_{k+1} = x_{k+1,1} + x_{k+1,3}$ and $w_{k+1} \in \{-2, 0, 2\}$. For this 3×3 response matrix, with an input of $\{x_{k+1,1}, x_{k+1,2}, x_{k+1,3}\}$, the number of possible input symbols is 8 and the trellis needs 8×8 states. However, with an input $\{x_{k+1,2}, w_{k+1}\}$, the number of possible input symbols is 6 and the trellis only needs 6×6 states. Therefore, we can obtain a reduced complexity trellis for channel detection.

To perform channel detection on this reduced complexity trellis with an input of $\{x_{k+1,2}, w_{k+1}\}$, the forward

Fig. 3. Asymmetric iterative multi-track detection scheme.

and backward recursions of the reduced-BCJR algorithm have the same expressions as those of the standard BCJR algorithm

$$\alpha_k(s_k) = \sum_{s_{k-1}} \alpha_{k-1}(s_{k-1}) \gamma(s_k, s_{k-1}), \quad (7)$$

$$\beta_k(s_k) = \sum_{s_{k+1}} \beta_{k+1}(s_{k+1}) \gamma(s_k, s_{k+1}), \quad (8)$$

where $\alpha_k(s_k)$ and $\beta_k(s_k)$ are the state probabilities of forward and backward, respectively, and $\gamma(s_k, s_{k-1})$ is branch transition probability from state s_{k-1} to s_k , it can be computed by

$$\gamma(s_k) = p(x_{k,2}, w_k) p_s(r_k | s_{k-1}, s_k, x_{k,2}, w_k), \quad (9)$$

where $p(x_{k,2}, w_k)$ is the a priori probability (APP) of a detected symbol to be $\{x_{k,2}, w_k\}$. For the recording with three islands per symbol, since there exist eight equiprobability recording patterns, $p(x_{k,2}, w_k)$ will be initialized with $p(-1, 0) = p(-1, -1, 1) + p(1, -1, -1) = 2/8$, $p(1, 0) = p(-1, 1, 1) + p(1, 1, -1) = 2/8$, where $p(-1, -1, 1)$ and $p(1, -1, -1)$ are the APPs of a recorded symbol to be $(-1, -1, 1)$ or $(1, -1, -1)$, and the others equal $1/8$. Then the posteriori probability of a detected symbol to be $\{x_{k,2}, w_k\}$ can be computed as

$$P(x_{k,2}, w_k | \mathbf{r}) = \frac{1}{P(\mathbf{r})} \sum_{s_{k-1}} \alpha_{k-1}(s_{k-1}) \gamma(s_{k-1}, s_k) \beta_k(s_k), \quad (10)$$

where $P(\mathbf{r})$ is a probability of the received block. The extrinsic information at the output of this detector is

$$p^e(x_{k,2}, w_k) = P(x_{k,2}, w_k | \mathbf{r}) / p(x_{k,2}, w_k). \quad (11)$$

D. Symbol Probability Mapping Between Detectors and Decoders

According to Fig. 3, the inputs and the outputs of the detectors are 6-tuple APPs and 6-tuple extrinsic information for a detected symbol over GF(8), respectively, and the inputs and the outputs of the decoders are 4-tuple APPs and 4-tuple extrinsic information, 4-tuple posteriori probabilities for a detected symbol over GF(4), respectively. Therefore, from the detectors to the decoders, we need to map 6-tuple extrinsic information to 4-tuple APPs, and

from the decoder to the detectors, we need to map 4-tuple extrinsic information to 6-tuple APPs. Thus, these mappings are asymmetric.

Let us first investigate the symbol probability mappings from the detectors to the decoders. Suppose $v(x_{k,1}, x_{k,2})$ and $V(x_{k,1}, x_{k,2})$ are the APP and the *posterior* probability of the k -th detected symbol to be $\{x_{k,1}, x_{k,2}\}$ before decoding and after decoding, respectively, where $\{x_{k,1}, x_{k,2}\} \in \{\pm 1, \pm 1\}$. Thus, the extrinsic information at the output of the NB-LDPC decoder can be computed as

$$v^e(x_{k,1}, x_{k,2}) = V(x_{k,1}, x_{k,2}) / v(x_{k,1}, x_{k,2}). \quad (12)$$

In the following, we use subscript “ u ” and “ l ” to denote corresponding variables for the upper and the lower detections, respectively. According to Fig. 3, the marginal probabilities of $V_u(x_{k,1}, x_{k,2})$ and $V_l(x_{k,3}, x_{k,4})$ are

$$\begin{aligned} V_u(x_{k,2} = 1) &= V_u(-1, 1) + V_u(1, 1) \\ V_u(x_{k,2} = -1) &= 1 - V_u(x_{k,2} = 1) \\ V_l(x_{k,3} = 1) &= V_l(1, -1) + V_l(1, 1) \\ V_l(x_{k,3} = -1) &= 1 - V_l(x_{k,3} = 1), \end{aligned} \quad (13)$$

respectively. Then the symbol probability mappings from the upper detector to the upper decoder are

$$\begin{aligned} v_u(-1, -1) &= p_u^e(-1, -2) + p_u^e(-1, 0) \cdot V_l(x_{k,3} = 1) \\ v_u(-1, 1) &= p_u^e(-1, -2) + p_u^e(-1, 0) \cdot V_l(x_{k,3} = -1) \\ v_u(1, -1) &= p_u^e(1, -2) + p_u^e(1, 0) \cdot V_l(x_{k,3} = 1) \\ v_u(1, 1) &= p_u^e(1, -2) + p_u^e(1, 0) \cdot V_l(x_{k,3} = -1), \end{aligned} \quad (14)$$

and the symbol probability mappings from the lower detector to the lower decoder are

$$\begin{aligned} v_l(-1, -1) &= p_l^e(-1, -2) + p_l^e(-1, 0) \cdot V_u(x_{k,2} = 1) \\ v_l(-1, 1) &= p_l^e(-1, -2) + p_l^e(-1, 0) \cdot V_u(x_{k,2} = -1) \\ v_l(1, -1) &= p_l^e(1, -2) + p_l^e(1, 0) \cdot V_u(x_{k,2} = 1) \\ v_l(1, 1) &= p_l^e(1, -2) + p_l^e(1, 0) \cdot V_u(x_{k,2} = -1), \end{aligned} \quad (15)$$

For the upper detection, since the lower detection does not output any extrinsic information at the first conversion, both $V_l(x_{k,3} = 1)$ and $V_l(x_{k,3} = -1)$ are initialized to $1/2$.

For iterative multi-track detection between the detectors and the decoders, the extrinsic information at the output of the NB-LDPC decoders is used to update the APPs of symbols for the reduced-BCJR detectors. The symbol

probability mappings from the upper NB-LDPC decoder to the upper reduced-BCJR detector are

$$\begin{aligned}
 p_u(-1, -2) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = -1) \\
 p_u(-1, -1) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = 1) \\
 p_u(-1, 0) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = -1) \\
 p_u(-1, 1) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = 1) \\
 p_u(0, -2) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = -1) \\
 p_u(0, -1) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = 1) \\
 p_u(0, 0) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = -1) \\
 p_u(0, 1) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = 1) \\
 p_u(1, -2) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = -1) \\
 p_u(1, -1) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = 1) \\
 p_u(1, 0) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = -1) \\
 p_u(1, 1) &= v_u^e(-1, -1) \cdot V_l(x_{k,3} = 1)
 \end{aligned} \tag{16}$$

and the symbol probability mappings from the lower NB-LDPC decoder to the lower reduced-BCJR decoder are

$$\begin{aligned}
 p_l(-1, -2) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = -1) \\
 p_l(-1, -1) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = 1) \\
 p_l(-1, 0) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = -1) \\
 p_l(-1, 1) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = 1) \\
 p_l(0, -2) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = -1) \\
 p_l(0, -1) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = 1) \\
 p_l(0, 0) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = -1) \\
 p_l(0, 1) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = 1) \\
 p_l(1, -2) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = -1) \\
 p_l(1, -1) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = 1) \\
 p_l(1, 0) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = -1) \\
 p_l(1, 1) &= v_l^e(-1, -1) \cdot V_u(x_{k,2} = 1)
 \end{aligned} \tag{17}$$

E. Iterative Detection and Extrinsic Information Exchange

As shown in Fig. 2, we can directly use the outputs from the detectors of the upper three tracks and the lower three tracks to calculate the bit probabilities of track 2 and track 3, respectively. The bit probability of track 2 is

$$\begin{aligned}
 P(x_{k,2} = 1) &= P(x_{k,2} = 1, w_{k,u} = -2) + P(x_{k,2} = 1, w_{k,u} = 0) \\
 &\quad + P(x_{k,2} = 1, w_{k,u} = 2),
 \end{aligned} \tag{18}$$

where $w_{k,u} = x_{k,1} + x_{k,3}$, and the bit probability of track 3 is

$$\begin{aligned}
 P(x_{k,3} = 1) &= P(x_{k,3} = 1, w_{k,l} = -2) + P(x_{k,3} = 1, w_{k,l} = 0) \\
 &\quad + P(x_{k,3} = 1, w_{k,l} = 2),
 \end{aligned} \tag{19}$$

where $w_{k,l} = x_{k,2} + x_{k,4}$. With the aid of $P(x_{k,3})$ and $P(x_{k,2})$, we can now distinguish the symbols $(-1, -1, 1)$ and $(1, -1, -1)$, $(-1, 1, 1)$ and $(1, 1, -1)$, at the outputs from detectors of the upper three tracks and the lower three tracks, respectively. Therefore, the proposed A-IMD scheme can distinguish all eight possible symbols from the readback signal of the channel. Furthermore, this joint collaborative detection method between the upper three tracks and the lower three tracks improves the detection performance of the sidetracks.

TABLE I
COMPARISON OF NB-LDPC DECODING COMPLEXITY PER ITERATION (12 TRACKS AS A UNIT)

	The A-IMD scheme	The IMD scheme
q	4	8
Number of FFT-Based BP	6	4
$O(q \log_2(q))$	8	24

F. Complexity Analysis

Since the proposed A-IMD scheme can simultaneously recover 4 tracks' data while the conventional IMD scheme recover 3 tracks' data, to make fair comparison, we analyze the computational complexity of recovering 12 tracks' data for each scheme. Let us first investigate the computational complexity of NB-LDPC decoding. Here, we consider FFT-Based BP algorithm in which the check node computation can be reduced to $O(q \log_2(q))$ additions and multiplications [14]. For the A-IMD scheme, it needs to perform 6 times FFT-Based BP over GF(4) per iteration, while the IMD scheme needs to perform 4 times FFT-Based BP over GF(8) per iteration. The number of FFT-Based BP performed in each iteration and the corresponding complexity are shown in Table I. The result of multiplying the number of FFT-Based BP and $O(q \log_2(q))$ indicates that the NB-LDPC decoding complexity in the proposed A-IMD scheme is much lower than that in the conventional IMD scheme. For channel detection complexity, the proposed A-IMD scheme needs to perform 6 times reduced-BCJR detection per iteration, while the conventional IMD needs to perform 4 times standard BCJR detection per iteration. This indicates that the proposed A-IMD scheme has higher detection complexity as compared with the conventional IMD scheme. However, when we consider that the reduced-BCJR uses 36 states while the standard BCJR uses 64 states, the detection complexity in the proposed A-IMD scheme is also reduced as compared with that in the conventional IMD scheme. All these make the proposed A-IMD scheme have much lower computational complexity as compared with the conventional IMD scheme. Although here we only consider the FFT-Based BP algorithm, the other low-complexity algorithm, such as extended min-sum (EMS) [15], also can be used to further reduce the computational complexity of the proposed A-IMD scheme.

Another point that we need to consider is the latency and the hardware implementation. In order to reduce the latency of detection and decoding, the proposed A-IMD scheme can be implemented in parallel. This needs two reduced-BCJR detectors and two NB-LDPC decoders. To lower hardware cost, serial implementation of this multi-track detection scheme also is feasible, it only needs one reduced-BCJR detector and one NB-LDPC decoder.

IV. SIMULATION RESULTS

In the simulations, two types of binary LDPC codes are used to evaluate the bit error rate (BER) performance of the proposed A-IMD scheme. The first is a (576, 1152) rate 1/2 quasi cyclic LDPC (QC-LDPC) code [16], and the other is a (222, 1998) rate 8/9 LDPC code [17]. The NB-LDPC codes are constructed by replacing the nonzero

Fig. 4. BER performance comparison between the proposed A-IMD scheme and the conventional IMD scheme over the (576, 1152) NB-LDPC coded BPMR channels when H matrix is employed (The dashed lines are for sidetracks and the solid lines are for center tracks).

elements of the corresponding binary parity matrix with uniformly distributed nonzero elements over $\text{GF}(q)$ ($q > 2$). For the conventional IMD scheme, the information bits are first encoded by the NB-LDPC code encoder over $\text{GF}(8)$ and recorded on three adjacent tracks, then the symbol-based detector concatenated with the symbol-based decoder are used to perform iterative detection [6].

We use the following 3×3 channel response matrix as described in [7].

$$H = \begin{pmatrix} 0.06 & 0.2 & 0.06 \\ 0.20 & 0.6 & 0.20 \\ 0.06 & 0.2 & 0.06 \end{pmatrix}, \quad (20)$$

which means that the head partially spans the outer sub-tracks. The signal-to-noise ratio (SNR) in decibels is defined as

$$\text{SNR} = 10 \log\left(\frac{\sum_{m,n} h_{m,n}^2}{2R_c\sigma^2}\right), \quad (21)$$

where R_c is the LDPC code rate. For both the proposed A-IMD scheme and the conventional IMD scheme, the maximum number of iterations for LDPC decoding is set to 10, and the maximum number of iterations between the detectors and the decoders is denoted by $Iter$. In all simulations, $q = 4$ is used for the proposed A-IMD scheme and $q = 8$ for the conventional IMD scheme.

Figure 4 shows the BER performance comparison between the proposed A-IMD scheme and the conventional IMD scheme over the (576, 1152) NB-LDPC coded BPMR channels. For the proposed A-IMD scheme, since the tracks 1 and 4 have the same BER performance, and the tracks 2 and 3 have the same BER performance, we use the dashed line and the solid line to represent the averaged BER of the tracks 1 and 4, and the tracks 2 and 3, respectively. Similarly, for the conventional IMD scheme, the dashed line represents the averaged BER of the tracks 1 and 3, and the solid line represents the BER of the track 2. To facilitate the comparison, we also show the BER performance of the reduced-BCJR detector without using iterative detection in Fig. 4. For both the two schemes,

Fig. 5. BER performance comparison between the proposed A-IMD scheme and the conventional IMD scheme over the (222, 1998) NB-LDPC coded BPMR channels when H matrix is employed (The dashed lines are for sidetracks and the solid lines are for center tracks).

the dashed lines and the solid lines are also used to represent the BER of the sidetracks and the center tracks, respectively. From Fig. 4, we can see that the BER performance of the sidetracks and the center tracks is greatly improved by using the proposed A-IMD scheme. Specifically, the proposed A-IMD scheme with $Iter=5$ achieves about 4dB gain as compared with the conventional IMD scheme with $Iter=5$ at $BER=2 \times 10^{-5}$. This performance gain can be attributed to the fact that the proposed scheme enhances the detection capability of sidetracks and eliminates the indistinguishable symbols. From Fig. 4, we also see that the proposed A-IMD scheme has a slower convergence rate than the conventional IMD scheme when $Iter$ is set too small. This can be explained by the fact that the proposed A-IMD scheme needs the exchange of the extrinsic information between the upper detection and the lower detection to enhance the detection capability of sidetracks. For the A-IMD scheme, with $Iter$ increase, we see that the performance gain provided by the number of iterations is quickly reduced.

Figure 5 shows the BER performance comparison between the proposed A-IMD scheme and the conventional IMD scheme over the (222, 1998) NB-LDPC coded BPMR channels. For this high rate NB-LDPC coded BPMR channel, from Fig. 5, we see that the proposed A-IMD scheme still can achieve a good BER performance with a waterfall characteristic. However, this cannot be achieved by the conventional IMD scheme. Furthermore, with SNR increase, the conventional IMD scheme does not provide much more perform gain as compared with that of the reduced-BCJR detector. This can be explained with the fact that the number of errors caused by the weak signals from the sidetracks and the indistinguishable symbols is beyond the error correction capability of the NB-LDPC code with high code rate.

To further investigate the performance of the proposed A-IMD scheme on channel with asymmetry response

Fig. 6. BER performance comparison between the proposed A-IMD scheme and the conventional IMD scheme over the (576, 1152) NB-LDPC coded BPMR channels when H_A matrix is employed (The dashed lines are for sidetracks and the solid lines are for center tracks).

matrix, we modify channel response matrix (20) to

$$H_A = \begin{pmatrix} 0.06 & 0.2 & 0.06 \\ 0.20 & 0.6 & 0.20 \\ 0.0618 & 0.22 & 0.0618 \end{pmatrix}, \quad (22)$$

where the readback signal from $x_{k,3}$ is increased about 10%, and the readback signals from $x_{k-1,3}$ and $x_{k+1,3}$ are increased proportionally. Theoretically, the eight-symbol-based decoder can distinguish all symbols from readback signals of this channel.

The BER performances of the proposed A-IMD scheme and the conventional IMD scheme over this channel are shown in Fig. 6, where the (576, 1152) NB-LDPC code is employed. From Fig. 6, we see that the proposed A-IMD still achieves much better performance as compared with the conventional IMD scheme, and the BER performance of the conventional IMD scheme do not be greatly improved by using this asymmetric channel response matrix. This verifies that the weak asymmetry cannot make the symbol-based decoder easily distinguish all symbols from readback signals corrupted by ISI, ITI and AWGN. Moreover, the conventional IMD scheme cannot enhance the detection capability of adjacent sidetracks.

The complexity of detection, in terms of detection-time per track, of the proposed A-IMD scheme and the IMD scheme over the (576, 1152) NB-LDPC coded BPMR channels are shown in Fig. 7. It can be seen that the proposed A-IMD scheme for both $Iter=2$ and 5 requires much less detection-time per track compared with the IMD scheme. We also note that the proposed A-IMD scheme with $Iter=5$ saves more and more detection-time per track with SNR increase. This reduction of detection-time is mainly due to the reduced average number of iterations for both the joint iterative detection between the detector and the decoder and the iterative decoding of LDPC codes.

Fig. 7. Complexity comparison (in terms of detection-time per track) between the proposed A-IMD scheme and the conventional IMD scheme over the (576, 1152) NB-LDPC coded BPMR channels when H matrix is employed.

V. CONCLUSIONS

In this paper, we have considered a “multiple islands per read head” model for BPMR channels and proposed an asymmetric multi-track detection scheme with the aim to improve the performance of the channels. We have shown that the proposed scheme can enhance the detection capability of adjacent sidetracks and eliminate the indistinguishable symbols. We have presented the simulation results over the 2D NB-LDPC coded BPMR channels and shown that the proposed scheme with $Iter=5$ provides about 4dB gain as compared with the conventional IMD scheme with $Iter=5$ at $BER=2 \times 10^{-5}$. Furthermore, the proposed A-IMD scheme has much lower computational complexity compared with the IMD scheme. Our work has indicated that the multi-track read channel model can be used to simultaneously recover data from several tracks and we do need not to mitigate ITI. It is hopeful to further improve the detection performance by combining the proposed scheme with the redesigned read heads and the optimized response matrices.

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