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A Multiple-DOF Piezoelectric Energy Harvesting Model

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ABSTRACT

Conventional vibration energy harvesters have been usually studied as single-degree-of-freedom (1DOF) models. The fact that such harvesters are only efficient near sole resonance limits their applicability in frequency-variant or random vibration scenarios. In this paper, a novel multiple-DOF piezoelectric energy harvesting model (PEHM) is presented. First, a 2DOF model is analyzed and its two configurations are characterized. In the first configuration, the piezoelectric element is placed between one mass and the base, and in the second configuration it is placed between the two masses. It is shown that the former is advantageous over the latter since with a slight increase of overall weight to the 1DOF model, we can achieve two close and effective peaks in power response, or one effective peak with significantly enhanced magnitude. The first configuration is then generalized to an n -DOF model and its analytical solution is derived. This solution provides a convenient tool for parametric study and design of a multiple-DOF PEHM. Finally, the equivalent circuit model (ECM) of the proposed n -DOF PEHM is developed via the analogy between the mechanical and electric domains. With the ECM, system-level electric simulation can be performed to evaluate the system performance when sophisticated interface circuits are attached.

Keywords: piezoelectric, energy harvesting, multiple DOF, resonance, broadband

1. INTRODUCTION

In recent years, vibration energy harvesting has aroused immense research interests from various disciplines as it provides a promising solution for self-powered wireless sensing electronics. Conventional vibration energy harvesters reported in the literature have been usually designed as 1DOF models, which are only efficient near sole resonance. Unfortunately, the vast majority of practical vibration sources are present in the frequency-variant or random form. Hence, a critical issue in vibration energy harvesting research is how to improve the functionality of energy harvesters in various practical vibration scenarios.

One approach to enable a vibration energy harvester adaptive in frequency-variant scenarios is the resonance tuning technique. Mechanical preload (Leland and Wright, 2006; Hu *et al.*, 2007) and magnetic force (Challa *et al.*, 2011) have been frequently exerted to tune the stiffness and thus the fundamental resonant frequency of a harvester. Besides, broadband energy harvesting can be achieved by introducing nonlinearity into the harvester. Stanton *et al.* (2009) proposed a monostable nonlinear energy harvester, in which the bent hardening or softening response curve can provide a wide bandwidth. Erturk *et al.* (2009a) investigated the bistable mechanism of a broadband piezo-magneto-elastic generator under sinusoidal excitations. Cottone *et al.* (2009), Ferrari *et al.* (2010) and Andò *et al.* (2010) further studied the energy harvesting performance of

bistable cantilevers with repulsive magnets under wide-spectrum vibrations. Soliman *et al.* (2009) and Liu *et al.* (2012) investigated the broadband performance of the energy harvesters with piecewise-linear stiffness by introducing mechanical stoppers. To date, however, how to enable the harvester to persist in high energy orbits for large-amplitude oscillations remains a critical issue in nonlinear energy harvester design.

On the other hand, a system with multiple modes is also capable of harvesting broadband vibration energy. Roundy *et al.* (2005) first proposed the idea of multiple-DOF system incorporating multiple proof masses to achieve wider bandwidth. Based on this idea, Yang *et al.* (2009) developed an electromagnetic energy harvesting beam with multiple magnets as proof masses and voltage inducing components. Ou *et al.* (2010) theoretically modeled a two-mass cantilever beam for broadband energy harvesting. Tadesse *et al.* (2009) presented a cantilever harvester integrated with hybrid energy harvesting schemes, each of which is efficient for a specific mode. However, in these designs, the high-order modes are far away from the fundamental mode.

When designing a multimodal energy harvester, having its multiple modes close to each other is rationally preferable for practical random or frequency-variant vibrations. Aldraihem and Baz (2011) and Arafa *et al.* (2011) studied a 2DOF piezoelectric energy harvester with one mass served as a dynamic magnifier. Although close resonances could be achieved, the magnifier required a huge weight, which places some limitations in practice. Erturk *et al.* (2009b) introduced an L-shaped cantilever harvester where the second natural frequency approximately doubles the first. Jang *et al.* (2010) and Kim *et al.* (2011) developed 2DOF electromagnetic and piezoelectric energy harvesting devices in which translation and rotation vibration modes of a single mass were exploited and the two natural frequencies could be designed to be very close to each other. Another way to achieve multiple close resonant frequencies is to assemble an array of cantilever harvesters on a common rigid base (Shahruz, 2006; Xue *et al.*, 2008; Ferrari *et al.*, 2008; Lien and Shu, 2011), which renders certain flexibility in design and deployment. For example, a large piece of cantilever harvester can be cut into multiple small pieces and arranged vertically in the applications with area constraint for installing. The geometry and proof mass of the cantilevers can be carefully selected such that their resonant frequencies can be close to each other. However, such configuration of achieving broad bandwidth significantly sacrifices the power density with the fact that only a single cantilever is active for energy harvesting while others are relatively 'silent'. Hence, a fine multimodal energy harvesting system is expected to achieve multiple close and effective resonant peaks in targeted bandwidth with least sacrifice of power density (i.e., with slight increase of system weight or volume).

In this paper, a novel multiple-DOF piezoelectric energy harvesting model (PEHM) is presented to overcome the bandwidth issue of conventional PEHM. First, a 2DOF model is investigated and its two configurations are characterized. In the first configuration, the piezoelectric element is placed between the one mass and the base, and in the second configuration, it is placed between the two masses. The advantage of the former is that with slight increase of overall weight to the conventional 1DOF model, we can achieve two close and effective peaks in power response (though unable to outperform the 1DOF model in magnitude), or one peak with significantly enhanced magnitude. Thus, this configuration overcomes the limitations in the previous 2DOF

models. The second 2DOF configuration can provide two close and enhanced peaks with a huge increase of overall weight, which may not improve or even deteriorate the system efficiency in terms of power density. Even if the power density is not concerned, it can only significantly outperform the 1DOF model with unpractical and unfavorable damping. Based on the characterization of the 2DOF model, the first configuration is subsequently generalized to an n -DOF model and its analytical solution is provided. With such n -DOF model and parametric study, one can design a PEHM to have multiple close peaks with effective magnitude to contribute to energy harvesting, or to have one effective peak with substantially enhanced magnitude, with only slight increase of system weight. Finally, the equivalent circuit model (ECM) of the proposed n -DOF PEHM is developed via the analogy between mechanical and electric domains. With the ECM, system-level electric simulation can be performed to evaluate the energy harvesting performance of the n -DOF PEHM when sophisticated interface circuits are attached, which is difficult to address in analytical modeling.

2. ANALYTICAL MODELING

2.1 Conventional 1DOF Model

A PEHM is conventionally designed as a 1DOF lumped parameter model to simplify the formulation procedure (DuToit *et al.* 2005). This model is widely adopted in the literature especially when sophisticated interface circuits are considered (Guyomar *et al.*, 2005; Shu and Lien, 2006; Lien *et al.*, 2010; Tang and Yang, 2011; Liang and Liao, 2011, 2012). Since the 1DOF model provides the fundamental insights of the electromechanical coupled characteristics of a PEHM, it is quickly reviewed here.

Figure 1 1DOF PEHM

The schematic of 1DOF PEHM under base excitation is depicted in Figure 1. The governing equations of the system can be written as,

$$\begin{cases} m_1 \ddot{u}_1 = -k_1(u_1 - u_0) - \eta_1(\dot{u}_1 - \dot{u}_0) - \theta V \\ -\theta(\dot{u}_1 - \dot{u}_0) + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (1)$$

where m_1 , k_1 , η_1 are the mass, stiffness and mechanical damping of the system, respectively; C^S is the clamped capacitance of the piezoelectric element; θ is the electromechanical coupling coefficient of the system; R_l is the electric load; V is the voltage across R_l ; $-\theta V$ is the force induced by backward electromechanical coupling in the dynamic equation; u_0 and u_1 are the displacements of base and mass m_1 , respectively. Setting $x = u_1 - u_0$ and applying the Laplace transform for (1), we have

$$\begin{cases} m_1 s^2 \hat{X} + \eta_1 s \hat{X} + k_1 \hat{X} + \theta \hat{V} = -s^2 m_1 \hat{U}_0 \\ -\theta s \hat{X} + C^S s \hat{V} + \hat{V}/R_l = 0 \end{cases} \quad (2)$$

where s is the Laplace variable; the cap denotes a variable or function in the Laplace domain. Solving (2) and setting $s = j\omega$, we obtain the dimensionless voltage across R_l as,

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{1}{(1 - \Omega^2 + j2\zeta_1 \Omega) \frac{j r \Omega + 1}{j r k_e^2 \Omega} + 1} \right| \quad (3)$$

in which the dimensionless parameters are

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \zeta_1 = \frac{\eta_1}{2\sqrt{k_1 m_1}}, \Omega = \frac{\omega}{\omega_1}, r = \omega_1 C^s R_l, k_e^2 = \frac{\theta^2}{C^s k_1} \quad (4)$$

2.2 2DOF Model

A conventional 1DOF PEHM is only efficient to harness energy near its sole resonant frequency. This intrinsic drawback of 1DOF model limits its applicability in the majority of practical vibration scenarios. Though a distributed parameter model such as a cantilever beam with proof mass carries multiple modes, the contribution to energy harvesting from the high-order modes is usually neglected as they are far away from the fundamental one. Hence, a conventional distributed parameter model can be reduced to a single-mode harvester, which is actually an improved 1DOF model with some correction factor introduced to forcing amplitude (Erturk and Inman, 2008). Obviously, it is preferable not only that an energy harvesting system has multiple modes or multiple DOFs but also that the corresponding peaks in the response are close to each other and with effective magnitudes to contribute to energy harvesting. To design such a multiple-DOF PEHM is the motivation of this paper.

We begin the analysis of multiple-DOF PEHM from the 2DOF lumped parameter model. There are two configurations of the 2DOF model according to the location of the piezoelectric element, as shown in Figure 2. Configuration B is actually a minor extension of the 2DOF PEHM introduced by Aldraihem and Baz (2011). In their model, the damping η_1 between the base and the mass m_1 (termed as “dynamic magnifier” by Aldraihem and Baz) was ignored, but it will be considered here. However, Configuration B is not the focus in this Section. Although the difference between the two configurations of 2DOF PEHM is only the location of the piezoelectric element, their energy harvesting performances are totally different, which will be characterized in the following sections. To our best knowledge, Configuration A has not been investigated in the existing literature. More importantly, it will be shown that Configuration A is more advantageous over Configuration B.

Figure 2 Two configurations of 2DOF PEHM

2.2.1 Configuration A of 2DOF Model

For Configuration A in Figure 2(a), the piezoelectric element is placed between the base and mass m_1 . To our best knowledge, such configuration has not been studied in the literature. It comprises two subsystems. The primary subsystem, composed of the primary mass m_1 , spring k_1 , damper η_1 and piezoelectric element, is actually the same as the 1DOF model shown in Section 2.1. The parasitic subsystem is composed of the parasitic mass m_2 , spring k_2 and damper η_2 . Thus, we can regard the 2DOF model as the combination of original 1DOF model with an attached parasitic subsystem. By setting $y=u_2-u_1$, $x=u_1-u_0$, the governing equations of the model can be written as,

$$\begin{cases} m_2 \ddot{y} + \eta_2 \dot{y} + k_2 y = -m_2 \ddot{x} - m_2 \ddot{u}_0 \\ (m_1 + m_2) \ddot{x} + \eta_1 \dot{x} + k_1 x + \theta V + m_2 \ddot{y} + (m_1 + m_2) \ddot{u}_0 = 0 \\ -\theta \dot{x} + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (5)$$

Letting

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}, \zeta_1 = \frac{\eta_1}{2\sqrt{k_1 m_1}}, \zeta_2 = \frac{\eta_2}{2\sqrt{k_2 m_2}}, \mu = \frac{m_2}{m_1} \quad (6)$$

and applying the Laplace transform for (5), we obtain

$$\begin{cases} s^2 \hat{Y} + 2\zeta_2 \omega_2 s \hat{Y} + \omega_2^2 \hat{Y} = -s^2 \hat{X} - s^2 \hat{U}_0 \\ (1 + \mu) s^2 \hat{X} + 2\zeta_1 \omega_1 s \hat{X} + \omega_1^2 \hat{X} + (\theta/m_1) \hat{V} + \mu s^2 \hat{Y} + (1 + \mu) s^2 \hat{U}_0 = 0 \\ -\theta s \hat{X} + C^S s \hat{V} + \hat{V}/R_l = 0 \end{cases} \quad (7)$$

It should be mentioned that ω_1 and ω_2 are the natural frequencies when the primary and parasitic subsystems work separately. Solving (7), we have

$$\hat{V} = \frac{\left(\frac{\mu s^2}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2} - (1 + \mu) \right) s^2 \hat{U}_0}{\left((1 + \mu) s^2 + 2\zeta_1 \omega_1 s + \omega_1^2 - \frac{\mu s^4}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2} \right) \frac{R_l C^S s + 1}{R_l \theta s} + \frac{\theta}{m_1}} \quad (8)$$

Setting $s = j\omega$, the dimensionless voltage across the R_l is written as

$$\left| \tilde{V} \right| = \left| \frac{\hat{V}}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{(1 + \mu) + \frac{\mu \Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)}}{\left(1 - (1 + \mu) \Omega^2 + j2\zeta_1 \Omega - \frac{\mu \Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2 \alpha \Omega)} \right) \frac{j r \Omega + 1}{j r k_e^2 \Omega} + 1} \right| \quad (9)$$

and the dimensionless power is

$$|\tilde{P}| = \left| \frac{\hat{P}}{\frac{m_1(\omega^2 \hat{U}_0)^2}{\omega_1}} \right| = \frac{rk_e^2 \Omega^2}{(r\Omega)^2 + 1} \left| \frac{(1+\mu) + \frac{\mu\Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2\alpha\Omega)}}{\left(1 - (1+\mu)\Omega^2 + j2\zeta_1\Omega - \frac{\mu\Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2\alpha\Omega)}\right) + \frac{jr k_e^2 \Omega}{jr\Omega + 1}} \right|^2 \quad (10)$$

where the dimensionless parameters are

$$\alpha = \frac{\omega_2}{\omega_1}, \Omega = \frac{\omega}{\omega_1}, r = \omega_1 C^S R_l, k_e^2 = \frac{\theta^2}{C^S k_1} \quad (11)$$

Given $\mu \rightarrow 0$, we can obtain the same expression of $|\tilde{P}|$ as (3), which implies that the 2DOF model degrades to the 1DOF model.

Two Resonant Frequencies

To be versatile in frequency-variant or random vibrations with known dominant bandwidth, the two resonant frequencies of the 2DOF model are preferred to be close to each other. Thus, we need to understand how to tune the system parameters to achieve this. For open circuit condition (i.e., $r \rightarrow \infty$), the dimensionless open circuit voltage of Configuration A can be derived from (9) as,

$$|\tilde{V}_{oc}| = \left| \frac{k_e^2 \left((1+\mu) + \frac{\mu\Omega^2}{(\alpha^2 - \Omega^2 + j2\zeta_2\alpha\Omega)} \right)}{\left(1 - (1+\mu)\Omega^2 + j2\zeta_1\Omega - \frac{\mu\Omega^4}{(\alpha^2 - \Omega^2 + j2\zeta_2\alpha\Omega)} \right) + k_e^2} \right| \quad (12)$$

For undamped condition, the following equation from (12)

$$(1 - \Omega^2)(\alpha^2 - \Omega^2) - \mu\alpha^2\Omega^2 + k_e^2(\alpha^2 - \Omega^2) = 0 \quad (13)$$

yields the two undamped open circuit resonant frequencies as

$$\Omega_{1,2} = \sqrt{\frac{(1+\mu)\alpha^2 + (1+k_e^2)}{2} \pm \frac{\sqrt{((\mu+1)\alpha^2 + 1 + k_e^2)^2 - 4\alpha^2(1+k_e^2)}}{2}} \quad (14)$$

Figure 3 shows the difference of the two resonant frequencies $\Delta\Omega_{1,2}$ versus α and μ for different coupling coefficient k_e . Though $\Delta\Omega_{1,2}$ tends to increase with μ , we can tune α to an optimal value to minimize $\Delta\Omega_{1,2}$ for each specific μ . We note that the optimal α (for minimal $\Delta\Omega_{1,2}$) decreases with the increase of μ and shifts to the right of α axis with the increase of k_e . According to these results, when designing a 2DOF PEHM, we should carefully select α according to μ and k_e to reduce $\Delta\Omega_{1,2}$. Moreover, with a relatively larger μ , the region near optimal α is much flatter. Thus, the selection of α can be relatively flexible.

Figure 3 $\Delta\Omega_{1,2}$ versus α and μ for different k_e : (a) $k_e=0.02$ (b) $k_e=0.3$ (c) $k_e=0.6$

Effects of α and μ on Peak Magnitude

For a 2DOF model, tuning the two resonant frequencies to be close to each other is only one aspect in system design. To ensure advantageous performance, another aspect is to ensure that both peaks in the power response have significant magnitudes for energy harvesting. Thus, it is equally important to understand how the system parameters affect the magnitudes of the two peaks.

In the following case studies, we select $k_e=0.02$, $\zeta_1=0.02$ and $\zeta_2=0.004$. The dimensionless optimal power (impedance matching is achieved at each frequency) versus Ω and μ for different α is illustrated in Figure 4. For the optimal $\alpha \approx 1$ when $\mu \rightarrow 0$, we note in Figure 4(b) that the two peaks gradually approach each other and merge into one peak (1DOF response). With the increase of μ , the magnitude of the first peak initially decreases then constantly increases, while the magnitude of the second peak monotonically decreases. For $\alpha > 1$ (Figure 4(a)), the first peak of the 2DOF model degrades to the 1DOF peak when $\mu \rightarrow 0$. With the increase of μ , the magnitude of the first peak initially decreases then constantly increases, while the second peak appears, increases and finally decreases. The trend of the second peak may be difficult to discern in the contour plot (Figure 4(a)). However, due to its much smaller magnitude compared to the first peak, the second peak is not concerned. For $\alpha < 1$ (Figure 4(c) and (d)), the second peak of the 2DOF model degrades to the 1DOF peak when $\mu \rightarrow 0$. With the increase of μ , the first peak of power appears and monotonically increases, while the second peak monotonically decreases. In general, we can conclude that except for the initial limited range, the magnitude of the first peak increases and the magnitude of the second peak decreases with the increase of μ , regardless of α . Besides, based on these observations, we can conclude that given $\alpha < 1$ and certain small μ , it is possible to achieve both peaks with equally significant magnitudes. This is one important aspect for the design of a 2DOF model.

(c)

(d)

Figure 4 Dimensionless optimal power output versus Ω and μ for different α . (a) $\alpha=1.05$ (b) $\alpha=1$ (c) $\alpha=0.95$ (d) $\alpha=0.9$

Given specific μ , Figure 5 further depicts how the two peaks in power response are affected by α . For a very small μ (e.g. 0.04), it is observed that the magnitudes of both peaks are sensitive to α . Besides, the two peaks can be close and have significant contribution to energy harvesting if α is carefully tuned (e.g., $\alpha=0.85$ can give two peaks with nearly equal magnitudes). While, for a relative larger μ (e.g. 0.2), the magnitude of the first peak barely varies with α . Though the first peak has much larger magnitude as compared to that for $\mu=0.04$, the second peak is drastically lower than the first peak and thus its contribution to energy harvesting is negligible. In a word, if one's intension is to have one larger peak response, a relatively larger μ can be selected (i.e., use bigger parasitic mass) and α can be flexibly chosen. While, if one's intension is to have two close peaks with significant magnitudes to contribute to broadband energy harvesting, a relatively small μ should be selected and α should be carefully tuned.

Figure 5 Dimensionless optimal power output versus Ω for different α . (a) $\mu=0.04$ and (b) $\mu=0.2$

Besides, compared to the dimensionless power response of the 1DOF model, for $\mu=0.04$, the two peaks of the 2DOF model have the magnitude at the same order; while for $\mu=0.2$, the first peak is significantly enhanced, as shown in Figure 5. Here, we should emphasize that these dimensionless power responses are normalized with respect to the mass of the primary subsystem of Configuration A (i.e., m_1). When the mass ratio is not small, a more fair comparison should be based on the power density. Therefore, we provide a further example for quantitative comparison. We assume that the parameters in Configuration A (Figure 2(a)) are $m_1=0.04\text{kg}$, $k_1=100\text{Nm}^{-1}$, $\eta_1=0.08\text{Nsm}^{-1}$, $m_2=0.008\text{kg}$, $k_2=14.45\text{Nm}^{-1}$, $\eta_2=0.00272\text{Nsm}^{-1}$, $\theta=3.1623\text{e-}5\text{NV}^{-1}$ and $C^S=25\text{nF}$. The corresponding dimensionless parameters are $\mu=0.2$, $\alpha=0.85$, $k_e=0.02$ and $\zeta_1=0.02$, $\zeta_2=0.004$. Under the excitation level of 1m/s^2 , the first peak in the power response has the magnitude of $250.4\mu\text{W}$ and the power density of 5.22mW/kg . While, for the 1DOF model (m_2 , k_2 and η_2 are removed), the power magnitude and power density at the peak are obtained as $99.04\mu\text{W}$ and 2.48mW/kg , respectively. Obviously, for a relatively large $\mu=0.2$, the power density of the 2DOF model is still significantly larger than that of the 1DOF model. Furthermore, the above observations of the peak magnitudes of the 1DOF model and Configuration A are only valid with the given damping ratios of $\zeta_1=0.02$ and $\zeta_2=0.004$. Whether they can be extended to other cases

with different damping ratios should be further studied, which is given in the following section.

In addition, it should be mentioned that the conditions for small $\Delta\Omega_{1,2}$ and those for two effective peaks or one enhanced peak may conflict. For relatively large μ , fortunately, the selection of α can be flexible to achieve both small $\Delta\Omega_{1,2}$ and one enhanced peak. For relatively small μ , the values of α to achieve small $\Delta\Omega_{1,2}$ and two effective peaks may be inconsistent. For example, with $\mu=0.04$ and $k_e=0.02$, the minimal $\Delta\Omega_{1,2}$ requires $\alpha=0.96$ while two peaks with equal magnitudes requires $\alpha=0.85$. In such case, certain tradeoff should be made.

Effects of Damping on Peak Magnitude

Other than α and μ , the damping in the system is another factor affecting the magnitudes of the two peaks in power response. We assume $k_e=0.02$, $\alpha=1$. Two cases are studied, that is, (1) $\zeta_2=0.004$ and ζ_1 varies and (2) $\zeta_1=0.02$ and ζ_2 varies.

For Case (1), it is noted in Figure 6 that ζ_1 significantly affects the performance of both the 2DOF and the original 1DOF model. Fortunately, the influence of ζ_1 is almost equivalent to both models. Hence, with various ζ_1 , for $\mu=0.04$, the 2DOF model always exhibits two peaks that have the same level magnitude as the 1DOF model; and for $\mu=0.2$, the first peak of the 2DOF model always has significantly larger magnitude than the 1DOF model. Even in terms of the power density, these conclusions still hold. For Case (2), Figure 7 shows that ζ_2 has very minor influence on the performance of the 2DOF model. The 1DOF model does not include damper η_2 thus is not affected by ζ_2 . Based on these results, when designing a 2DOF PEHM, one should devote efforts to reducing ζ_1 to improve the performance of the harvester rather than ζ_2 .

Figure 6 Dimensionless optimal power output versus Ω for different ζ_1 . (a) $\mu=0.04$ and (b) $\mu=0.2$

Figure 7 Dimensionless optimal power output versus Ω for different ζ_2 . (a) $\mu=0.04$ and (b) $\mu=0.2$

2.2.2 Configuration B of 2DOF Model

For Configuration B of the 2DOF model shown in Figure 2(b), the piezoelectric element is placed between m_1 and m_2 . Now the primary subsystem of the PEHM comprises the primary mass m_2 , spring k_2 , damper η_2 and piezoelectric element, and the parasitic subsystem comprises parasitic mass m_1 , damper η_1 and spring k_1 .

The governing equations of Configuration B are written as,

$$\begin{cases} m_2 \ddot{y} + \eta_2 \dot{y} + k_2 y + \theta V = -m_2 \ddot{x} - m_2 \ddot{u}_0 \\ (m_1 + m_2) \ddot{x} + \eta_1 \dot{x} + k_1 x + m_2 \ddot{y} + (m_1 + m_2) \ddot{u}_0 = 0 \\ -\theta \dot{y} + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (15)$$

Solving the above equations in a similar way to Configuration A in Section 2.2.1, we obtain

$$\hat{V} = \frac{-\frac{2\zeta_1 \omega_1 s + \omega_1^2}{(1+\mu)s^2 + 2\zeta_1 \omega_1 s + \omega_1^2} s^2 \hat{U}_0}{\left(s^2 + 2\zeta_2 \omega_2 s + \omega_2^2 - \frac{\mu s^4}{(1+\mu)s^2 + 2\zeta_1 \omega_1 s + \omega_1^2} \right) \frac{R_l C^S s + 1}{R_l \theta s} + \frac{\theta}{m_2}} \quad (16)$$

The dimensionless voltage can be written as

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_2 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{\frac{\alpha'^2 + j2\zeta_1 \alpha' \Omega'}{\alpha'^2 - (1+\mu)\Omega'^2 + j2\zeta_1 \alpha' \Omega'}}{\left(\left(1 - \Omega'^2 + j2\zeta_2 \Omega' - \frac{\mu \Omega'^4}{\alpha'^2 - (1+\mu)\Omega'^2 + j2\zeta_1 \alpha' \Omega'} \right) \frac{j r' \Omega' + 1}{j r' k_e'^2 \Omega'} + 1 \right)} \right| \quad (17)$$

and the dimensionless power is

$$|\tilde{P}| = \left| \frac{\hat{P}}{\frac{m_2 (\omega^2 \hat{U}_0)^2}{\omega_2}} \right| = \frac{r' k_e'^2 \Omega'^2}{(r' \Omega')^2 + 1} \left| \frac{\frac{\alpha'^2 + j2\zeta_1 \alpha' \Omega'}{\alpha'^2 - (1+\mu)\Omega'^2 + j2\zeta_1 \alpha' \Omega'}}{\left(\left(1 - \Omega'^2 + j2\zeta_2 \Omega' - \frac{\mu \Omega'^4}{\alpha'^2 - (1+\mu)\Omega'^2 + j2\zeta_1 \alpha' \Omega'} \right) + \frac{j r' k_e'^2 \Omega'}{j r' \Omega' + 1} \right)} \right|^2 \quad (18)$$

where the dimensionless parameters are

$$\alpha' = \frac{1}{\alpha} = \frac{\omega_1}{\omega_2}, \Omega' = \frac{\omega}{\omega_2}, r' = \omega_2 C^S R_l, k_e'^2 = \frac{\theta^2}{C^S k_2} \quad (19)$$

Given $\mu \rightarrow 0$, if we regard mass m_1 as the base and u_1 as the base displacement, it is not difficult to find

$$\left| \frac{\hat{V}}{\frac{m_2 \omega^2 \hat{U}_1}{\theta}} \right| \rightarrow \left| \frac{1}{\left(1 - \Omega'^2 + j2\zeta_2 \Omega'\right) \frac{j\alpha' \Omega' + 1}{j\alpha' k_e'^2 \Omega'} + 1} \right| \quad (20)$$

Thus, Configuration B of the 2DOF model is also degraded to 1DOF model when $\mu \rightarrow 0$.

Two Resonant Frequencies

Similar to Configuration A, two undamped open circuit resonant frequencies can be obtained as

$$\Omega'_{1,2} = \sqrt{\frac{(1+\mu)(1+k_e'^2) + \alpha'^2}{2}} \pm \sqrt{\frac{\left((\mu+1)(1+k_e'^2) + \alpha'^2\right)^2 - 4\alpha'^2(1+k_e'^2)}{2}} \quad (21)$$

Figure 8 shows the difference of the two resonant frequencies $\Delta\Omega'_{1,2}$ versus α' and μ for different coupling coefficient k_e' . Compared to $\Delta\Omega_{1,2}$ of Configuration A (Figure 3), we note that $\Delta\Omega'_{1,2}$ increases faster with μ . Besides, the optimal α' for minimum $\Delta\Omega'_{1,2}$ shifts to the right of axis α' with the increase of k_e' but does not vary with μ , as shown in Figure 8. Moreover, the two resonant frequencies can also be tuned close to each other with a small μ . However, contrary to Configuration A, this means that a much heavier parasitic mass m_1 has to be added to the original 1DOF model to achieve this purpose, which not only sacrifices the system performance in terms of power density but also limits its application. Even we select a practical μ , for example $\mu=1$, the difference of the two resonant frequencies is much larger than that of Configuration A, as compared in Figures 3 and 8. In a word, Configuration A is preferred to have two close resonant frequencies as compared to Configuration B.

Figure 8 $\Delta\Omega'_{1,2}$ versus α' and μ for different coupling k_e' : (a) $k_e'=0.02$ (b) $k_e'=0.3$ (c) $k_e'=0.6$

Effects of α' and μ on Peak Magnitude

Similar to Configuration A, the magnitudes of the two peaks are also concerned. In the following case studies, we assume $k_e'=0.02$, $\zeta_1=0.02$ and $\zeta_2=0.004$. The dimensionless optimal power (impedance matching is achieved at each frequency) for different α' and μ are illustrated in Figure 9. Since an extremely small μ is impractical (infinite large mass added to original 1DOF harvester), only $\mu \geq 0.1$ are considered. Different from Configuration A, we note in Figure 9 that regardless of α' , the magnitudes of both peaks monotonically decrease with the increase of μ . In other words, a smaller μ can always provide better performance, but the apparent larger power output may be depreciated in terms of the power density. Additionally, to illustrate how α' affects the performance, the power for various α' at specific mass ratios $\mu=0.2$ and $\mu=1$ is shown in Figure 10. For both $\mu=0.2$ and $\mu=1$, the magnitude of the first peak is quite sensitive to α' and increases with

α' , while the second is insensitive to α' . Since the magnitude of the second peak is substantially smaller than the first one, the overall performance is attributed to the first peak and thus sensitive to α' . Hence, Configuration B always requires a larger α' to ensure a better performance. However, similar to Configuration A, a tradeoff may be needed to ensure a small $\Delta\Omega'_{1,2}$ at the same time.

Effects of Damping on Peak Magnitude

Other than α' and μ , the damping effect on the magnitudes of the two peaks in the power response

is also our concern. In the following case studies, we assume $k_e'=0.02$, $\alpha'=1$. Two cases are studied, that is, (1) $\zeta_2=0.004$ and ζ_1 varies and (2) $\zeta_1=0.02$ and ζ_2 varies.

For Case (1), Figure 11 illustrates the significant influence of ζ_1 on the performance of Configuration B. As compared to the 1DOF response (under base excitation with parasitic subsystem removed), Configuration B cannot guarantee to provide advantageous performance. Neglecting ζ_1 in the modeling may greatly overestimate the performance of Configuration B. For Case (2), since the 1DOF model also includes damper η_2 , we observe in Figure 12 that ζ_2 has pronounced influence on the performances of both Configuration B and 1DOF model. However, ζ_2 obviously affects the latter more significantly. In other words, if a small ζ_2 is feasible (a small ζ_2 is always preferred to have large output in practice), Configuration B has no obvious advantage or even worse performance than the 1DOF model, as depicted in Figure 12. Configuration B can only provide significantly advantageous performance when a large ζ_2 exists. It should be mentioned that these dimensionless power responses are normalized with respect to the mass of the primary subsystem of Configuration B (i.e., m_2). In terms of the power density, the superiority of Configuration B for amplitude magnification will be further depreciated.

Figure 11 Dimensionless optimal power output versus Ω' for different ζ_1 . (a) $\mu=0.2$ and (b) $\mu=1$

Figure 12 Dimensionless optimal power output versus Ω' for different ζ_2 . (a) $\mu=0.2$ and (b) $\mu=1$

2.2.3 Summary of 2DOF Model

In Sections 2.2.1 and 2.2.2, we have discussed the characteristics of two configurations of 2DOF

model in terms of $\Delta\Omega_{1,2}$ (or $\Delta\Omega'_{1,2}$) and magnitudes of two peaks in their power responses, as summarized in Table 1.

These characteristics provide important guidelines for designing a 2DOF PEHM. In general, the advantage of Configuration A is that with slight increase of overall weight (parasitic mass added to original 1DOF model), the system can be tuned to provide: **(a)** two close and effective peaks (though their magnitudes may not be greater than that of the 1DOF model), or **(b)** one effective peak with significantly larger magnitude than the 1DOF model. Configuration B can only have two close and effective peaks with huge increase of overall weight, which may not help improve and even deteriorate the performance in terms of power density. Even if the power density is not concerned, it can only significantly outperform the 1DOF model when ζ_1 is very small (difficult to achieve) and ζ_2 is very large (not desirable for large power output).

Table 1 Summary of two configurations of 2DOF model

	Configuration A	Configuration B
$\Delta\Omega_{1,2}$ (or $\Delta\Omega'_{1,2}$)	<ul style="list-style-type: none"> ◆ A small $\Delta\Omega_{1,2}$ requires a small μ (slight increase of weight to original 1DOF model provides two close resonances) ◆ Slowly increases with μ at optimal α ◆ Optimal α for minimum $\Delta\Omega_{1,2}$ increases with k_e and decreases with μ ◆ For larger μ, $\Delta\Omega_{1,2}$ versus α is flat around optimal α (minor mistuning of α still ensures $\Delta\Omega_{1,2}$ close to minimum) 	<ul style="list-style-type: none"> ◆ A small $\Delta\Omega'_{1,2}$ requires a small μ (huge weight should be added to original 1DOF model for two close resonances) ◆ Fast increases with μ even at optimal α' ◆ Optimal α' for minimum $\Delta\Omega_{1,2}$ increases with k_e' but does not change with μ
Magnitudes of two peaks	<ul style="list-style-type: none"> ◆ Except for the limited range close to $\mu \rightarrow 0$, the magnitude of first peak increases while the second peak decreases with μ. ◆ For a very small μ, the performance is sensitive to α and both peaks can have significant contribution to energy harvesting (though the magnitudes may not be larger than that of the 1DOF model). While for a relatively large μ, the performance is insensitive to α and only the first peak has significant contribution to energy harvesting and can outperform the 1DOF model. ◆ The performance is sensitive to ζ_1 but insensitive to ζ_2. Besides, ζ_1 has nearly equivalent influence on Configuration A and the 1DOF model. 	<ul style="list-style-type: none"> ◆ Generally, the magnitudes of both peaks decrease with μ. ◆ For different μ, the performance is always sensitive to α' (thus α' should be always carefully tuned) and only the first peak significantly contributes to energy harvesting. ◆ The performance is sensitive to both ζ_1 and ζ_2. ζ_2 has more substantial influence on the 1DOF model than that on Configuration B. Hence, Configuration B cannot guarantee an advantageous performance over the 1DOF model unless ζ_1 is negligible and ζ_2 is large. However, in practice, a negligible ζ_1 is very difficult to achieve and small ζ_2 is always preferred for large output.

2.3 Generalized n -DOF Model

Based on the characterization of the 2DOF model, Configuration A is preferred in this paper due to the fact that two close and effective peaks can be achieved with slight increase of overall weight as compared to the original 1DOF model. With more small parasitic masses attached, we can generalize Configuration A of the 2DOF model to an n -DOF model, as shown in Figure 13. It is expected that more effective peaks can be obtained with minor increase of overall weight.

Figure 13 Generalized n -DOF PEHM

The governing equations of the n -DOF PEHM can be written as

$$\begin{cases} m_1 \ddot{u}_1 = -\eta_1 (\dot{u}_1 - \dot{u}_0) - k_1 (u_1 - u_0) - \theta V \\ \quad + \eta_2 (\dot{u}_2 - \dot{u}_1) + k_2 (u_2 - u_1) + \eta_3 (\dot{u}_3 - \dot{u}_1) + k_3 (u_3 - u_1) + \dots + \eta_n (\dot{u}_n - \dot{u}_1) + k_n (u_n - u_1) \\ m_2 \ddot{u}_2 = -\eta_2 (\dot{u}_2 - \dot{u}_1) - k_2 (u_2 - u_1) \\ m_3 \ddot{u}_3 = -\eta_3 (\dot{u}_3 - \dot{u}_1) - k_3 (u_3 - u_1) \\ \vdots \\ m_n \ddot{u}_n = -\eta_n (\dot{u}_n - \dot{u}_1) - k_n (u_n - u_1) \\ -\theta (\dot{u}_1 - \dot{u}_0) + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (22)$$

Setting $x=u_1-u_0$, $y_1=u_2-u_1$, $y_2=u_3-u_1$, $\dots$, $y_{n-1}=u_n-u_1$ and rearranging (22), we have

$$\begin{cases} \sum_{p=1}^n m_p \ddot{x} + \eta_1 \dot{x} + k_1 x + \theta V + \sum_{p=2}^n m_p \ddot{y}_{p-1} + \sum_{p=1}^n m_p \ddot{u}_0 = 0 \\ m_2 \ddot{y}_1 + \eta_2 \dot{y}_1 + k_2 y_1 = -m_2 \ddot{x} - m_2 \ddot{u}_0 \\ m_3 \ddot{y}_2 + \eta_3 \dot{y}_2 + k_3 y_2 = -m_3 \ddot{x} - m_3 \ddot{u}_0 \\ \vdots \\ m_n \ddot{y}_{n-1} + \eta_n \dot{y}_{n-1} + k_n y_{n-1} = -m_n \ddot{x} - m_n \ddot{u}_0 \\ -\theta \dot{x} + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (23)$$

Assuming that all the parasitic masses have the same weight, i.e., $m_2=m_3=\dots=m_n$ and setting,

$$\mu = \frac{m_2}{m_1}, \omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}, \dots, \omega_n = \sqrt{\frac{k_n}{m_n}}, \zeta_1 = \frac{\eta_1}{2\sqrt{k_1 m_1}}, \zeta_2 = \frac{\eta_2}{2\sqrt{k_2 m_2}}, \dots, \zeta_n = \frac{\eta_n}{2\sqrt{k_n m_n}} \quad (24)$$

we can write

$$\begin{cases} (1 + (n-1)\mu) \ddot{x} + 2\zeta_1 \omega_1 \dot{x} + \omega_1^2 x + (\theta/m_1) V + \mu \sum_{p=2}^n \ddot{y}_{p-1} + (1 + (n-1)\mu) \ddot{u}_0 = 0 \\ \ddot{y}_1 + 2\zeta_2 \omega_2 \dot{y}_1 + \omega_2^2 y_1 = -\ddot{x} - \ddot{u}_0 \\ \ddot{y}_2 + 2\zeta_3 \omega_3 \dot{y}_2 + \omega_3^2 y_2 = -\ddot{x} - \ddot{u}_0 \\ \vdots \\ \ddot{y}_{n-1} + 2\zeta_n \omega_n \dot{y}_{n-1} + \omega_n^2 y_{n-1} = -\ddot{x} - \ddot{u}_0 \\ -\theta \dot{x} + C^S \dot{V} + V/R_l = 0 \end{cases} \quad (25)$$

Applying the Laplace transform for (25), it is not difficult to find

$$\hat{V} = \frac{\left(-(1 + (n-1)\mu) + \mu s^2 \sum_{p=2}^n \frac{1}{s^2 + 2\zeta_p \omega_p s + \omega_p^2} \right) s^2 \hat{U}_0}{\left((1 + (n-1)\mu) s^2 + 2\zeta_1 \omega_1 s + \omega_1^2 - \mu s^4 \sum_{p=2}^n \frac{1}{s^2 + 2\zeta_p \omega_p s + \omega_p^2} \right) \frac{R_l C^S s + 1}{R_l \theta s} + \frac{\theta}{m_1}} \quad (26)$$

Setting $s=j\omega$, the dimensionless voltage across the R_l is obtained as

$$|\tilde{V}| = \left| \frac{\hat{V}}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{(1 + (n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega}}{\left(1 - (1 + (n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) \frac{j r \Omega + 1}{j r k_e^2 \Omega} + 1}} \right| \quad (27)$$

and the dimensionless power is

$$|\tilde{P}| = \left| \frac{\hat{P}}{\frac{m_1 (\omega^2 \hat{U}_0)^2}{\omega_1}} \right| = \frac{r k_e^2 \Omega^2}{(r \Omega)^2 + 1} \left| \frac{(1 + (n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega}}{\left(1 - (1 + (n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) + \frac{j r k_e^2 \Omega}{j r \Omega + 1}} \right|^2 \quad (28)$$

where the dimensionless parameters are

$$\alpha_1 = \frac{\omega_2}{\omega_1}, \alpha_2 = \frac{\omega_3}{\omega_1}, \dots, \alpha_{n-1} = \frac{\omega_n}{\omega_1}, \Omega = \frac{\omega}{\omega_1}, r = \omega_1 C^S R_l, k_e^2 = \frac{\theta^2}{C^S k_1} \quad (29)$$

Multiple Resonant Frequencies

For open circuit condition (i.e., $r \rightarrow \infty$), the voltage is obtained as,

$$|\tilde{V}_{oc}| = \left| \frac{\hat{V}_1}{\frac{m_1 \omega^2 \hat{U}_0}{\theta}} \right| = \left| \frac{k_e^2 \left((1 + (n-1)\mu) + \mu \Omega^2 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right)}{\left(1 - (1 + (n-1)\mu)\Omega^2 + j2\zeta_1 \Omega - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2 + j2\zeta_p \alpha_{p-1} \Omega} \right) + k_e^2} \right| \quad (30)$$

The undamped open circuit resonant frequencies can be determined with the following equation from (35),

$$\left(1 - (1 + (n-1)\mu)\Omega^2 - \mu \Omega^4 \sum_{p=2}^n \frac{1}{\alpha_{p-1}^2 - \Omega^2} \right) + k_e^2 = 0 \quad (31)$$

It is difficult to express the multiple resonant frequencies explicitly from the above equation, but they can be determined with numerical method.

Case Study

In the following case studies, a 3DOF model is analyzed. Firstly, we assume $k_e=0.02$, $\zeta_1=0.02$ and $\zeta_2=\zeta_3=0.004$. In this case and the case studies in Section 2.2, we have $k_e^2/\zeta \ll 1$, corresponding to a weak coupling system (Shu and Lien, 2006; Shu *et al.*, 2007). The dimensionless optimal power versus Ω and μ with various combinations of α_1 and α_2 is shown in Figure 14. Obviously, the effects of μ , α_1 and α_2 on the three peaks in the power response are more complicated than the 2DOF model. First, for $\alpha_1=1$ or $\alpha_2=1$, two of the three peaks will merge when $\mu \rightarrow 0$, as shown in Figures 14(a) and (b). If μ is not extremely small, for all cases the magnitude of first peak increases while the magnitudes of other two decreases with increase of μ . Moreover, among the four combinations of α_1 and α_2 , only for $\alpha_1=1$ and $\alpha_2 < 1$ (Figure 14(b)), all three peaks have significant magnitudes at certain μ (swap between α_1 and α_2 , i.e., $\alpha_2=1$ and $\alpha_1 < 1$ does not affect this conclusion).

With fixed $\alpha_1=1$, Figure 15 illustrates the dimensionless optimal power for various α_2 at specific mass ratios $\mu=0.04$ and $\mu=0.2$. The damping is still assumed as $\zeta_1=0.02$ and $\zeta_2=\zeta_3=0.004$. It is observed that, for both $\mu=0.04$ and $\mu=0.2$, only the second peak in power response is quite

sensitive to α_2 . When α_2 decreases from 1.1 to 1 then further to 0.8, the second peak declines, disappears (for $\alpha_1=\alpha_2$), reappears and increases. For $\mu=0.04$, though the magnitudes of the three peaks may not be larger than that of the 1DOF model, they can be tuned to have significant values that contribute to energy harvesting (for example, $\alpha_2=0.8$), as shown in Figure 15(a). For a relatively large $\mu=0.2$, the first peak contributes most to energy harvesting and its magnitude can drastically surpass that of the 1DOF model. Thus, we can design a multiple-DOF system to have either multiple effective peaks with reasonable magnitudes, or one peak with significantly enhanced magnitude. The former will improve the energy harvesting by broadening the frequency bandwidth and the latter by enhancing the power output.

Figure 14 Dimensionless optimal power output of 3DOF model with weak coupling. (a) $\alpha_1=1.1$, $\alpha_2=1$ (b) $\alpha_1=1$, $\alpha_2=0.8$ (c) $\alpha_1=0.9$, $\alpha_2=0.8$ (d) $\alpha_1=1.2$, $\alpha_2=0.8$

(a)

(b)

Figure 15 Dimensionless optimal power output of 3DOF model with weak coupling versus Ω for different α_2 . (a) $\mu=0.04$ and (b) $\mu=0.2$

We present a further example involving a strong coupling. All the parameters are the same as those in the previous 3DOF model except $k_e=0.45$. Hence, we have $k_e^2/\zeta>10$, corresponding to a strong coupling system (Shu and Lien, 2006; Shu *et al.*, 2007). The power response at specific mass ratios $\mu=0.04$ and $\mu=0.2$ are shown in Figure 16. Obviously, the main trends of power with different α_2 and μ are similar to those of the weak coupling system. The only difference is that the power response near each mode may appear as a flat peak, or split into two peaks as the 1DOF system does in the case of strong coupling.

Figure 16 Dimensionless optimal power output of 3DOF model with strong coupling versus Ω for different α_2 . (a) $\mu=0.04$ and (b) $\mu=0.2$

Besides, it should be noted in Figures 15 and 16 (also in Figure 5) that there are deep valleys between the two neighboring peaks, which correspond to the anti-resonance points of the system. This behavior is similar to that in the old vibration absorber problem. However, different from the vibration absorber problem, throughout this paper, our efforts are devoted into enhancing the magnitudes of the resonant peaks and tuning these peaks close to each other through a comprehensive parametric study. The design guidelines based on this parametric study are different from those for the design of vibration absorbers. These deep valleys are obviously favorable in vibration absorber problem but they are harmful for energy harvesting performance. To overcome this limitation, the power reduction near the valleys can be improved by certain sophisticated interface circuits, including the SSHI (Shu *et al.*, 2007; Lien and Shu, 2011) and SCE (Tang and Yang, 2011) interfaces. For instance, in Lien and Shu (2011), for an array configuration of piezoelectric cantilevers with medium coupling, the parallel-SSHI interface could significantly widen the bandwidth by enhancing the responses in off-resonance frequency range.

Finally, it should be mentioned that with more DOFs introduced, the effects of parameters on system behavior become more complicated. Fortunately, the analytical solution given in (28) provides a way to investigate the system performance through parametric study, which would greatly assist in designing a multiple-DOF PEHM.

3. EQUIVALENT CIRCUIT MODELING

In previous analytical modeling of a multiple-DOF PEHM, a resistor R_l is attached to represent the electric load. In practice, however, the energy harvesting interface circuits can be much more complicated than a resistor. To improve the energy harvesting efficiency, interface circuits such as SSHI (Lien *et al.*, 2010; Liang and Liao, 2011, 2012) and SCE (Tang and Yang, 2011) have been enthusiastically pursued. When these sophisticated interfaces are considered, it becomes very challenging to obtain the analytical solution of the proposed multiple-DOF system. System-level electric simulation is thus resorted to. The equivalent circuit model of the proposed n -DOF PEHM is presented in this section to address this issue.

In (22), the current through the resistor is represented by V/R_l . With the consideration of a sophisticated interface, we use I to replace V/R_l . Setting $u_1^* = u_1 - u_0$, $u_2^* = u_2 - u_0, \dots, u_n^* = u_n - u_0$, we rearrange (22) as

$$\begin{cases} m_1 \ddot{u}_1^* + \eta_1 \dot{u}_1^* + k_1 u_1^* + \sum_{p=2}^n (\eta_p (\dot{u}_1^* - \dot{u}_p^*) + k_p (u_1^* - u_p^*)) + \theta V + m_1 \ddot{u}_0 = 0 \\ m_2 \ddot{u}_2^* + \eta_2 (\dot{u}_2^* - \dot{u}_1^*) + k_2 (u_2^* - u_1^*) + m_2 \ddot{u}_0 = 0 \\ m_3 \ddot{u}_3^* + \eta_3 (\dot{u}_3^* - \dot{u}_1^*) + k_3 (u_3^* - u_1^*) + m_3 \ddot{u}_0 = 0 \\ \vdots \\ m_n \ddot{u}_n^* + \eta_n (\dot{u}_n^* - \dot{u}_1^*) + k_n (u_n^* - u_1^*) + m_n \ddot{u}_0 = 0 \\ -\theta \dot{u}_1^* + C^S \dot{V} + I = 0 \end{cases} \quad (32)$$

Considering the analogy between the mechanical and electric domains as listed in Table 2, (32) can be further written as

$$\begin{cases} L_1 \ddot{q}_1 + R_1 \dot{q}_1 + q/C_1 + \sum_{p=2}^n (R_p (\dot{q}_1 - \dot{q}_p) + (q_1 - q_p)/C_p) + NV + v_1 = 0 \\ L_2 \ddot{q}_2 + R_2 (\dot{q}_2 - \dot{q}_1) + (q_2 - q_1)/C_2 + v_1 = 0 \\ L_3 \ddot{q}_3 + R_3 (\dot{q}_3 - \dot{q}_1) + (q_3 - q_1)/C_3 + v_3 = 0 \\ \vdots \\ L_n \ddot{q}_n + R_n (\dot{q}_n - \dot{q}_1) + (q_n - q_1)/C_n + v_n = 0 \\ -N \dot{q}_1 + C^S \dot{V} + I = 0 \end{cases} \quad (33)$$

which represents the constitutive equations of the circuit network shown in Figure 17. The first equation in (33) represents the main loop of the equivalent circuit model, satisfying the Kirchhoff voltage law. The second to n^{th} equations in (33) represent the $n-1$ parasitic circuit loops (equivalent to $n-1$ parasitic subsystems in Figure 13), independent of each other but coupled to the main circuit loop. They also satisfy the Kirchhoff voltage law. The last equation in (33) represents the circuit part at the right-hand-side of the transformer (Figure 17), satisfying the Kirchhoff current law. With this equivalent circuit model shown in Figure 17, the system performance with any type of energy harvesting interface circuits attached can be evaluated through system-level electric simulation (Yang and Tang, 2009).

Table 2 Analogy between mechanical and electrical domains

Mechanical Parameters	Equivalent Circuit Parameters
Relative coordinate $u_1^*, u_2^*, \dots, u_n^*$	Charge $q_1, q_2, \dots, q_n$
Relative velocity $\dot{u}_1^*, \dot{u}_2^*, \dots, \dot{u}_n^*$	Current $i_1, i_2, \dots, i_n$
Mass $m_1, m_2, \dots, m_n$	Inductor $L_1, L_2, \dots, L_n$
Damper $\eta_1, \eta_2, \dots, \eta_n$	Resistor $R_1, R_2, \dots, R_n$
Reciprocal of spring stiffness $1/k_1, 1/k_2, \dots, 1/k_n$	Capacitor $C_1, C_2, \dots, C_n$
Inertia force on mass $m_1\ddot{u}_0, m_2\ddot{u}_0, \dots, m_n\ddot{u}_0$	Ideal voltage source $v_1, v_2, \dots, v_n$
Electromechanical coupling θ	Turn ratio of ideal transformer N

Figure 17 Equivalent circuit model of proposed n -DOF PEHM with sophisticated interfaces

Validation Case

A 3DOF PEHM is again taken as example to validate the proposed equivalent circuit modeling method, with the parameters $m_1=1\text{kg}$, $m_2=m_3=0.04\text{kg}$, $k_1=400\text{Nm}^{-1}$, $k_2=16\text{Nm}^{-1}$, $k_3=10.24\text{Nm}^{-1}$, $\eta_1=0.8\text{Nsm}^{-1}$, $\eta_2=0.0064\text{Nsm}^{-1}$, $\eta_3=0.00512\text{Nsm}^{-1}$, $\theta=6.325\text{e-}5\text{NV}^{-1}$ and $C^S=25\text{nF}$. The corresponding dimensionless parameters are obtained as $\mu=0.04$, $\alpha_1=1$, $\alpha_2=0.8$, $\zeta_1=0.02$, $\zeta_2=\zeta_3=0.004$, $k_e=0.02$. A unity root mean square (RMS) base acceleration is assumed. Recalling the analogy between the mechanical and electric domains, the equivalent circuit parameters and the circuit model of the 3DOF PEHM are depicted in Figure 18(a). By electric simulation in SPICE software, the open circuit voltage is evaluated and compared with the analytical result, as shown in Figure 18(b). The simulation result is observed consistent with the analytical solution. Hence, with the proposed equivalent circuit model, the system performance with AD/DC standard (Shu and Lien, 2006), SCE (Tang and Yang, 2011) or SSHI interfaces (Lien *et al.*, 2010; Liang and Liao, 2011, 2012) can be evaluated.

Figure 18 Evaluation of dimensionless open circuit voltage of a 3DOF PEHM: (a) Equivalent circuit model and (b) comparison between equivalent circuit simulation and analytical solution.

4. CONCLUSIONS

In this paper, a multiple-DOF PEHM under base excitations is proposed and analyzed. The main findings of this paper can be summarized as follows:

- ◆ First, two 2DOF PEHM configurations are analyzed and characterized. Configuration B is a minor extension from the work of Aldraihem and Baz (2011) and Configuration A is a new alternative configuration which has not been discussed in the existing literature. For each configuration, parametric study is conducted to evaluate and characterize the system performance in terms of $\Delta\Omega_{1,2}$ (or $\Delta\Omega'_{1,2}$) and magnitudes of two peaks in their power response, as summarized in Table 1. In general, the advantage of Configuration A is that with slight increase of overall weight to conventional 1DOF model, it can be tuned to provide two close and effective peaks, or one peak with significant enhanced magnitude. Configuration B can only have two close and enhanced peaks with a huge increase of overall weight, which may not help improve or even deteriorate the performance in terms of power density. Even if the power density is not concerned, it can only significantly outperform the 1DOF model when ζ_1 is very small (which is unable or difficult to achieve in practice) and ζ_2 is very large (which is not preferred for efficient power output). Based on the characterization of the 2DOF model, Configuration A is preferred and generalized to an n -DOF PEHM and its analytical solution of power output is derived. With this n -DOF model for parametric study, we can design a harvester to have multiple close and effective peaks, or to have one peak with significantly enhanced magnitude, with only slight increase of system weight. The results of the parametric study in this work provide important guidelines for the design of a useful n -DOF PEHM, which are different from those for the design of conventional vibration absorbers.
- ◆ The equivalent circuit model of the proposed n -DOF PEHM is also developed via the analogy between the mechanical and electric domains. With such model, system-level electric simulation can be performed to evaluate the system performance when sophisticated interface circuits are attached, which is difficult to address by analytical modeling.

In the future, a prototype of the proposed multiple-DOF piezoelectric energy harvester will be devised and tested for practical vibration scenarios.

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