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Intrinsic Dynamical Fluctuation Assisted Symmetry Breaking in Adiabatic Following

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Classical adiabatic invariants in actual adiabatic processes possess intrinsic dynamical fluctuations. The magnitude of such intrinsic fluctuations is often thought to be negligible. This widely believed physical picture is contested here. For adiabatic following of a moving stable fixed-point solution facing a pitchfork bifurcation, we show that intrinsic dynamical fluctuations in an adiabatic process can assist in a deterministic selection between two symmetry-connected fixed-point solutions, with the outcome independent of the duration of the adiabatic process. Using a classical model Hamiltonian also relevant to a two-mode quantum system, we further demonstrate the formation of an adiabatic hysteresis loop in purely Hamiltonian mechanics and the generation of a Berry phase via changing one single-valued parameter only.

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Introduction.—The adiabatic theorem is about the dynamical behavior of a Hamiltonian system whose parameters are changing slowly with time. It constitutes a fundamental topic in Hamiltonian mechanics [1]. For example, Einstein was among the first to recognize the importance of classical adiabatic invariants in understanding quantization [2]. In recent years there is still considerable interest in several aspects of the adiabatic theorem in both quantum mechanics [3] and classical mechanics [4]. The relation between the quantum adiabatic theorem and the classical adiabatic theorem is also an interesting topic [5].

Classical adiabatic following is the subject of this study, but our findings are also relevant to certain quantum systems. We start from the fact that the classical adiabatic theorem is not an exact theorem: an actual adiabatically evolving trajectory fluctuates around an idealized solution predicted by the adiabatic theorem. Adiabatic invariants hence possess intrinsic dynamical fluctuations (IDFs) [4,6–9]. The magnitude of such fluctuations, typically proportional to the rate of change of adiabatic parameters, becomes extremely small in truly slow adiabatic processes. Therefore, except for special quantities that can accumulate IDF during an adiabatic process [4,9] (e.g., in calculations of dynamical angles), IDF does not seem to be interesting or physically relevant. As shown in this Letter via both theory and computational examples, this perception is about to change.

Bifurcation phenomenon is ubiquitous in nonlinear systems and it is of fundamental interest to many topics in physics, among which we mention localization-delocalization phase transitions and symmetry breaking [10–15]. Here we consider the adiabatic following of a

stable fixed-point solution, which, as a result of a varying adiabatic parameter, moves towards a supercritical pitchfork bifurcation. As schematically shown in Fig. 1(a), two new stable fixed points and one unstable fixed point emerge after the adiabatic parameter (denoted R) slowly passes the bifurcation point at $R = R_1$. It is then curious to know among the three fixed points which fixed-point solution the system will land on and whether the selection is predictable. It is found, both theoretically and numerically, that

FIG. 1. A supercritical pitchfork bifurcation in a classical Hamiltonian system parametrized by R . The solid (dashed) lines represent the phase space locations of stable (unstable) fixed points as a function of R . (a) When R exceeds R_1 , a stable fixed point splits into two stable and one unstable fixed points. One then wonders which branch the system will land on as R adiabatically increases beyond R_1 . (b) The bifurcated fixed points merge back when R exceeds R_2 . If R first increases from $R < R_1$ to $R > R_2$ and later returns to $R < R_1$, then an adiabatic hysteresis loop may be formed.

IDF is crucial for a *deterministic* and *intrinsic* selection between the symmetry-connected pair of stable fixed-point solutions. As such, through crossing the bifurcation point, a tiny IDF is amplified to a macroscopic level after the bifurcation: it determines the fate of the trajectory afterwards by “forcing” the system to make a selection between symmetry-breaking solutions. We term this as deterministic symmetry breaking because there is no need for external noise to initiate the symmetry breaking. We also note that the IDF-based symmetry breaking is independent of the duration of the adiabatic process. This differs from Ref. [15], where symmetry-breaking processes associated with a nonmoving fixed point are considered and the outcome is sensitive to a time-varying angle variable [15]. Figure 1(b) depicts an interesting situation that involves a second bifurcation at $R = R_2$, after which the three fixed-point solutions merge back to one stable fixed point. We shall show that this case may allow us to generate a Berry phase by manipulating one single-valued parameter only.

Preliminaries.—We recently developed a general description of IDF in classically integrable systems [4], which can be reduced to a rather simple form for stable fixed-point solutions in phase space. In particular, let us consider a one-dimensional Hamiltonian $H(q, p; R)$, with (q, p) being the canonical variables, R a system parameter to be tuned slowly, and its stable fixed-point solutions denoted by $[\bar{q}(R), \bar{p}(R)]$. According to the traditional picture offered by classical adiabatic theorem, a stable phase space fixed point has zero action, so when R varies slowly, the system must retain its zero action as an adiabatic invariant and therefore must follow the instantaneous fixed point $[\bar{q}(R), \bar{p}(R)]$. This is, however, a picture without IDF. The actual time evolving state (q, p) deviates from $[\bar{q}(R), \bar{p}(R)]$,

$$q = \bar{q} + \delta q, \quad p = \bar{p} + \delta p, \quad (1)$$

where $(\delta q, \delta p)$ are IDF on top of the idealized adiabatic solution $[\bar{q}(R), \bar{p}(R)]$ with $R = R(t)$. It is straightforward to see why $(\delta q, \delta p)$ has to be nonzero: were it indeed zero, then by definition of a fixed point, we have $\frac{\partial H(q, p, R)}{\partial q} = \frac{\partial H(q, p, R)}{\partial p} = 0$, indicating that the current state (q, p) cannot evolve and hence the system can never do adiabatic following with a moving fixed point. This indicates that nonzero $(\delta q, \delta p)$ is not due to nonadiabaticity. Rather, because a state slightly off a stable fixed point tends to rotate around that fixed point, there must be nonzero δq or δp in order to generate a movement of the actual state such that the actual state may adiabatically follow a moving fixed point [16]. Our general theory applied to the case of a moving stable fixed point [4] gives (see also [9])

$$\begin{pmatrix} \langle \delta p \rangle \\ \langle \delta q \rangle \end{pmatrix} = \Gamma^{-1} \begin{pmatrix} \frac{\partial \bar{p}}{\partial R} \\ \frac{\partial \bar{q}}{\partial R} \end{pmatrix} \frac{dR}{dt}, \quad (2)$$

where

$$\Gamma = \begin{pmatrix} -\frac{\partial^2 H}{\partial q \partial p} & -\frac{\partial^2 H}{\partial q \partial q} \\ \frac{\partial^2 H}{\partial p \partial p} & \frac{\partial^2 H}{\partial p \partial q} \end{pmatrix}_{p=\bar{p}, q=\bar{q}}, \quad (3)$$

and $\langle \cdot \rangle$ denotes an average over all possible initial conditions of $(\delta q, \delta p)$. As seen from Eq. (2), so long as Γ^{-1} exists, the scale of IDF is proportional to $\frac{dR}{dt} \equiv V$, the speed of adiabatic manipulation. The magnitude of $(\delta q, \delta p)$ is then deceptively small (for a sufficiently small V), but nonzero in general.

We now examine how nonzero $(\delta q, \delta p)$ impacts the adiabatic following of $[\bar{q}(R), \bar{p}(R)]$ that eventually undergoes a supercritical pitchfork bifurcation at $R = R_1$. First, right before the adiabatic parameter R reaches the bifurcation point R_1 , the system’s actual state (q, p) must deviate from the instantaneous solutions $[\bar{q}(R), \bar{p}(R)]$ by $(\delta q, \delta p)$. Without loss of generality and based on Eq. (2), the actual time-evolving state (q, p) is assumed to be slightly shifted to the left side of the instantaneous fixed point $[\bar{q}(R), \bar{p}(R)]$. This is illustrated in Fig. 2(a), where the periodic orbits (associated with a fixed R) around $[\bar{q}(R), \bar{p}(R)]$ are also shown. The second stage is illustrated by Fig. 2(b), where R only slightly exceeds R_1 . There the bifurcation already occurs, but the actual state does not “feel” the bifurcation yet: it continues to stay on the left side of all the three new fixed points. So the actual left-shifted state is located on an orbit (if R were fixed) surrounding all the fixed points (this is confirmed in our numerical studies). The line shown in Fig. 2(b) passing through the unstable fixed point represents the separatrix. In the last stage, R further increases, the two stable fixed points split further, with the stable fixed point moving to the left capturing the actual state [see Fig. 2(c)]. During the ensuing adiabatic following, the actual state then adiabatically follows the instantaneous fixed point on the left. Clearly then, if, as R approaches a bifurcation point, IDF can induce a definite shift (for a qualitative explanation, see [16]) of the actual state with respect to $[\bar{q}(R), \bar{p}(R)]$, then the system will be trapped, deterministically, by one of the two stable fixed-point solutions after the bifurcation. Note that, exactly

FIG. 2 (color online). Impact of intrinsic dynamical fluctuations on the crossing of a pitchfork bifurcation. (a) The system’s actual state is on the left of the instantaneous fixed point. (b) Immediately after the bifurcation, the actual state is still shifted to the left of all the fixed points. (c) As two stable fixed points split further, the actual state gets trapped by, and starts to adiabatically follow, the stable fixed-point solution on the left.

because of IDF, the actual state always stays away from the vicinity of the unstable fixed point (the extremely slow part of a separatrix). As a result the diverging time scale associated with the whole separatrix does not affect adiabatic following here. This understanding will be confirmed in our following numerical experiments.

Model Hamiltonian.—Here we turn to a concrete Hamiltonian system with supercritical pitchfork bifurcations. Specifically, we choose

$$H = -\frac{c}{2}q^2 - R\sqrt{1-q^2}\cos(p) + \Delta\sqrt{1-q^2}\sin(p) \quad (4)$$

in dimensionless units, with $c > \Delta > 0$. Interestingly, this system has two bifurcation points at $R_1 = -\sqrt{c^2 - \Delta^2}$ and $R_2 = \sqrt{c^2 - \Delta^2}$. Define $\eta = \sqrt{1 - (R^2 + \Delta^2)/c^2}$ and $\mu = \arctan(-\Delta/R)$. In the negative regime of R , the stable fixed point is $(\bar{q}, \bar{p}) = (0, \mu - \pi)$ for $R < R_1$, which then bifurcates into two stable fixed points at $(\mp \eta, \mu - \pi)$. In the positive regime of R , the two stable fixed points are at $(\mp \eta, \mu)$ for $R < R_2$ and then merge back to one stable fixed point at $(0, \mu)$. Note that this model Hamiltonian is invariant under the joint operation of space reflection ($\mathcal{P}$) $q \rightarrow -q$ and time reversal ($\mathcal{T}$) $t \rightarrow -t$. Therefore, for cases with only one stable fixed point, the solution itself has the same $\mathcal{PT}$ symmetry, and for cases with two stable fixed points, the pair is transformed to each other under the $\mathcal{PT}$ operations, with each individual fixed point being a symmetry-breaking solution. The phase space locations of these stable fixed-point solutions for various fixed values of R are shown by the solid lines in Fig. 3.

Another motivation to choose H in Eq. (4) is that it describes a two-mode many-body quantum system on the mean-field level. Consider the following mean-field Hamiltonian in dimensionless units ($\hbar = 1$),

$$\hat{H}_m = \begin{pmatrix} c(|b|^2 - |a|^2) & -R - i\Delta \\ -R + i\Delta & -c(|b|^2 - |a|^2) \end{pmatrix}, \quad (5)$$

where a and b are quantum amplitudes on two modes, c represents the self-interaction strength, and $R \pm i\Delta$ represents intermode coupling. By rewriting $a = |a|e^{i\phi_a}$, $b = |b|e^{i\phi_b}$, $p = \phi_b - \phi_a$, and $q = |b|^2 - |a|^2$, the time evolution of q and p , as obtained from the Schrödinger equation for this quantum model, becomes precisely that under the classical model Hamiltonian H in Eq. (4). The fixed points of H become eigenstates of $\hat{H}_m$; the adiabatic following when crossing a bifurcation for H is mapped to the issue of adiabatic following as degenerate eigenstates of $\hat{H}_m$ emerge. One possible realization of $\hat{H}_m$ is a Bose-Einstein condensate in a double-well potential, with the imaginary coupling constant Δ implemented via phase imprinting on one well [17]. $\hat{H}_m$ may also be realized in nonlinear optics by using two nonlinear optical waveguides with biharmonic longitudinal modulation of the refractive

FIG. 3 (color online). Solid lines depict phase space locations of stable fixed-point solutions for our model Hamiltonian in Eq. (4) as values of R are scanned. The solid and empty dots represent the actual states of the system in numerical experiments of an adiabatic process, where R is first increased from $R = -0.25 < R_1$ to $R = 0.25 > R_2$, and then returns to $R = -0.25$, at a constant speed $V = 10^{-6}$. Long arrows indicate the moving direction of the actual states as R changes. The four double-well plots schematically show the symmetry-breaking consequence in the context of a two-mode Bose-Einstein condensate Hamiltonian. $c = 0.2$, $\Delta = 0.1$. Variables plotted here are in dimensionless units.

index [18]. Therefore, our detailed results below are relevant to both classical and quantum physics.

Theory and numerical experiments.—Applying the theory of IDF to the Hamiltonian in Eq. (4), one obtains $\langle \delta p \rangle = 0$ and $\langle \delta q \rangle = -\frac{\Delta}{\sqrt{R^2 + \Delta^2}} \frac{1}{R^2 + \Delta^2 - c\sqrt{R^2 + \Delta^2}} \frac{dR}{dt}$ for $R < R_1$ [19]. Therefore, as R increases from $R < R_1$, $\frac{dR}{dt} = V > 0$, and $\langle \delta q \rangle$ is definitely negative. Returning to the phase space plot in Fig. 3, this means that if R increases from $R < R_1$, the actual state (on average, to be more precise) is slightly left shifted from the fixed point at $(0, \mu - \pi)$. So after passing the bifurcation at $R = R_1$, the system is expected to adiabatically follow the left symmetry-breaking solution $(\bar{q}, \bar{p}) = (-\eta, \mu - \pi)$ for $R_1 < R < 0$ and $(\bar{q}, \bar{p}) = (-\eta, \mu)$ for $0 < R < R_2$. Similar theoretical results show that, if R decreases from $R > R_2$, then the actual state must be slightly right shifted from the fixed point at $(0, \mu)$, a prediction consistent with the above-mentioned $\mathcal{PT}$ symmetry of the system. So after crossing the bifurcation point at $R = R_2$, the system should adiabatically follow the other symmetry-breaking solution $(\bar{q}, \bar{p}) = (\eta, \mu)$ for $0 < R < R_2$ and $(\bar{q}, \bar{p}) = (\eta, \mu - \pi)$ for $R_1 < R < 0$. As shown in Fig. 3 (solid or empty dots), our numerical results based only on Hamilton's equation of motion confirm this prediction. For the results shown we have set $V = 10^{-6}$ to ensure a slow process. Indeed, at all times the difference between the actual states (dots) and the instantaneous fixed points (solid lines) is invisible to our naked eyes. Yet, the small IDF does assist in a symmetry-breaking choice regarding which of two stable fixed-point

solutions is adiabatically followed by the system. In the language of $\hat{H}_m$, after the system passes the bifurcation at $R = R_1$ owing to an increasing R , $q = |b|^2 - |a|^2$ becomes appreciably negative, so one mode develops more population than the other, signaling a clear delocalization-localization transition induced by IDF. The opposite population imbalance occurs when the system passes the bifurcation at $R = R_2$ with a decreasing R . Furthermore, joining these two manipulation steps together so that R returns to its very original value in the end (which is already the case in Fig. 3), we clearly observe the formation of an adiabatic “hysteresis” loop in phase space. That is, by increasing and then decreasing R , a navigation loop in phase space is formed because the adiabatic following in the forward step and the backward step lands on different symmetry-breaking branches [20].

There is one subtle point to be clarified: Our theory of IDF [see (Eq. (2))] is about quantities $\langle \delta q \rangle$ averaged over all initial conditions of $(\delta q, \delta p)$, but what determines the adiabatic following is the actual δq in a single process. The Supplemental Material [16] contains a detailed analysis on this point. In particular, we show that if initially there is a small deviation of δq from $\langle \delta q \rangle$, then the difference $\delta q - \langle \delta q \rangle$ oscillates with time, and later, as R approaches the bifurcation point, this deviation becomes negligible as compared with $\langle \delta q \rangle$. It is for this reason that the definite sign of $\langle \delta q \rangle$ leads to a definite sign of δq , which hence justifies our theory based on $\langle \delta q \rangle$.

Berry phase generation via one single-valued parameter.—As a final interesting concept, we discuss the generation of a Berry phase using one single-valued adiabatic parameter R . This is made possible by IDF and bifurcations. In particular, the navigation loop in Fig. 3 shows that the time-evolving states of $\hat{H}_m$ trace out a nontrivial geometry after increasing R from $R < R_1$ to $R > R_2$ and then returning R to its initial value. Let $|\psi_{\text{left}}\rangle$ and $|\psi_{\text{right}}\rangle$ be the eigenstates of $\hat{H}_m$ (with $R_1 < R < R_2$) mapped from the left and right fixed-point solutions of H . Assuming exact adiabatic following with the instantaneous adiabatic eigenstates, the Berry phase generated along the navigation loop is analytically found to be

$$\begin{aligned} \beta_{\text{Berry}} &= i \int_{R_1}^{R_2} dR \left(\langle \psi_{\text{left}} | \frac{d}{dR} | \psi_{\text{left}} \rangle - \langle \psi_{\text{right}} | \frac{d}{dR} | \psi_{\text{right}} \rangle \right) \\ &= \pi(1 - \Delta/c). \end{aligned} \quad (6)$$

We have also carried out numerical experiments by integrating the Berry connection using the actual states during the physical process with $V = 10^{-6}$ (this simple method will not account for the nonlinear Berry phase correction studied in Ref. [9]). Fair agreement between theory and simulations is obtained, but with some small differences. For example, for $c = 0.15$ and $\Delta = 0.1$, the theoretical result is $\beta_{\text{Berry}} = 1.047$, and our simulation yields $\beta_{\text{Berry}} \approx 1.076$. This tiny difference reminds us that a direct

numerical integration of the Berry connection along the actual time evolution path is not necessarily reliable because it could accumulate the effect of IDF [4]. Nevertheless, the obtained agreement still confirms our analytical treatment, demonstrates the feasibility of Berry phase generation using only one single-valued adiabatic parameter, and verifies from another angle that IDF is important for understanding adiabatic following in the presence of bifurcation.

Conclusion.—Bifurcation greatly amplifies subtle intrinsic fluctuations that are beyond classical adiabatic invariants. This leads to a selection rule regarding which of two symmetry-connected stable fixed-point solutions may be adiabatically followed. In cases of multiple bifurcation points, adiabatic hysteresis loops in phase space and the generation of Berry phase by manipulating one single-valued parameter are also shown to be possible. Our findings are of fundamental interest to both classical systems and quantum many-body systems on the mean-field level. The implications of this work for symmetry breaking in fully quantum many-body systems should be a fascinating topic in our future studies.

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