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Comparison of Turbulence Models in Near Wake of Transport Plane C-130H Fuselage

Samuel K. H. Pang*, E.Y.K. Ng† and W.S. Chiu‡
Nanyang Technological University, Singapore 639798, Singapore

ABSTRACT

This paper presents a CFD analysis of the DDES-SST on a 1:16 scale simplified C-130H fuselage model in flight conditions and with a closed cargo ramp to observe the turbulent flow structures that evolve behind its empennage region. The results of which are also compared against already existing x-vorticity plots of the DDES-SARC. With vorticity magnitude of 750 rotations/s, the DDES-SST is shown to display a lot more detail in terms of the formation and unsteadiness of the upsweep and detached vortices. The simulations are performed with the finite volume solver Cobalt with up to sixty-four processors on a high performance computer.

Nomenclature

\[ \Delta = \text{distance between neighbor cell center with cell center in concern} \]
\[ d = \text{distance to closest wall} \]
\[ \varepsilon_{jmn} = \text{Levi-Civita tensor} \]
\[ \varepsilon = \text{turbulence dissipation rate} \]
\[ \gamma = \text{specific heat ratio} \]
\[ k = \text{turbulence kinetic energy} \]
\[ M = \text{Mach number} \]
\[ \Omega = \text{vorticity magnitude} \]
\[ \Omega_{ij} = \text{components of vorticity} \]
\[ P = \text{pressure} \]

* Research Student, Division of Thermal and Fluids Engineering, Sch. of Mech. & Aero. Engg.
† Associate Professor, Division of Thermal and Fluids Engineering, Sch. of Mech. & Aero. Engg.
‡ Head of Aerodynamic Branch, Air Logistic Dept., RSAF, Singapore
\[ R = \text{gas constant} \]
\[ \rho = \text{atmospheric density} \]
\[ S = \text{strain rate magnitude} \]
\[ S_{ij} = \text{components of strain rate tensor} \]
\[ \sigma = \text{Prandtl number} \]
\[ T = \text{temperature} \]
\[ \omega = \text{specific dissipation rate} \]
\[ \nu_t = \text{eddy viscosity} \]
\[ \nu = \text{molecular viscosity} \]

I. Introduction

The Lockheed C-130 Hercules is arguably one of the most versatile fixed-wing transport aircraft available in any modern air force's arsenal. First introduced in 1974 as an improvement to the E variant, the Hercules, and its sub-variants, is the most widely used and exported C-130 aircraft. The cargo compartment is 12.31 m long, 3.12 m wide and 2.74 m high and most of the loading and unloading is assisted via a cargo ramp, known also as the tailgate, that is 3.12 m long by 3.02 m wide located aft of the aircraft. It is also the main platform on which tactical mid-flight ejections of goods are performed.

During the standard airdrop method, materials and goods are released at altitudes of 50 to 1500 ft at aircraft velocities between 120 to 150 knots to avoid enemy radar detection and anti-aircraft artillery counter-action\(^1\). A mid-air ejection as such always entails high risk for the materials dropped as well as the C-130H. In addition, priority is given to securing the materials' integrity. At such flight speeds, the mouth of the cargo bay is faced with zones of massive flow separations and recirculation. Not to mention, the upsweep draft may have a force large enough to propel the extraction parachutes, together with its cargo, upwards, veering it off course from the drop zone.

In more destructive scenarios, both aircraft and goods might be damaged as the parachutes are brought close the tail end, while the turbulent vortices perpetually slam the goods against the underside aircraft body or tail. With such potentially undesirable circumstances to be considered, accurate prediction of these turbulent structures is essential to imposing safer protocols for airdrop procedures to achieve improved resupply operation success rates.
The delayed detached-eddy simulation implementation of the Menter’s shear stress transport (DDES-SST) turbulence model is applied to a simplified C-130H fuselage to observe the vortices evolved at the empennage region. At the same time the results of DDES-SST are compared with the DDES counterpart using Spalart-Allmaras with rotation and curvature correction (DDES-SARC) for accuracy, efficiency and feasibility.

II. Background

This paper is part of the research by the Republic of Singapore Air Force (RSAF) to improve airdrop procedures on the C-130H. It is a parallel study to the wind tunnel and CFD analyses conducted by the United States Air Force Academy (USAFA), France’s Institut Supériuer de l’Aéronautique et de Espace (ISAE), the German Federal Office for Defense Technology and Procurement (IABG) and United Kingdom’s Joint Air Transport Evaluation Unit (JATEU) under NATO’s Aircraft Influence on Airdrop (AIA) project which commenced in 2002. In the CFD segment of the AIA project, Claus et al. worked on a half simplified C-130H CAD model and ran the detached-eddy simulation (DES) on it. The team found good resemblance between the CFD simulation and the wind tunnel PIV captures of upsweeping vortices propagating along the underside of the empennage into the wake. The DES also captured a downwash interaction of the wing tip vortices with the upsweep occurring slightly away from the fuselage.

Morton et al. used a 1:16 scale CAD model of simplified closed tailgate fuselage model of the C-130H which corresponded to their wind tunnel model and subjecte it to a speed of 40 m/s and at zero angle of attack. According to Morton et al., the rationale for the fuselage model was that with the main wings on, the model would not be able to fit the width of the wind tunnel. Furthermore, the main wings provided an additional undesirable downwash which had to be negated by increasing the angle of attack to about 2 to 8˚ so that the airflow angle seen from the cargo bay would be zero.

In the same paper, SARC, including the DES implementation of it, was used in their CFD analysis over the SST because it was computationally 14% quicker only 1% less accurate in comparison to the lift and drag findings from their wind tunnel experiments. The SARC, not the Spalart-Allmaras (SA), was selected due to the fact that it had the ability to portray curvature effects of convex walls better. In the end, the results proved that there was virtually no lift and drag difference between DES and its Reynolds-Averaged Navier-Stokes (RANS) turbulence model counterpart, but the DES was far superior in portraying the instantaneous breaking up of vortices in the wake.
Morton et al. also noticed that when the tailgate was opened, a revelation of altered upsweep vortices and a disappearance of detached vortices due to a change in the geometry on the underside of the fuselage.

Bury et al. went further to show that the upsweep vortices followed the empennage geometry very closely and induced vortices appeared right under the empennage’s horizontal wings. Induced vortices are created from the upsweep-wall interactions.

More recently, Bergeron et al. used the same 1:16 CAD model as Morton et al. did, replacing the DES with DDES. They discovered that the DDES surpassed the DES in that its predictions of orientations and shapes of vortices corresponded almost exactly to the PIV pictorials gathered. The vortex strength at the cargo ramp unfortunately was over-predicted because, as Bergeron et al. postulates, of the lack of the main wings in place or due to the over-sensitivity which the SARC brings to a grid that has been sufficiently resolved for rotational compensation.

III. Turbulence Modeling

Reynolds-Averaged Navier-Stokes (RANS) equations turbulence models conventionally have been the method of choice for most CFD simulations. Models like the SA and Menter’s SST are although suited for attached flow studies, they do not predict flow physics well in massively detached flow regions, meaning that RANS is adequate in the boundary layers but not so in high Reynolds wakes. Conversely, large-eddy simulation (LES) functions opposite to the RANS. The DES formulation was proposed by Spalart et al. as a RANS-LES hybrid that would serve both purposes by critically selecting the RANS’s and LES’s strengths. The SA equation, given below, formed the backbone of the original DES method:

$$\frac{D\tilde{v}}{Dt} = c_{b1} \tilde{S} \tilde{v} + \frac{1}{\sigma} \left( \nabla \cdot (\tilde{v} + \bar{v}) \nabla \tilde{v} \right) + c_{h1} \tilde{v}^2 \frac{1}{\bar{d}}$$

(1)

where $\tilde{v}$ is the working variable and the constituents to the equation are

$$\chi = \frac{\tilde{v}}{v}$$

$$v_i = \tilde{v} f_{v1}$$

$$f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}$$

$$\tilde{S} = S + \frac{\tilde{v}}{k^2 d^2} f_{v2}$$

$$f_{v2} = 1 - \frac{\chi}{1 - \chi f_{v1}}$$

$$f_w = g \left[ 1 + c_{w3} \right]^{1/6}$$
\[ g = r + c_{w2}(r^6 - r) \quad \quad r = \frac{\bar{v}}{S\kappa^2 d^2} \]

In formulating the transport equation, Spalart and Allmaras proposed this set of constants given below:

\[
\sigma = \frac{2}{3} \quad \quad c_{b1} = 0.1355 \quad \quad c_{b2} = 0.622
\]

\[
c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{1 + c_{b2}}{\sigma}
\]

\[
\kappa = 0.41 \quad \quad c_{v1} = 7.1
\]

\[
c_{w2} = 0.3 \quad \quad c_{w3} = 2
\]

The Spalart-Shur correction to accommodate rotations and curvatures\(^5\) simply modifies the production term of the SA by multiplying it with the rotational function

\[
f_r \left( r^*, \bar{r} \right) = (1 + c_{r1}) \left( \frac{2r^*}{1 + r^*} \left[ 1 - c_{r3} \tan^{-1}(c_{r2}\bar{r}) \right] - c_{r1} \right)
\]

(2)

and its constituting variables are

\[
r^* = \frac{S}{\Omega} \quad \quad \bar{r} = 2\Omega_{ik}S_{ik} \left[ \frac{DS_{ij} + \left( E^{imm}S_{jm} + E^{jmn}S_{in}\right)Q_m}{D^4} \right] \quad \quad S^2 = 2S_{ij}S_{ij}
\]

\[
\bar{\Omega}^2 = 2\Omega_{ij}\Omega_{ij} \quad \quad \Omega_{ij} = \frac{1}{2} \left[ \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) + 2E_{mij}Q_m \right] \quad \quad D^2 = \frac{1}{2} \left( S^2 + \bar{\Omega}^2 \right)
\]

while the rest of the constants in the rotational function are \( c_{r1} = 1.0, \quad c_{r2} = 12 \) and \( c_{r3} = 1.0 \).

In order for the DES to switch from RANS in the boundary layer to LES in the wake, a limiter is placed to transform the distance to the closest wall, \( d \) variable into

\[
\bar{d} = \min(d, C_{DES} \Delta) \quad C_{DES} = 0.65
\]

(3)

The DES can incur problematic premature separations in thick boundary layers due to modeled stress depletion (MSD) at ambiguous grids\(^{10-12}\). Menter et al.\(^{13}\) introduced the DDES to ‘protect’ the boundary layer and delay the
separation further downstream. The DDES was initially implemented on the SST but the concept could also be applied to the SA by further refining the $r$ ratio into the delayed ratio $r_d$ (the subscript $d$ indicates delayed),

$$ r_d = \frac{v_t + v}{S \kappa^2 d^2} \quad (4) $$

This is required in the function

$$ f_d = 1 - \tanh \left[ 8 r_d \right] \quad (5) $$

which in turn was part of the new length scale:

$$ \tilde{d} = d - f_d \max \left( 0, d - C_{\text{DES}} \Delta \right) \quad (6) $$

In a similar manner, Menter’s $k$-$\omega$ SST turbulence model can be implemented into the DDES. The SST is a two transport equation turbulence model which is a hybrid of the established $k$-$\varepsilon$ and $k$-$\omega^{14}$. The $k$-$\omega$ is very sensitive to the arbitrary freestream $\omega$ values that are designated outside the boundary and is therefore unable to adequately predict turbulence outside boundary layer regions well. The $k$-$\varepsilon$ conversely is less sensitive to the arbitrary freestream values, but reacts drastically within the boundary layer where adverse pressure gradients are present.

Menter blended them both and thus his new model took the form:

$$ \frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_i k)}{\partial x_i} = \tilde{P}_k - \beta^* \rho k \omega + \frac{\partial}{\partial x_i} \left[ \left( \mu + \sigma_{\varepsilon} v \right) \frac{\partial k}{\partial x_i} \right] \quad (7) $$

$$ \frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho u_i \omega)}{\partial x_i} = \alpha \rho S^2 - \beta \rho \omega^2 + \frac{\partial}{\partial x_i} \left[ \left( \mu + \sigma_{\omega} v \right) \frac{\partial \omega}{\partial x_i} \right] + 2 \left( 1 + F_1 \right) \rho \sigma_{\omega} + \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i} \quad (8) $$

Menter’s production term, $\tilde{P}_k$, already has in place a rotation correction modification that involves substituting the strain-rate tensor with a vorticity tensor.

The length scale used was $l = \sqrt{k} / (\beta^* \omega)$ and first blending function was introduced:

$$ F_1 = \tanh \left( \arg^4 \right) \quad (9) $$

whereby
\[
\arg_1 = \min \left[ \max \left( \frac{\sqrt{k}}{0.09 \omega d}, \frac{500 \nu}{d^2 \omega} \right), \frac{4 \rho \sigma_{w2} k}{D_k \omega d^2} \right], \quad CD = \max \left( 2 \rho \sigma_{w2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, 10^{-10} \right)
\]

He also reconstructed the eddy-viscosity term into

\[
v_\tau = \frac{a_{1k}}{\max(a_{1k}, SF_2)}
\]

with the second blending function given as

\[
F_2 = \tanh(\arg_2), \quad \arg_2 = \max \left( \frac{2 \sqrt{k}}{0.09 \omega d}, \frac{500 \nu}{d^2 \omega} \right)
\]

Strelets\textsuperscript{15} implemented the DES on the SST by changing the length scale to \( \tilde{l} = \min(l, C_{DES} \Delta) \) and introduced a DES blending function in the dissipative term of the SST equations:

\[
D_{DES} = \beta^* \rho \omega F_{DES}
\]

\[
F_{DES} = \max \left( \frac{l}{C_{DES} \Delta}, 1 \right), \quad C_{DES} = (1 - F_1) C_{DES}^{k_{-\omega}} + F_1 C_{DES}^{k_{-\epsilon}}
\]

whereby \( C_{DES}^{k_{-\omega}} = 0.78 \) and \( C_{DES}^{k_{-\epsilon}} = 0.61 \). The rest of the constants are given as \( \alpha_1 = 5/9, \alpha_2 = 0.44, \beta^* = 0.09, \beta_1 = 3/40, \beta_2 = 0.0828, \sigma_{k1} = 0.85, \sigma_{k2} = 1, \sigma_{\omega1} = 0.5 \) and \( \sigma_{\omega2} = 0.856 \).

To counter the problem of premature separation caused by MSD, Menter \textit{et al.}\textsuperscript{13} “shielded” the boundary layer with the DDES limiter

\[
F_{DDES} = \max \left[ \frac{1}{C_{DES} \Delta} \left( 1 - F_{SST} \right), 1 \right], \quad F_{SST} = 0, F_1, F_2
\]

essentially converting the DES to DDES.

\section*{IV. Grid and Numerical Setup}

The grid was provided by USAFA, one which was similar to Bergeron \textit{et al.’s}, with a flow domain of \( 40 \times 20 \times 10 \) m and a fuselage model of 1.7 m, which is approximately 1/16 of the actual size. The domain contains
12.8\times10^6 \text{ tetrahedral cells and the mesh at the empennage and in the wake areas have been carefully refined to capture the best possible predictions of the turbulent flow structures occurring behind the aircraft. In addition to this, there were 12 prismatic layers and an average y+ value of 0.1 in the boundary layer to ensure that the velocity profiles would be resolved properly. Fig. 1 shows the fuselage model, its placing in the computational domain as well as the mesh generated at the empennage region.}

No grid convergence test was conducted in this study because Morton et al. had already done so with grid meshes ranging from 3.05\times10^6 to 11.27\times10^6 \text{ cells. They concluded that increasing the number of cells above 6\times10^6 \text{ cells yielded insignificant effects on the lift (C}_L\text{) and drag (C}_D\text{) coefficients.}

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**Fig. 1** Flow domain, C-130H fuselage CAD model and grid in the wake of the aircraft.
The simulation was performed on Nanyang Technological University’s high performance computer at the High Performance Computing Centre. For a single case, it used eight dx360 M2 cores on the IBM iDATAPlex platform, amounting to 64 processors with a memory capability of 192 GB. **Solutions were all done with Cobalt v5.0. Cobalt is the commercial derivative of the military’s Cobalt 60**, a compressible flow cell-centered, finite volume solver developed by the Air Force Research Laboratory. In the newest version, Cobalt v5.0, released in 2010, all the DES algorithms from the previous version had been updated, among numerous other updates such as implementation of a time-step ramp for transient solution manipulation and overset grid compatibility. Other details of the solver updates may be found at the Cobalt Solutions, LLC news website, URL: www.cobaltcfd.com/index.php/site/news/.

Flight conditions were analogous to normal atmospheric conditions to the wind tunnel setup by Morton *et al.*, where $\rho = 1.18448 \text{ kg/m}^3$, $\gamma = 1.4$, $T = 298 \text{ K}$, $R = 287 \text{ kJ/(kg K)}$, $P = 101325 \text{ Pa}$. The Mach number was set at $M = 0.1155$, which is equivalent to 40 m/s at an angle of attack (AOA) of 0˚. The AOA would range from -5˚ to 5˚, with 5˚ intervals. The fuselage surface was taken as an adiabatic non-slip wall without any trips.

V. **Results**

Time-averaged $C_L$ and $C_D$ values over the range of AOA tested showed the simulated $C_L$ for both numerical schemes adhered closely to the experimental values from Morton *et al.* report while the simulated $C_D$ was perpetually below expectation. It could be due to the additional number of cells particularly close to the fuselage surface which would require more iterations in order to bring the $C_D$ values to within the same magnitude. The SARC implementation of the DDES was also sensitive towards massively separated flows where pressure gradients are sufficiently high as reported by Bergeron *et al.* Comparing the DDES-SST and DDES-SARC, $C_L$ was similar throughout (Fig. 2) however there was an exception at 5˚ where the DDES-SARC measured 0.002 less in $C_D$ value than the DDES-SST’s. This in turn gave a significant 20% increase in error over the DDES-SST against the baseline experimentation (Fig. 3).
Fig. 2 Lift and drag comparisons between DDES-SARC and DDES-SST with respect to angle of attack.
The difference in the two turbulence models however surfaced more clearly in the way they capture the vortices evolving from the fuselage. Morton et al.\textsuperscript{4} and Bergeron et al.\textsuperscript{7} in their $x$-vorticity plots revealed two symmetrical upsweep, empennage wing tip and detached vortices evolving in the flow field slightly aft of the tail. The detached and upsweep vortices are fed by two vorticity pockets travelling up the underside of the fuselage. A single trail of detached vortex accompanies either upsweep vortex line.

With the DDES-SST, these vorticity pockets of magnitude 750 rotations/s were larger and more pronounced (Fig. 4). This led to each upsweep vortex possessing at least three detached vortices. Similar effects were seen at $5^\circ$ and $-5^\circ$ in Figs. 4(b&c), where the DDES-SST produced in greater details the unsteadiness of vortices seen developing along the trailing edge of the horizontal tail wings than the SARC equivalent. Such observations were in line with Zhong’s\textsuperscript{17} RANS comparison of the SARC and SST in his study on vortices borne from wing-fuselage configurations at various angles of attacks. In his research, he found that the SST graphical resolution of vortices was almost congruent to his wind tunnel test on a fighter jet aircraft at all angles of attack. The SARC worked well at higher angles but fared worse than its SST counterpart in the lower angles.

As Zhong\textsuperscript{17} had pointed out, the SA equation tended to induce large dissipation near the aircraft surface, reducing the clarity of vortex cores and was inadequate in resolving the vortices as accurately as Menter’s SST.

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**Fig. 3 Morton et al.’s CFD results of (a) lift and (b) drag coefficients with wind tunnel data [4].**
Since the SARC is built upon the SA by adding the rotational correction function $f_{r1}$, there is no change in the dissipation modeling, governed by $\nu_r$, from wall surfaces and would remain as high as with the SA. This could explain the lack of representation of vortices by the DDES-SARC as opposed to the DDES-SST. Nonetheless, experimentations on the SARC have proved that it has remarkably improved the results from the SA due to the modification\textsuperscript{5,17}.

(a) 0˚ angle of attack

(b) 5˚ angle of attack
Airflow around the empennage experiences adverse pressure gradients, particularly in the absence of the main wings which provide a downwash “shield”. As Bergeron et al.\textsuperscript{7} mentioned, the SARC implementation of the DDES was sensitive towards massively separated flows where pressure gradients are especially high. Furthermore, the result of a highly refined grid around the empennage region in fact induced problematic inaccuracies as the SARC tries to resolve the shear stresses production.

Another reason to the difference between both turbulence models could be that Menter\textsuperscript{14} in his SST formulation had already accounted for the greater production of turbulent shear stress over its dissipation in boundary layer. He ensured that the eddy-viscosity observes Bradshaw’s assumption of the proportional shear stress to turbulent kinetic energy was maintained. In the rest of the free shear flow, the eddy-viscosity would express to its original value of $\nu_t = k/\omega$. The effect to this formulation was that regions of adverse pressure gradients were a lot better managed than many other eddy-viscosity models, and perhaps even the SARC.

**VI. Conclusion**

The SARC and SST versions of the DDES were pit together on their performance in the simulation of a C-130H fuselage with a closed cargo hatch. Both SST and SARC did not produce any major differences in the lift and drag...
coefficients with respect to the fuselage angle of attack, however in the process, the DDES-SST was found to have predicted a far greater detail of vortex production and unsteadiness as the vortices emerged from the tail regions.

This research has served to be a platform for opened cargo hatch studies utilizing the DDES-SST as the numerical model due to the concerns of the SARC with inaccuracies in high pressure gradients, especially at the mouth of the cargo bay. With the details that the SST promises, it will be highly beneficial for future work pertaining to airdrop situations and parachute release from the cargo bay.

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