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Localization for Mixed Near-Field and Far-Field Sources Using Data Supported Optimization

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Abstract—Recently, localization for the coexistence of the far-field and near-field sources has received more attentions. In this paper, a maximum likelihood (ML) localization method using data supported optimization is considered. The range and direction of arrival (DOA) of the sources are estimated sequentially. Since a two step estimation method is used, the proposed method is applicable for the near-field sources, far-field sources or the mixture of these two kinds of sources. Furthermore, the proposed method is applicable for far-field and near-field source classification. Simulations are implemented to verify the performance of the proposed method.

Index Terms—Maximum likelihood, near-field source, far-field source, data supported optimization

I. INTRODUCTION

Source localization using passive sensor arrays is a fundamental problem in array signal processing. Many techniques have been proposed for localization of either far-field or near-field sources. Recently, localization for the coexistence of the far-field and near-field sources has received more attentions, which is common in some practical applications. For example, coexistence of the far-field and near-field sources may be encountered for speaker localization using microphone arrays, as mentioned in [1]. Therefore, it is of practical importance to develop methods for the far-field and near-field mixture scenarios. Far-field sources are parameterized by direction of arrival (DOA). Meanwhile, both DOA and range are used to describe near-field sources.

In [1], the authors propose a cumulant based two-stage multiple signal classification (MUSIC) localization technique for mixed near-field and far-field sources. However, the higher order cumulant based method would fail when the Gaussian sources are presented. Furthermore, the computational complexity is high to calculate the cumulant of the signals. In [2], the authors propose a second order statistics based MUSIC algorithm, the directions of the far-field and near-field are sources are estimated separately. It is efficient compared with the higher order statistics based methods.

The maximum likelihood (ML) method is one of the widely used techniques for direction of arrival (DOA) estimation, which has excellent performance in both asymptotic and threshold regimes. However, multi-dimensional search is needed to obtain the global maximum of the likelihood function. Stochastic ML and deterministic ML estimators are obtained by making different assumptions on the source signals. Here deterministic ML estimator is considered. The source signals are unknown, but nonrandom.

Many techniques have been proposed to obtain the ML estimator. Method of direction estimation (MODE) is a computationally simple approach, which is a large sample realization of ML estimator [3]. However, the threshold signal-to-noise ratio (SNR) is high or large number of snapshots is needed. Expectation maximization (EM) [4], alternating projection [5], space alternating generalized EM (SAGE) [6], and gradient based techniques are typical examples of local search approach. An eigen-decomposition based alternating projection method with reduced computational complexity compared with the direct alternating projection method is proposed in [7]. Comparative convergence analysis of EM and SAGE algorithms is given in [8]. Recursive EM and SAGE-inspired algorithms are proposed in [9]. The main drawback for local search approach is that it can only guarantee convergence to a local maximum; the performance depends on the initial DOA values used in the search.

Nature inspired optimization algorithms are also used obtain the global optimal solution of the ML estimator, which are global search based approaches. Such as simulated annealing (SA) [10], genetic algorithms (GA) [11], particle swarm optimization (PSO) [12]. For the nature inspired optimization based global search methods, the estimation error is low but the computational burden is high. Furthermore, the performance critically depends on the choice of the parameters of these algorithms.

Data supported optimization (DSO) is an interesting approach emerging in the statistical literature. The basic idea of this approach is to partition the data sample in a large number of elemental sets. Parameters are estimated for each elemental data set in parallel mode. These estimates are then viewed as a data supported grid over which the likelihood function can be maximized. A DSO approach is first proposed in [13] to obtain ML DOA estimates for far-field sources. They show that the DSO approach despite its conceptual and computational simplicity is quite an effective tool to compute a good approximation to the ML DOA estimator.

In this paper, we propose a DSO based ML estimation technique which is applicable for the near-field sources, far-
field sources or the mix of these two kinds of sources. To obtain the data supported grid points, a two step estimation technique is used, which is proposed in [14]. We assume that the number of sources is known. However, the number of far-field and near-field sources is unknown. To apply the MUSIC algorithm, the number of sources should be less than the number of sensor elements. For the proposed method far-field and near-field source classification is also considered, which can be done based on the estimated parameters.

Notations Scalars are denoted by italic letters, column vectors by lower-case bold-face letters and matrices by uppercase boldface letters. Blackboard bold letters \( \mathbb{R} \) and \( \mathbb{C} \) denote the real and complex numbers, respectively. \((\cdot)^T\) and \((\cdot)^H\) denote transpose and Hermitian transpose, respectively. \(\det[\cdot]\) is the determinant of a square matrix. Operator diag[\(\cdot\)] creates a diagonal matrix with elements of a vector.

II. MIXED NEAR-FIELD AND FAR-FIELD SIGNAL MODEL

We consider a mixed near-field and far-field scenario of \( N \) uncorrelated narrowband sources impinging on a uniform linear array (ULA) with \( 2M+1 \) elements. The inter-element spacing is \( d \) and the center of the array is selected as the phase reference point. The system model for near-field source is shown in Fig.1.

![Fig. 1. System model for near-field sources.](image)

The received signal of the \( m \)th sensor at time index \( k \) can be expressed as

\[
x_m(k) = \sum_{n=1}^{N} s_n(k)e^{j\tau_{mn}} + w_m(k),
\]

where \( s_n(k) \) is the \( n \)th source signal, \( w_m(k) \) is the additive Gaussian noise for the \( m \)th sensor element. \( \tau_{mn} \) is the phase shift associated with the propagation time delay between sensor 0 and sensor \( m \) of the \( n \)th source signal.

The \( n \)th source within the Fresnel region is considered as near-field source, which is given by

\[
r_n \in \left[ 0.62 \sqrt{D^3 \lambda, 2D^2 \lambda} \right],
\]

where \( D \) is the aperture of the array and \( \lambda \) is the wavelength of the source signal [14].

For near-field source \( n \), \( \tau_{mn} \) is a function of source signal parameters, range \( r_n \) direction of arrival \( \theta_n \in [-\frac{\pi}{2}, \frac{\pi}{2}] \) and interelement spacing \( d \), which is given by

\[
\tau_{mn} = \frac{2\pi}{\lambda} \left( \sqrt{r_n^2 + (md)^2} - 2r_n md \sin(\theta_n) - r_n \right),
\]

where \( \theta_n \) is measured with respect to the broadside of the array. \( \tau_{mn} \) can be approximated by using the second-order Taylor expansion.

\[
\tau_{mn} \approx m\omega_n + m^2\phi_n, \quad (4)
\]

where \( \omega_n \) and \( \phi_n \) are called electric angles and are given by

\[
\omega_n = -2\pi \frac{d}{\lambda} \sin(\theta_n), \quad (5)
\]

\[
\phi_n = \pi \frac{d^2}{\lambda^2} \cos^2(\theta_n). \quad (6)
\]

The system model for far-field source is shown in Fig.2. For far-field sources, \( \tau_{mn} \) is obtained by first-order Taylor expansion of (3), which can be considered as a special case for the near-field source by letting \( \phi_n = 0 \). Rather than estimate the source parameters \( r_n \) and \( \theta_n \) for the near-field sources, \( \theta_n \) for the far-field sources directly. Parameters \( \{\omega_n, \phi_n\}, n = 1, 2, \cdots, N \) are estimated.

The received snapshots \( x(k) \) can be written as

\[
x(k) = As(k) + w(k) = \begin{bmatrix} x_{-M}(k) \\ \vdots \\ x_0(k) \\ \vdots \\ x_M(k) \end{bmatrix} \in \mathbb{C}^{(2M+1)\times 1}
\]

where \( s(k) = [s_1(k), \cdots, s_N(k)]^T \in \mathbb{C}^{N\times 1} \) is the signal vector and \( w(k) = [w_{-M}(k), \cdots, w_M(k)]^T \in \mathbb{C}^{(2M+1)\times 1} \) is the noise vector. Array manifold matrix \( A \in \mathbb{C}^{(2M+1)\times N} \) is given by

\[
A = [a(\omega_1, \phi_1) \ a(\omega_2, \phi_2) \cdots \ a(\omega_N, \phi_N)],
\]
where the array steering vector $\mathbf{a}(\omega_n, \phi_n)$ is given by

$$
\mathbf{a}(\omega_n, \phi_n) = \begin{bmatrix}
a_{n,-M} & \cdots & e^{-j\omega_n M - j\phi_n M^2} \\
a_{n,0} & \cdots & 1 \\
\vdots & \ddots & \vdots \\
a_{n,M} & \cdots & e^{j\omega_n M + j\phi_n M^2}
\end{bmatrix}.
$$

(9)

From (9), we can see that the elements are symmetric with respect to the second term. The second term $e^{j\omega_n M + j\phi_n M^2}$ of $a_{n,-m}$ and $a_{n,m}$ are equal, for $1 \leq m \leq M$.

III. DATA SUPPORTED DETERMINISTIC MAXIMUM LIKELIHOOD METHOD

Assuming that the signals $s(k)$ are deterministic and unknown. The ML estimation of $\{\omega, \phi\}$ is given by

$$
\{\hat{\omega}, \hat{\phi}\} = \arg \max_{\omega, \phi} \text{Tr} \{ \mathbf{P}_A \hat{\mathbf{R}} \},
$$

(10)

where

$$
\mathbf{P}_A = \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H.
$$

(11)

and $\hat{\mathbf{R}}$ is the covariance matrix which is estimated by

$$
\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k) \mathbf{x}^H(k).
$$

(12)

Instead of estimating $\{\hat{\omega}, \hat{\phi}\}$ directly by multidimensional search. For data supported optimization, a set of $\{\omega, \phi\}$ are estimated by using some computationally efficient estimators, and then the ML criterion was used to pick up the "best" estimates from the set. To obtain the set of estimators, the $K$ snapshots $\mathbf{X} \in \mathbb{C}^{(2M+1) \times K}$ are reformulated into $J$ sub matrices $\mathbf{X}_j \in \mathbb{C}^{(2M+1) \times L}$, $1 \leq j \leq J$. Generic matrix $\mathbf{X}_j$ is made by choosing $L$ snapshots from the $K$ snapshots available as follow:

$$
\mathbf{X}_j = [\mathbf{x}(k_1) \cdots \mathbf{x}(k_l) \cdots \mathbf{x}(k_L)] \in \mathbb{C}^{(2M+1) \times L},
$$

(13)

where $k_1 \in [1, K]$ and $k_1 < k_2 < \cdots < k_L$. The number of sub matrices is obtained by combination operation $J = \binom{K}{L}$.

Fig. 3 shows the relationship between the number of snapshots $K$, number of snapshots in each sub-matrices $L$ and the number of submatrices $J$. From Fig. 3, we can see that to make the computational complexity of the DSO method affordable, $L$ should be chose properly according to the total number of snapshots $K$. There are two choices, $L$ is chosen either slightly larger than 1 or smaller than $K$. However, if a smaller $L$ is used, there are less snapshots in each submatrices. The accuracy of the paralleled estimators is low, which will affect the final results. One of the suitable selection criterion is choosing $L$ slightly smaller than $K$. For example, if there are 20 snapshots ($K = 20$), then $L = 18$ is a good choice to make a compromise between performance and complexity.

For each $\mathbf{X}_j$, $\omega^{(j)}$ and $\phi^{(j)}$ are estimated sequentially, generalized ESPRIT is used to estimate $\omega^{(j)}$, where

$$
\omega^{(j)} = \begin{bmatrix}
\omega_1^{(j)} & \omega_2^{(j)} & \cdots & \omega_N^{(j)}
\end{bmatrix}^T.
$$

(14)

After that MUSIC parameter estimator is used for each $\phi^{(j)}$. $N$ times 1-D search are needed to estimate $\phi^{(j)}$, where

$$
\phi^{(j)} = \begin{bmatrix}
\phi_1^{(j)} & \phi_2^{(j)} & \cdots & \phi_N^{(j)}
\end{bmatrix}^T.
$$

(15)

Since there are $J$ snapshots matrices, parallel estimation is done for $\mathbf{X}_1, \cdots, \mathbf{X}_J$. After obtaining $\{\omega^{(j)}, \phi^{(j)}\}$, $1 \leq j \leq J$, For the $j$th estimation, two matrices $\mathbf{A}_j$ and $\mathbf{P}_{\mathbf{A}_j}$ are reconstructed as follows,

$$
\mathbf{A}_j = \begin{bmatrix}
\mathbf{a}(\omega_1^{(j)}, \phi_1^{(j)}) & \cdots & \mathbf{a}(\omega_N^{(j)}, \phi_N^{(j)})
\end{bmatrix},
$$

(16)

and

$$
\mathbf{P}_{\mathbf{A}_j} = \mathbf{A}_j (\mathbf{A}_j^H \mathbf{A}_j)^{-1} \mathbf{A}_j^H.
$$

(17)

The final $\{\hat{\omega}, \hat{\phi}\}$ are estimated from $\{\omega^{(j)}, \phi^{(j)}\}$, $1 \leq j \leq J$ by deterministic ML.

$$
\{\hat{\omega}, \hat{\phi}\} = \arg \max_{\omega, \phi} \{ \text{Tr} \{ \mathbf{P}_A \hat{\mathbf{R}} \}, 1 \leq j \leq J \}.
$$

(18)

Far-field and near-field sources are classified based on the estimated $\{\omega, \phi\}$. The classification criterion for the $n$th source is summarized as

$$
\begin{cases}
\hat{\phi}_n = 0 & \text{for far-field source} \\
\hat{\phi}_n \neq 0 & \text{for near-field source}
\end{cases}
$$

(19)

From (5), the direction of arrival $\theta_n$ of the $n$th source can be estimated by

$$
\theta_n = \arcsin \left( -\frac{\omega_n \lambda}{2\pi d} \right).
$$

(20)

Base on (5) and (6), the corresponding range $r_n$ for the $n$th source is obtained by

$$
r_n = \frac{(2\pi d + \omega_n \lambda)(2\pi d - \omega_n \lambda)}{4\pi \lambda \phi_n}.
$$

(21)

For far-field source, angle information $\theta_n$ is the parameter to be estimated, which is obtained from (20). However there is
satisfy the following constraint:

\[ \sum a \leq D \]

Details about \( \omega^{(j)} \) and \( \phi^{(j)} \) estimation are given as follows.

A. Generalized ESPRIT

Estimation of signal parameters via rotational invariance (ESPRIT) is a subspace based high resolution DOA estimation method. The high resolution property of ESPRIT is obtained by exploiting the rotational invariance among signal subspaces induced by an array of sensors with a translational invariance structure [15].

To apply the Generalized ESPRIT method, the ULA is divided into two subarrays. The first subarray is formed with the first \( Q \) sensors in ascending order (from sensor \( -M \) to sensor \( -M+Q-1 \)) and the second subarray is formed with the last \( Q \) sensors in descending order (from sensor \( M \) to sensor \( M+Q-1 \)). The length of the subarray \( Q \) should satisfy the following constraint: \( D \leq Q < 2M+1 \).

The array manifold matrix for the first subarray is given by

\[
A_1 = [a_1(\omega_1, \phi_1) \cdots a_1(\omega_N, \phi_N)]^T. \tag{22}
\]

The corresponding array steering vector is

\[
a_1(\omega_n, \phi_n) = [a_{n-M} \cdots a_{n-(M+Q-1)}]^T. \tag{23}
\]

The array manifold matrix for the second subarray is given by

\[
A_2 = [a_2(\omega_1, \phi_1) \cdots a_2(\omega_N, \phi_N)]. \tag{24}
\]

The corresponding array steering vector is

\[
a_2(\omega_n, \phi_n) = [a_{n,M} \cdots a_{n,M+Q-1}]^T. \tag{25}
\]

The symmetric property gives

\[
A_2 = [\Phi(\omega_1)a_1(\omega_1, \phi_1) \cdots \Phi(\omega_N)a_1(\omega_N, \phi_N)]. \tag{26}
\]

where \( \Phi(\omega_d) \) is a diagonal matrix and

\[
\Phi(\omega_d) = \begin{bmatrix}
e^{j2\omega_d M} & 0 & \cdots & 0 \\
0 & e^{j2\omega_d (M+1)} & \cdots & 0 \\
0 & 0 & \cdots & e^{j2\omega_d (M+1)} \\
\end{bmatrix}. \tag{27}
\]

The covariance matrix for snapshots \( X_j \) is given by

\[
\tilde{R}_j = E\{X_jX_j^H\}. \tag{28}
\]

Taking eigen-decomposition of \( R_j \), we obtain

\[
\tilde{R}_j = U_j \Sigma_j U_j^H + V_j A_j V_j^H, \tag{29}
\]

where \( \Sigma_j \) and \( A_j \) are the diagonal matrices that contain the signal and noise subspace eigenvalues of \( \tilde{R}_j \), respectively, whereas \( U_j \) and \( V_j \) are the orthonormal matrices composed of signal and noise subspace eigenvectors of \( \tilde{R}_j \), respectively.

Let \( U_j^{(1)} \) and \( U_j^{(2)} \) denote the first and last \( Q \) rows of \( U_j \). Then \( U_j^{(1)} = A_1 G \) and \( U_j^{(2)} = J A_2 G \), where \( G \) is a full rank matrix and \( J \in \mathbb{R}^{Q \times Q} \) is the exchange matrix with ones on the counter diagonal and zeros elsewhere.

To implement the generalized ESPRIT algorithm, we construct the following diagonal matrix

\[
\Psi(\omega) = \begin{bmatrix}
e^{j2\omega M} & 0 & \cdots & 0 \\
0 & e^{j2\omega (M+1)} & \cdots & 0 \\
0 & 0 & \cdots & e^{j2\omega (M+1)} \\
\end{bmatrix}. \tag{30}
\]

For \( \omega = \omega_n \), matrix \( JU_j^{(2)} \Psi(\omega)U_j^{(1)} \) is rank reduced. Electronic angles \( \omega^{(j)} \) are estimated from the \( N \) nulls of the following spectrum

\[
P_G^{(j)}(\omega) = \det [E^HE]. \tag{31}
\]

where \( E = (JU_j^{(2)} - \Psi(\omega)U_j^{(1)}) \).

Let

\[
\Psi(\omega) = \text{diag} [z^M \cdots z^{(M-Q+1)}], \tag{32}
\]

where \( z = e^{-j2\omega} \). From (31), we observe that the electronic angles \( \omega^{(j)} \) can be alternatively estimated from the roots of the polynomial.

B. MUSIC

The MUSIC algorithm is one of the widely used subspace based high resolution DOA estimation technique [16]. It achieves an excellent trade-off between the DOA estimation performance and computational cost. As a result, MUSIC is accepted in the literature as a benchmark approach. After obtaining \( \omega^{(j)} \), MUSIC is used to estimate \( \phi^{(j)} \). Calculate the noise subspace \( V \) by taking eigen-decomposition of the array covariance matrix \( \tilde{R} \).

\[
\tilde{R} = U \Sigma U^H + V \Lambda V^H. \tag{33}
\]

where \( \Sigma \) and \( \Lambda \) are the diagonal matrices that contain the signal and noise subspace eigenvalues of \( \tilde{R} \), respectively, whereas \( U \) and \( V \) are the orthonormal matrices composed of signal and noise subspace eigenvectors of \( \tilde{R} \), respectively.

For the \( n \)th source, substituting the estimated \( \omega_n^{(j)} \) back into the steering vector \( a(\omega_n^{(j)}, \phi_n) \) in (9). The following MUSIC spatial spectrum is obtained

\[
P_M^{(j)}(\phi_n) = \frac{1}{a^H(\omega_n^{(j)}, \phi_n)VV^Ha(\omega_n^{(j)}, \phi_n)}. \tag{34}
\]

\( \phi_n^{(j)} \) is estimated by finding the peak of the MUSIC spatial spectrum,

\[
\phi_n^{(j)} = \arg \max P_M^{(j)}(\phi_n). \tag{35}
\]

Since there are \( N \) sources, multiple MUSIC estimators are implemented in parallel to estimate \( \phi^{(j)} \).

A summary of the maximum likelihood source localization algorithm using data supported optimization is given in Algorithm 1.
Algorithm 1 DML Source Localization Using DSO

1: Parallel Processing
2: for $j = 1 \rightarrow J$
3: Estimate $\omega^{(j)}$ by using $X_j$ (31).
4: Parallel Processing
5: for $n = 1 \rightarrow N$
6: Estimate $\phi^{(j)}_n$ using MUSIC by 1-D search (34).
7: end for
8: $\phi^{(j)} = \begin{bmatrix} \phi^{(j)}_1 & \phi^{(j)}_2 & \ldots & \phi^{(j)}_N \end{bmatrix}^T$.
9: end for
10: Obtain $\{\omega, \phi\}$ by deterministic ML (18).
11: Parallel Processing
12: for $n = 1 \rightarrow N$
13: Far-field and near-field classification (19).
14: if far-field source then
15: $\omega_n \rightarrow \theta_n$ by (20).
16: end if
17: if near-field source then
18: $\{\omega_n, \phi_n\} \rightarrow \{\theta_n, r_n\}$ by (20) and (21).
19: end if
20: end for

IV. SIMULATION RESULTS

For the simulation, a ULA consisting of 5 elements ($M = 2$) with inter-element spacing $d = \lambda/4$ is considered. There are $N = 2$ uncorrelated narrowband sources signals with unit power impinging on the array, one near-field sources and one far-field source. The parameters of the near-field sources are $(r_1, \theta_1) = (0^\circ, 2\lambda)$. The far-field source is from $(\theta_3) = (15^\circ)$. The number of snapshots is $K = 20$. To construct the supported set $L = 19$ is used and $J = 20$.

Root mean square error (RMSE) of the proposed method versus signal-to-noise ratio (SNR) of DOA and range estimation are shown in Fig.4 and Fig.5, respectively. The figures are obtained by 500 Monte Carlo simulations. From Fig.4, we can see that the DOAs for both of the far-field and near-field sources can be estimated by the proposed method. Similarly, the range for the near-field source is obtained as shown in Fig.5.

V. CONCLUSION

Localization of mixed near-field and far-field sources is a challenging problem, which has been received more attentions recently. In this paper, we propose an effective DSO based ML estimation technique which is applicable for the near-field sources, far-field sources or the mixture of these two kinds of sources. To obtain the data supported grid points, a two step estimation technique is used. Far-field and near-field source classification is also considered.

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