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Noise reduction mechanisms of active noise barriers

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The sound field and acoustic power flow from different regions around an active noise barrier are analyzed using a two-dimensional numerical model. Results show that the noise reduction of the active noise barrier can be described as a combination of three mechanisms. They are acoustic energy transfer in space, acoustic energy absorption and radiated energy suppression by the secondary sources. At low frequencies, when the distance between the primary and secondary sources is well separated compared to the wavelength, the coupling is weak and the energy reflected from the shadow zone to adjacent regions by the secondary source. Acoustic energy transfer in space is the dominant mechanism. When the primary source and the secondary source are close compared to a wavelength, the mechanisms for noise reduction can be sound absorption by the secondary source and/or the radiated power of primary source is suppressed by the secondary source. The radiated power of secondary source is negative. Furthermore, both the radiated power of primary and secondary sources fluctuates acutely with the variation of distance between them. At high frequencies, the mechanism for noise reduction can be sound absorption by the secondary source. The radiated power of secondary source is negative. © 2013 Institute of Noise Control Engineering.

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1 INTRODUCTION

The mechanisms of active noise control vary with different control system configurations. According to prior work, the noise reduction mechanisms in duct and free fields are well known. In one-dimensional duct systems, when the distance between the primary and secondary sources is large compared with the wavelength, the main mechanism may be that the primary source radiated power is reflected at the location of the secondary source. However, when the secondary source is near the primary source, the main mechanism changes to that the primary source power output are suppressed by the secondary one\textsuperscript{1,2}. If a unidirectional secondary source is used in the duct, sound power absorption is usually the dominant mechanism\textsuperscript{3}. In three-dimensional free fields, the Kirchhoff–Helmholtz integral equation shows that the pressure in a given volume can be completely suppressed by introducing a layer of monopoles and dipoles on the surface of the volume. The mechanism may be either active absorption or reflection with continuous secondary source layer\textsuperscript{4}. From the studies of Mangiante and Nelson et al. in free field, it is found that when the primary and secondary sources are spaced less than half wavelength, they are well coupled and the suppression of primary source power output is the dominant mechanism\textsuperscript{5,6}. If the secondary sources are far away from the primary source, the weak coupling between these two kinds of sources causes that the secondary sources barely affect the power output of the primary source. Therefore, the control can be the radiated power from the primary source that is absorbed or reflected by the secondary sources\textsuperscript{7,8}. A recently developed active control methodology, difference potential method shows that the only data needed for the active control are acoustic quantities (the total sound pressure and the normal component of the particle velocity) at the perimeter of the shielded domain, and no detailed knowledge of the structure and location of the primary sources are needed. In this situation, the radiated power characteristic of primary sources and secondary sources varies with the structure of secondary sources, and the noise reduction mechanism is varied accordingly\textsuperscript{9,10}.\textsuperscript{R1 R2 R3 R4 R5 R6 R7 R8 R9 R10}

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Active noise barriers (ANB) has received attention since 1993, when Omoto et al. applied a multichannel control system to cancel the noise around a semi-infinite barrier top. Guo et al. and Niu et al. claimed that canceling the sound around the barrier top had the equivalent effect of increasing the height of the noise barrier. Han et al. used sound intensity as the cost function to enhance the insertion loss of the ANB system. The results show that this method is more useful than other cost functions in ANB system. Recently, Hart et al. pointed out that the method of controlling the pressure gradient on the top of the barrier is more efficient than that of controlling the sound pressure. Although the above-mentioned studies on ANB systems have continued to improve the performance of the ANB systems, the noise reduction mechanisms of ANB are still not explained clearly.

A comprehensive understanding of the mechanisms of the ANB system will be beneficial for further optimization of the parameters of the system, such as locations and types of the secondary sources and the error sensors. This paper is aimed at revealing the noise reduction mechanisms of active noise barrier.

Based on a two-dimensional model, the noise reduction mechanisms of the ANB system will be investigated, the sound field of the control system at different regions will be analyzed, and the radiated power of the primary and secondary sources with and without active control will be compared.

2 THEORY

To simplify the problem, the geometry of barrier, primary source row and the secondary source row can be simplified to be 2D. In this situation, an infinite line source in the 3D space can be regarded as a “monopole” source in 2D. The two-dimensional ANB system is shown in Fig. 1, where the ground reflection is omitted to emphasize the main physics. The top of the barrier along the positive y-direction is at (0, y_b) and the rigid barrier extends to infinity in the negative y-direction. A primary monopole source P is on the left side of the barrier, a secondary monopole source S is above the barrier, and N error sensors are located on the right side of the barrier. According to the principle of geometrical acoustics, the plane can be divided into three separated regions by the noise barrier with the dotted lines at \( \pi - \theta_p \) and \( \pi + \theta_p \). When the receiver is in region I, the total sound field is composed of the direct wave \( p_d \) from the primary source, the reflected wave \( p_r \) by the barrier, and the diffracted wave \( p_D \) along the edge of the barrier. When the receiver is in region II, the total sound field is composed of \( p_d \) and \( p_D \), while for the shadow zone (region III), only the diffracted wave \( p_D \) exists. The direct and reflected waves are given by:

\[
\begin{align*}
  p_d &= -\frac{j\omega \rho q_p}{4} H_0^{(2)}(kR) e^{i\omega t}, \\
  p_r &= -\frac{j\omega \rho q_p}{4} H_0^{(2)}(kR') e^{i\omega t},
\end{align*}
\]

where \( \omega \) is the angular frequency of the sound, \( k \) is the wave number, \( \rho \) is the air density, \( q_p \) is the strength of the primary source, and \( H_0^{(2)} \) is the zeroth-order Hankel function of the second kind. \( R \) and \( R' \) are respectively the distance from the receiver to the source and to its mirror image in the barrier. The diffracted wave \( p_D \) can be calculated with the Hadden and Pierce solution.

The sound pressure at the locations of \( N \) error sensors can be expressed in matrix form as:

\[
p_{tot} = p_p + Z_s q_s,
\]

where \( p_p \) is the vector of pressure produced by the primary source at \( N \) error sensors, \( Z_s \) is the transfer-impedance matrix from the secondary source to the \( N \) error sensors, and \( q_s \) is the strength of the secondary source. The sum of the squared sound pressure at the error sensor positions is selected as the cost function, and the optimal strength of the secondary source is:

\[
q_s = -\frac{1}{1 + \beta I} Z_s^H p_p,
\]

where \( I \) is the identity matrix, \( \beta \) is a constraint factor for the secondary source strength, and the superscript \( H \) denotes a Hermitian transpose. Taking the top of the barrier as the center, the pressure \( p \) produced by the primary and secondary sources on a circle with radius \( r \) can be calculated according to Eqns. (2) and (3), and

![Fig. 1 — Schematic diagram of a two-dimensional active noise barrier.](image-url)
IIR, III

As shown in Fig. 1, the barrier is rigid and the coordinates of the barrier top are (0, 1.3) m. The primary and secondary sources are at (−4, 0.1) m and (0, 1.38) m respectively, and only one error sensor is defined at (2, 0.5) m. The strength of primary source is $q_p = 0.25$ m$^2$/s with a frequency of 200 Hz. The radiated power can be further calculated as:

$$W = \frac{1}{2} \int_0^{2\pi} \Re[p^*(r, t)u_r(r, t)]r \, dr,$$

where $\alpha$ denotes the integral angle, and $u_r(r, t)$ is the normal velocity on the circle expressed as:

$$u_r(r, t) = -\frac{1}{\rho} \int \frac{\partial p(r, t)}{\partial r} \, dt.$$

Fig. 2—(a) Phase of the pressure along a circle with radius 20 m. (b) Directivity of the pressure along a circle with radius 20 m. $p_D$ is the diffracted wave produced by the primary source, $p_s$ is the pressure produced by the secondary source, $\Phi_D$ is the phase of $p_D$, $\Phi_s$ is the phase of $p_s$, and $\Delta \Phi$ is the phase difference between $\Phi_D$ and $\Phi_s$.

The sound power radiated from the secondary source can be obtained by integrating along a circle centered at point $S$ with a radius of $r_s$. The sound power radiated from the secondary source can be obtained by integrating along a circle centered at point $P$ with a radius of $r_p$, and the power radiated from the secondary source can be obtained by integrating along a circle centered at point $P$ with a radius of $r_p$. The sound power flow from region II to region III can be calculated by Eqn. (4) with the integral path from the top of the barrier to infinite along the line $\alpha = \pi + \theta_p$. As the normal intensity alone the line $\alpha = \pi + \theta_p$ decreases as the distance increases between the point and the barrier top, the infinite integral in Eqn. (4) can be approximated by a finite one in practice.

### 3.1 The Distribution of the Sound Pressure

Figure 2(a) shows the phases of the diffraction wave $p_D$ and the pressure of the secondary source $p_s$ on the circle with radius $r_t = 20$ m, and Fig. 2(b) illustrates the directivities of $p_D$, $p_s$ and the sum of the two terms. In Fig. 2(a), the phase of $p_D$ has three $\pi$-phase jumps at $\alpha = \pi - \theta_p$, $\alpha = \pi$, and $\alpha = \pi + \theta_p$. By observing the symmetry of the sound field, the plane can be divided into four regions: region I, the left region of II ($\pi - \theta_p < \alpha \leq \pi$, abbreviated as IIL), the right region of II ($\pi < \alpha \leq \pi + \theta_p$, abbreviated as IIR) and region III.

In region III, $p_D$ and $p_s$ are almost out of phase, so the sum is less than $p_D$. In region IIL, though $p_D$ and $p_s$ are also almost out of phase, the large difference between their magnitudes makes the sum decrease only in the zone near $\alpha = \pi - \theta_p$. In regions I and IIR, $p_D$ and $p_s$ are almost in-phase, the sum of the two terms is larger than $p_D$ alone. Therefore, the mechanism of diffraction wave reduction shows that the diffraction energy in the shadow zone is reflected to region IIR, resulting in a local quiet zone.

The directivity of the total pressure $p_t$ on the circle with radius $r_t = 20$ m is shown in Fig. 3. When the control system is working, the total pressure in the shadow zone (region III) decreases significantly while the total pressure in regions I and II changes little, because the...
Fig. 3—Directivity of the total pressure along a circle with radius 20 m.

3.2. The Radiated Power of the Primary and Secondary Sources

Figure 4(a) shows the radiated power of secondary source \( W_s \) as a function of the location of primary source \( x_p \) with active control at four different frequencies. Figure 4(b) gives the ratio \( R_{\text{flow}} \) of after-to-before control power which flows from region IIR to III as a function of \( x_p \). It can be seen that all the curves in Fig. 4(a) are quasi-sinusoidal fluctuation with the variation of \( x_p \). The fluctuation of \( W_s \) is mainly due to the interaction between the primary and secondary sources. The mutual radiation resistance of the two sources is proportional to \( H_0^2 (kr) \), where \( k \) is the wave number and \( r \) is the distance of the sources.

In low frequencies (200 or 300 Hz in Fig. 4(a)), when the primary source is far away from the secondary source, the interaction between the two sources is weak. The radiated power of secondary source is positive. The \( R_{\text{flow}} \) in Fig. 4(b) is less than 0.2. It indicates that the diffraction acoustic energy in region III is almost reflected back to region IIR after control. Therefore, energy transfer in space might be the dominant mechanism of active control. With the increasing of \( x_p \), the distance between primary and secondary source is decreased. At several positions, the radiated power of secondary source is approaching to zero, but Fig. 4(b) shows that the power flows into region III are still less than that of before control. It can be regarded as the radiation resistance of secondary source is approaching zero, but the radiation reactance is not equal to zero and it restrains the radiated power of primary source. Therefore, radiated energy suppression by the secondary source might be the dominant mechanism at these positions. As \( x_p \) increases, the interaction between primary and secondary source is enhanced. The radiated power \( W_s \) fluctuates abruptly. The value of \( R_{\text{flow}} \) in Fig. 4(b) increases. The radiated power of secondary source transforms to negative for the most positions of \( x_p \). Sound absorption of secondary source might be the dominant mechanism of active control.

For higher frequencies (500 or 800 Hz in Fig. 4(a)), the radiated power from the secondary source is negative even though the distance between primary and secondary source is large. Furthermore, the value of \( R_{\text{flow}} \) in Fig. 4(b) is larger than that at low frequencies. It means that energy transfer in space is no more the dominant mechanism of active control. However, sound absorption of the secondary source is the dominant mechanism of active control.

In order to show the detailed variation of the radiated power of primary source, Fig. 4(c) displays the ratio of after-to-before control of \( W_p \) at four different frequencies. The fluctuation of the curves reflects the interaction intensity between primary and secondary source. The lower the frequency, the more intense the interaction, when \( x_p \) is the same magnitude as the wavelength at different frequencies.

3.3 Simulation with the Finite Element Method

A commercial finite element software package COMSOL was used for the calculation of the pressure and intensity distribution. All the parameters are the same as those described in section 3.1. Figure 5 shows the pressure distribution and sound intensity flow without and with active control behind the barrier at 200 Hz. In order to investigate the details of the intensity, the amplitude of intensity in Fig. 5(a) larger than 0.2 \( \text{w/m}^2 \) is set to 0.2 \( \text{w/m}^2 \), and smaller than 0.08 \( \text{w/m}^2 \) is set to 0.08\( \text{w/m}^2 \). When the amplitude of intensity in Fig. 5(b) is larger than 0.02 \( \text{w/m}^2 \) it is set to 0.02 \( \text{w/m}^2 \), and smaller than 0.008 \( \text{w/m}^2 \) it is set to 0.008 \( \text{w/m}^2 \). Comparing Fig. 5(a) with 5(b), it can be seen that the pressure with active control in shadow zone decreases. In Fig. 5(b), the direction of intensity after control has changed from almost horizontal to up right in the circle zone on the left of the error sensor, which indicates that the energy is transferred to region IIR before flowing into the shadow zone.

Figure 6 shows the pressure distribution and sound intensity flow without and with active control behind the barrier at 800 Hz. Comparing Fig. 6(b) with 5(b), it shows that the direction of intensity in Fig. 6(b) changes less than that of Fig. 5(b) in the circle zone on the left of the error sensor. The result demonstrated that acoustic energy transfer in space is not the
dominant mechanism of active control in high frequency. It might be energy absorption and/or energy suppression.

4 CONCLUSIONS

By analyzing the pressure distributions and the acoustic power flow associated with both primary and secondary sources of an ANB system, the noise reduction mechanisms of the system can be summarized as: acoustic energy transfer in the space, acoustic energy absorption and radiated energy suppression by the secondary sources.

At low frequencies, when the primary source is far from the secondary source, acoustic energy transfer in space is the dominant mechanism, and the diffraction energy in the shadow zone is reflected to adjacent region, resulting in a local quiet zone. When the distance

Fig. 4—The radiated power as a function of primary source location. (a) The power radiated by the secondary source. (b) The ratio of after-to-before control power which flows from region IIR to III. (c) The ratio of after-to-before control power radiated by primary source.
between the primary source and the secondary source approaches an acoustic wavelength, the energy absorption is the dominant mechanism. At high frequencies, the energy absorption is the dominant mechanism. At several specific positions, due to the interaction between the primary and secondary sources, the radiation resistance of secondary source approaches zero. However, the radiation reactance is not equal to zero and radiated energy suppression by the secondary source might be the dominant mechanism at these positions.

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6 REFERENCES
