<table>
<thead>
<tr>
<th>Title</th>
<th>Lattice Boltzmann simulation of sound absorption of an in-duct orifice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Ji, Chenzhen; Zhao, Dan</td>
</tr>
<tr>
<td>Date</td>
<td>2013</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/10220/11939">http://hdl.handle.net/10220/11939</a></td>
</tr>
<tr>
<td>Rights</td>
<td>© 2013 Acoustical Society of America. This paper was published in Proceedings of Meetings on Acoustics and is made available as an electronic reprint (preprint) with permission of Acoustical Society of America. The paper can be found at the following official DOI: [<a href="http://dx.doi.org/10.1121/1.4799686">http://dx.doi.org/10.1121/1.4799686</a>]. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper is prohibited and is subject to penalties under law.</td>
</tr>
</tbody>
</table>
Lattice Boltzmann simulation of sound absorption of an in-duct orifice

Chenzhen Ji* and Dan Zhao

*Corresponding author's address: Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore, 639798, Singapore, cji1@e.ntu.edu.sg

Two-dimensional time-domain numerical investigation of sound-induced flow through an orifice with a diameter 6mm is conducted by using lattice Boltzmann method. Emphasis is placed on characterizing its acoustic damping behaviors. The main damping mechanism is identified as incident waves interact with the shear layers formed at the orifices rims and the acoustic oscillations destabilize the shear layers to form vortex rings. And acoustic energy is converted into vortical energy. To quantify the orifice damping effect, power absorption coefficient is used. It is related to Rayleigh conductivity and describes the fraction of incident acoustical energy being absorbed. Numerical simulations are conducted in time domain by forcing a fluctuating flow with multiple tones through the orifice. This is different from frequency-domain simulations, of which the damping is characterized one frequency at a time. Comparing our results with those from Howe theoretical model, good agreement is observed. In addition, orifice thickness effect on its damping is discussed.


1 INTRODUCTION

The occurrence of combustion instabilities has been a plaguing problem in the development of combustors for rockets, ramjets, and the afterburner of aero-engines. Combustion instability\(^1\) is generated by the interaction between unsteady heat release and acoustic waves. Under lean premixed conditions, a small change in the flow rate of inlet air or fuel may cause unsteady heat release. Unsteady heat release is an efficient monopole-like sound source, and generates acoustic waves. These pressure waves propagate within the combustor and partially reflected from the boundaries and arrive back at the combustion zone, where they can cause more unsteady heat release. This feedback can result in the pressure oscillation amplitude successively increasing. Eventually, some non-linearity in the system will limit the amplitude of the oscillations.

To mitigate combustion instabilities, the coupling between unsteady heat release and acoustic waves must somehow be interrupted.\(^2\)\(^-\)\(^4\) Perforated liners along the bounding walls of a combustor are widely used as acoustic dampers to dissipate the acoustic waves. They are usually metal sheets, which have tiny orifices in them. In practice, a cooling air flow through the orifices (known as bias flow) is needed to prevent the liners from being damaged under high temperature. Over the last few decades, perforated liners have been the subject of intense research activity, aiming to better understand and predict their damping performance. The main mechanism of such liner, involves vortex shedding generated over the rims of the orifices, when fluids flow through them. An unsteady 'jet' will be generated and undergoing subsequent viscous dissipation to convert acoustic fluctuations into non-radiating vertical fluctuations. The need to improve perforated liners designs with current low-emission engines leads to a resurgence of liner focused research. Both numerical and experimental investigations are conducted. However, experimental investigations focus on measuring the liners impedance or power absorption coefficient,\(^5\)\(^-\)\(^7\) since they are easier to measure in comparison with the flow field in and around the small-size orifice, typically around 1 mm in diameter. Ingard & Labate\(^5\) found in their experiments that the sound amplitude, frequency, orifice diameter and thickness affected the induced motion of the fluid near the orifice. Hughes & Dowling\(^6\) studied the acoustic damping performance of the perforated plate with regular array of the slits and circular perforations with bias flow. Jing & Sun’s experiments indicated the thickness and the bias flow Mach number were the critical factors impacting the liners’ performance.\(^7\)

To shed lights on their damping mechanism and to predict their damping, numerical investigations of orifices are also conducted. The majority modeling work has been carried out in frequency domain. Howe\(^8\) modeled the acoustic energy dissipated by the periodic shedding of vorticity for a single orifice at a high Reynolds-number by using Rayleigh conductivity. Eldredge and Dowling\(^9\) recently developed a 1D duct model in frequency domain to simulate the absorption of axial plane wave by a double-liner with a bias flow. The damping mechanism of vortex shedding was embodied using a homogeneous liner compliance adapted from the Rayleigh conductivity. In comparisons with frequency-domain modeling,\(^10\) time-domain modeling\(^1\)\(^1\) becomes more popular. This is most likely due to the high performance computer and more efficient computational methods. Tam \textit{et al.}\(^12\) carried out direct numerical simulation (DNS) of a single aperture, showing that vortex shedding was the dominant mechanism of absorption for incident waves of high amplitude. Mendez & Eldredge\(^13\) conducted compressible large-eddy simulations (LES) in time domain to study the flow through a single perforated hole or multiple holes. Zhang & Bodony\(^14\) investigated the acoustics behavior in a honeycomb liner using DNS, with a focus on the interaction between the orifice and the boundary layer.

Numerical simulations described above\(^12\)\(^-\)\(^14\) attempt to solve the Navier-Stokes equations by using different methods, such as finite volume method or finite difference method. As an alternative and promising computational tool, lattice Boltzmann method (LBM) is under the development of simulating fluid flows and modeling complex physics in fluids.\(^15\) Unlike conventional numerical schemes, LBM is based on mesoscopic models and kinetic equations. Simplified kinetic models are developed that assume the fluid flow composes of a collection of pseudo-particles. They can be represented by a set of density distribution functions. These particles are associated with collision and propagation over a discrete lattice mesh. The conserved variables such as density, momentum and internal energy are obtained by performing a local integration of the particle distribution. Compared with Navier-Stokes solvers via LES and DNS, LBM is easier to implement and code, has the potential for parallelization, and more successful in dealing with complex boundaries. Thus it has been applied to study not only fluid flows but also aeroacoustics, such as jet\(^16\)\(^-\)\(^18\), cavity\(^17\)\(^-\) and airfoil noise\(^19\). Much of the previous works\(^16\)\(^-\)\(^19\) has concluded that LBM possesses the required accuracy to capture weak acoustic pressure fluctuations.

In this work, lattice Boltzmann simulations are conducted to investigate the acoustic damping effect of an in-duct orifice, which is assumed to capture the main damping behavior of perforated liners with small open-area ratio. The numerical configuration is described in Sect. 2. A fluctuating flow with multiple tones is forced to pass through the
orifice. The governing unsteady compressible equations, combined with the Smagorinsky turbulence model, are solved by using lattice Boltzmann method (LBM). This is described in Sect. 3. To characterize the acoustic damping of the orifice, Rayleigh conductivity and power absorption coefficient are used. This is described in Sect. 4. Classical analytical models as proposed by Howe are also introduced. Finally, in Sec. 5, the numerical results are presented. The orifice damping effects are estimated both in terms of Rayleigh conductivity and power absorption coefficient. Comparison is then performed between the present results and the analytical ones. Good agreement is obtained. In addition, the orifice thickness effect on its acoustic damping is discussed.

2 DESCRIPTION OF NUMERICAL CONFIGURATION

In this work, the acoustic damping behavior of an orifice with acoustical excited bias flow is investigated numerically, as show in Fig. 1, to see whether LBM can be applied to shed lights on the damping mechanism of the orifice and to characterize the damping performance by estimating Rayleigh conductivity and power absorption coefficient. This has important implications for improving liners design. Any means of improving liners damping performance would be highly beneficial. The geometric configuration is similar with the experimental test rig reported by Belluci et al. The geometry of the plate and the computation domain are given as: its width is \( W = 50 \, \text{mm} \), and its thickness is fixed as \( h = 2 \, \text{mm} \). A single aperture in the middle of plate has a width of \( d = 6 \, \text{mm} \). An acoustically excited flow with an oscillating velocity \( u_{\text{in}} \) is applied upstream of the plate at \( l = 95 \, \text{mm} \). When the flow passes through the orifice, the mean velocity is approximately \( U \approx 5.5 \, \text{m/s} \), and the Reynolds numbers is about \( Re \approx 2200 \). The downstream boundary condition is assumed to be pressure outlet, since the end is far away from the orifice to avoid the interaction with the interest computational domain. Due to the assumption of periodicity of orifices and no interaction between neighboring ‘jets’, periodic boundary condition is used in both directions tangential to the plate to reduce the computation domain.

![FIGURE 1. Geometry of single-orifice liner.](image)

3 DESCRIPTION OF LATTICE BOLTZMANN METHOD

LBM is adopted to investigate the vorticity-generation mechanism and to predict the damping performance of the orifice. It is developed from the method of lattice gas automata. Let’s first consider the classical lattice Boltzmann equation as given:

\[
f_i(x, t + \Delta t) - f_i(x, t) = -\left( f_i(x, t) - f_i^{eq}(x, t) \right) / \tau,
\]

where \( \Delta t \), \( \tau \) and \( e_i \) are the lattice time step, single relaxation parameter and lattice discrete velocity, respectively, \( f_i \) is the particle distribution function, and \( f_i^{eq} \) is the equilibrium distribution function. The right term of Eq. (1), named as Bhatnagar-Gross-Krook (BGK) collision operator, represents the relaxation of the particles. To ensure that a particular discretization can simulate the flow, D2Q9 or D3Q19 lattice models are usually used. Since we are considering 2D flow through the orifice, D2Q9 is chosen in our work. The nine bit isothermal square lattice model is shown in figure 2. And the discrete velocity set of the particles for the D2Q9 lattice, is defined as:
where \( c = \Delta x / \Delta t \) (lattice space/time step) is the propagation speed, taken as \( c = 1 \) in most cases. The local equilibrium distribution function \( f^{eq}_i \), which is the second order truncated expansion of the Maxwell-Boltzmann equilibrium function, is given as:

\[
f^{eq}_i = \rho w_i \left[ 1 + \frac{3}{c^2} \mathbf{e}_i \cdot \mathbf{u} + \frac{9}{2c^2} (\mathbf{e}_i \cdot \mathbf{u})^2 - \frac{3}{2c^2} \mathbf{u} \cdot \mathbf{u} \right],
\]

where \( w_i \) is the weighting factor and given as:

\[
w_i = \begin{cases} 
4/9, & i = 0 \\
1/9, & i = 1,2,3,4, \\
1/36, & i = 5,6,7,8 
\end{cases}
\]

The macroscopic density and momentum on each lattice site are calculated using the distribution function, namely:

\[
\rho = \sum_{i=0}^{8} f_i, \quad \rho \mathbf{u} = \sum_{i=0}^{8} \mathbf{e}_i f_i.
\]

The lattice speed of sound in this model is \( c_s = 1/\sqrt{3} \) and the equation of state is that of an ideal gas, \( p = \rho c_s^2 \).

By using the Chapman-Enskog expansion, the lattice Boltzmann equations can be recovered to the compressible Navier-Stokes equation at the hydrodynamic limit.\(^{22}\) The kinematic viscosity in Navier-Stokes equation is calculated using the relaxation parameter and lattice sound speed: \( \nu = (\tau - 1/2) c_s^2 \).

FIGURE 2. 2-D, nine-velocity (D2Q9) lattice model.

In our lattice Boltzmann simulation, periodic boundary condition is applied at both sides of the orifice in horizontal direction, as shown in Fig. 1. And bouncing back condition is used at the wall as no-slip occurrence. In addition, the inlet and outlet are set with the velocity and pressure boundary conditions proposed by Zou & He.\(^{23}\)

Generally, many fluid flows of interest are turbulent. Thus a turbulent model is needed. For LBM, turbulent dissipation can be modeled through a locally enhanced collision, which effectively stabilizes the simulation. A common implementation is to combine the Smagorinsky\(^{24}\) sub-grid model into the collision term of lattice Boltzmann equation. Implementing the turbulent model involves two steps:

1) Evaluation the local stress tensor:

\[
\Pi_{m,n} = \sum_{i=0}^{8} \mathbf{e}_{i,m} \mathbf{e}_{i,n} (f_i - f^{eq}_i),
\]

where \((m, n) \in \{x, y\} \times \{x, y\} \).

2) Computation of the enhanced relaxation time with the turbulence model

\[
\tau_{total} = 3(\nu + C^2S) + 0.5,
\]
where $S = \left( \frac{v^2 + 18C^2}{\sqrt{\Pi_{m,n} \Pi_{m,n}^2}} - v \right) / (6C^2)$ and the $C$ is a Smagorinsky constant set as 0.18 in current simulation. Substituting Eq. (7) into Eq. (1) leads to:

$$f_i(x, \Delta t, t + \Delta t) - f_i(x, t) = -[f_i(x, t) - f_i^{eq}(x, t)] / \tau_{total}.$$  \hspace{1cm} (8)

### 4 ACoustic Models and Sound Absorption coefficient

The govern equations described are now solved to simulate the ‘jet’ generation and to characterize the orifice damping via estimating Rayleigh conductivity. The Rayleigh conductivity describes the acoustic behavior of the orifice, defined as:

$$K_R = \frac{j \rho \omega \hat{q}}{(\hat{p}_u - \hat{p}_d)},$$  \hspace{1cm} (9)

where $j = \sqrt{-1}$ is imaginary unit, $\omega = 2\pi f$ is the angular frequency of the pulsating perturbation, $\hat{q}$ is the volume flow rate fluctuations through the orifice, and $\hat{p}_u, \hat{p}_d$ are the amplitudes of pressure fluctuations, measured at the upstream and downstream of the orifice, respectively.

When fluid flows through the orifice, flow separation occurs at the rim of the orifice, forming a ‘jet’. The vorticity is supposed to concentrate in the axisymmetric vortex sheet separating two regions of the potential flow; jet and the remainder of the flow domain. The acoustic oscillations pulses the vortex sheet, resulting in the periodical shedding of vortex rings. The vortex shedding is assumed to have the diameter of the orifice and to be convected at the mean flow velocity in the orifice. With these assumptions, a detailed expression of Rayleigh conductivity was reported by Howe as given:

$$K_R = 2a(y - j\delta) \text{ with } y - j\delta = 1 + \frac{\pi \Gamma(I_1(S))e^{-St} - jK_i(S_t)sinh(S) + jK_i(S_t)cosh(S)}{S^2 \Gamma(I_1(S))e^{-St} + jK_i(S_t)cosh(S)}.$$  \hspace{1cm} (10)

where $a$ is the radius of the aperture, $\gamma$ and $\delta$ are functions of the Strouhal number, $I_1$ and $K_i$ are modified Bessel functions, and $St = o\omega / U$. $U$ is the mean velocity passing the aperture. To account for the thickness of the plate, an extension of above model is achieved by adding a term representing the effect of thickness. Now, the revised Rayleigh conductivity with plate thickness ($h$) is given as:

$$K_R = 2a(1/(y - j\delta) + 2h/(\pi a))^{-1}.$$  \hspace{1cm} (11)

The acoustic damping of the orifice can also be characterized by using power absorption coefficient. It is defined as:

$$\Delta = 1 - |R|^2,$$  \hspace{1cm} (12)

where $R$, the reflection coefficient of perforated plate, is derived as:

$$R = \left( z_t + 1 \right) / \left( z_t - 1 \right),$$  \hspace{1cm} (13)

$z_t$ is the normalized acoustic impedance of the whole system, and given as the sum of liner impedance $z_p$ and cavity impedance $z_c$: $z_t = z_c + z_p$. Assuming planar waves are propagating between the perforated plate and the back wall, the cavity impedance can be shown as: $z_c = -j\cot(kl)$, where $k$ is the acoustic wave number and $l$ is the cavity depth. The impedance of the perforated plate is related to Rayleigh conductivity: $z_p = (\hat{p}_d - \hat{p}_u) / \hat{u}_o = jod / (cK_R)$, where $c$ is the speed of sound. The total impedance of the system is obtained:

$$z_t = z_p + z_c = jod / (cK_R) - j\cot(kl).$$  \hspace{1cm} (14)

Substituting Eqs. (13) and (14) into Eq. (12), power absorption coefficient of system can then be calculated.
5 RESULTS AND DISCUSSION

In practice, perforate liners may consist of thousands of tiny size orifices. However if the open area ratio is large, it can be assumed that there is no interaction of the vorticity between neighboring orifices. Thus the damping behaviors of single orifice can be used to predict the liners, as studied by Mendez & Eldredge.\textsuperscript{13} The numerical simulations results are obtained by considering an acoustically excited bias flow with multiple tones as given:

\begin{equation}
   u_{in}(t) = 0.5 \left( 1 + 0.005 \sum_{k=1}^{28} \sin(200kt) \right). \tag{15}
\end{equation}

It can be seen that the simulation covers a broad frequency range, varying from 31.83 to 891.27 Hz ($200k/(2\pi)$). This is different from frequency-domain-simulations, of which the liners damping is characterized one frequency at a time. Fig. 3 shows the time evaluation of the pressure oscillations upstream and downstream of the orifice.

Fig. 4(a) and (b) shows the velocity and pressure contour respectively. As air flow through the orifice, the velocity distribution in the jet became uniform several aperture diameters downstream at the ‘vena contracta’. The “jet” flow rolled up the surrounding air and a shear layer was formed between the jet and surrounding flow.

![FIGURE 3. Fluctuations of pressure at upstream and downstream of the orifice measured in real time.](image)

![FIGURE 4. Velocity contour (a) and pressure contour (b) of single orifice liner.](image)

Fig. 5(a) illustrates the Rayleigh conductivity estimated by LBM. It can be seen that Rayleigh conductivity is varied with increased flow Strouhal number. Comparison is then made between the presented results and the modified Howe’s model (MHM), which takes the plate thickness into account. Good agreement is observed. The acoustic damping effect of the orifice depends on the frequency. Fig. 5(b) illustrates the variation of the power absorption coefficient with Strouhal number. It can be seen that the maximum power absorption occur at approximately $f = 350$ Hz, at this frequency, almost all the incident acoustic wave is absorbed by converting the acoustic energy into vorticity. Comparison is then conducted between the present model, Howe’s models (HM) and modified Howe’s model (MHM). Good agreement between LBM and MHM is obtained, although HM predicts a
larger damping at higher frequency. This indicates the plate thickness play an important role in influencing the liner’s acoustic damping, especially at high frequency.

**FIGURE 5.** (a) The real part ($\gamma$) and imaginary part ($\delta$) of Rayleigh conductivity on the Strouhal number; (b) Absorption coefficient ($\Lambda$) of the single orifice calculated by LBM, HM and MHM.

### 6 CONCLUSION

In summary, two-dimensional numerical investigation of the acoustic damping of an orifice is performed by using the lattice Boltzmann method. The damping effect is characterized by power absorption coefficient, which depends on the Rayleigh conductivity of the orifice. The effects of the bias flow and plate thickness are included. Comparing our results with the Howe models, good agreement is observed. Furthermore, the computational cost is lower, since the simulation is conducted in time domain and multiple tones are included to cover a broad frequency range. This shows the potential of applying LBM for aero-acoustic study. Experiment validation of the numerical results is being conducted.

### REFERENCES