<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>Joint optimization of sensing threshold and transmission power in wideband cognitive radio with energy detection</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Author(s)</strong></td>
<td>Liu, Xin; Bi, Guoan; Jia, Min; Guan, Yong Liang; Zhong, Weizhi; Lin, Rui</td>
</tr>
<tr>
<td><strong>Citation</strong></td>
<td>Liu, X., Bi, G., Jia, M., Guan, Y. L., Zhong, W., &amp; Lin, R. (2013). Joint optimization of sensing threshold and transmission power in wideband cognitive radio with energy detection. Radio Science, 48, 1–12.</td>
</tr>
<tr>
<td><strong>Date</strong></td>
<td>2013</td>
</tr>
<tr>
<td><strong>URL</strong></td>
<td><a href="http://hdl.handle.net/10220/12241">http://hdl.handle.net/10220/12241</a></td>
</tr>
<tr>
<td><strong>Rights</strong></td>
<td>© 2013 American Geophysical Union. This paper was published in Radio Science and is made available as an electronic reprint (preprint) with permission of American Geophysical Union. The paper can be found at the following official DOI: <a href="http://dx.doi.org/10.1002/rds.20043">http://dx.doi.org/10.1002/rds.20043</a>. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper is prohibited and is subject to penalties under law.</td>
</tr>
</tbody>
</table>
Joint optimization of sensing threshold and transmission power in wideband cognitive radio with energy detection

Xin Liu,1,2,3 Guoan Bi,2 Min Jia,3 Yong Liang Guan,2 Weizhi Zhong,1 and Rui Lin2

Received 10 March 2013; revised 8 April 2013; accepted 5 June 2013.

[1] In this paper, we consider a wideband cognitive radio system that operates over multiple idle subchannels. A joint optimization of sensing threshold and transmission power is proposed, which maximizes the total throughput subject to the constraints on the total interference, the total power, and the probabilities of false alarm and detection of each subchannel. An alternative joint optimization is proposed, which minimizes the total interference under the constraint of the total throughput. The bilevel optimization method is used to solve the proposed optimization problems with a minimized iteration complexity. The mixed-variable optimization problem is divided into two single-variable convex optimization subproblems: the upper level for threshold optimization and the lower level for power optimization. Weighed cooperative sensing is proposed to maximize the detection probability by choosing the optimal weighed factors. The simulations show that the proposed joint optimization algorithm can achieve desirable improvement on the throughput of cognitive radio at the same interference level to primary user, or vice versa within the limits on the probabilities of false alarm and miss detection, and the weighed cooperative sensing can considerably improve sensing performance compared with the unweighed cooperative sensing and single-user sensing.


1. Introduction

[2] Cognitive radio (CR) has been proposed to improve the spectrum utilization by exploiting the temporarily unused radio spectrum allocated to the primary user (PU) [Mitola and Maguire, 1999; Haykin, 2005; Hossain et al., 2009]. Before data transmission, CR must continuously sense the spectrum and identify the presence of PU in order to avoid causing any harmful interference to PU. Hence, the spectrum sensing has become an important research area [Cabric et al., 2004; Liang et al., 2008].

[3] Energy detection has been widely used as a single-user sensing method because of its simple implementation without using any prior information of PU’s signal [Urkowitz, 1967; Shen et al., 2008; Zhang et al., 2009]. However, the performance of energy detection may be degraded if PU is in a fading and shadowing environment as a hidden terminal [Digham et al., 2003]. Multiple CRs are designed to perform cooperative sensing in order to improve the accuracy of sensing the hidden PU. In cooperative sensing, a fusion center is needed to obtain a final decision on the presence of PU by combining the sensing results of individual CRs [Tan et al., 2012; Uchiyama et al., 2007; Ganesan and Li, 2005]. False alarm probability (the probability of detecting the presence of PU falsely) and detection probability (the probability of claiming the presence of PU accurately) are commonly used to measure the performance of spectrum sensing [Wei et al., 2008].

[4] Wideband CR can operate over multiple idle subchannels to make improvement on throughput. An optimal multiband joint detection for spectrum sensing is proposed in Fan and Jiang [2010] and Quan et al. [2009]. The spectrum sensing problem is formulated as a class of optimization problems about sensing threshold, which maximizes the aggregated throughput of CR under some constraints on the interference to PU. An iterative sensing threshold optimization is proposed in Liu et al. [2013] and Teo et al. [2010], and the optimal thresholds are selected in order to minimize the sum of the probabilities of false alarm and miss detection. However, the works reported in the above references all assume that each CR has a fixed transmission power and rate, and the gain of dynamic power allocation is not exploited.

[5] A joint optimization of detection and power allocation for multichannel CR is proposed in [Fan et al., 2011; Huang and Baltasar, 2010; Liu et al., 2013], where an efficient...
algorithm was reported to maximize the total throughput of CR by optimizing jointly both the detection operation and power allocation. However, this algorithm includes the interference as a part of the throughput, which is produced by the transmission of CR when the miss detection happens. Therefore, the interference to PU may be increased if the throughput is improved. In addition, the algorithm reported in Quan et al. [2009] and Fan et al. [2011] solves the proposed optimization problem based on the interior point method that requires high iteration complexity, because multiple iterations have to be implemented for each constraint condition.

[6] In this paper, a joint optimization of sensing threshold and transmission power is proposed, which maximizes the total throughput of wideband CR subject to the constraints on the total interference, the total power, and the probabilities of false alarm and detection of each subchannel. An alternative optimization algorithm is also proposed, which minimizes the total interference under the constraints of the total throughput, the total power, and the sensing probabilities. The weighted cooperative sensing is then proposed to maximize the detection probability by selecting the optimal weighed factors, and the joint optimization of sensing threshold and transmission power based on weighted cooperative sensing is also analyzed and solved. The bilevel optimization method with reduced iteration complexity is used to solve the proposed optimization problems. The simulations have shown that the proposed joint optimization algorithm can obtain desirable improvement on the throughput of CR and decrease the interference to PU greatly.

[7] The rest of the paper is organized as follows. The system models including energy detection model and wideband CR model are described in section 2. In section 3, we develop the joint optimization algorithms of sensing threshold and transmission power including maximizing throughput and minimizing interference. The joint optimization of weighted cooperative sensing is formulated in section 4. The advantages of the proposed joint optimization algorithm are illustrated by simulations in section 5, and conclusions are finally drawn in section 6.

2. System Model

2.1. Energy Detection Model

[8] Energy detection is widely used in CR as an effective method of single-user sensing. There are also some other detection methods such as matching filter detection and cyclic feature detection. However, matching filter detection needs to know the signal’s knowledge such as modulation type, signal wave, and frame structure, etc. Cyclic feature detection is only used to detect the periodic signal such as sinusoidal signal, where the cyclic power spectrum should also be known before detection. In addition, the cyclic feature detection needs to search for the circular frequency by using longer observed time and larger computation complexity. Hence, in this paper, we use the energy detector for single-user detection due to its simple implementation without knowing any prior information of PU’s signal.

[9] Energy detection compares the signal energy received in a certain frequency band to a properly set decision threshold. If the signal energy lies above the threshold, the band is declared to be busy. Otherwise, the band is assumed to be idle, which can be accessed by CR users. Owing to its operating principle that measures the energy of the PU’s signal, the performance of energy detection is independent on the prior information of the PU’s signal being detected.

[10] The structure of energy detection is shown in Figure 1. The sampled signal $y(m)$ is first obtained by passing the received signal through a band-pass filter. The energy statistic $\kappa(y)$ of the PU’s signal is then obtained by the operations of squaring and accumulating. The energy statistic is finally compared with a threshold $\lambda$ to make a decision on the presence of PU, i.e., $H_1$, if $\kappa(y) \geq \lambda$ or $H_0$ if $\kappa(y) < \lambda$.

[11] The spectrum detection for CR can be seen as a binary hypotheses testing, which is described as follows

$$y(m) = \begin{cases} n(m), & H_0 \\ s(m) + n(m), & H_1 \end{cases}, \quad m = 1, 2, \ldots, M$$  \hspace{1cm} (1)$$

where $s(m)$ is the sampled PU’s signal with a variance of $\sigma_n^2$, $n(m)$ is a Gaussian noise with a zero mean and a variance of $\sigma_n^2$, and $M$ is the number of the sampling nodes. The energy statistic is obtained as follows

$$\kappa(y) = \frac{1}{M} \sum_{m=1}^{M} |y(m)|^2$$  \hspace{1cm} (2)$$

[12] Supposing that the PU’s signal is band-pass with the bandwidth $W$, according to the Nyquist sampling theorem, the sampling frequency $f_s$ must satisfy

$$f_s \geq 2W$$  \hspace{1cm} (3)$$

[13] During the detection time $T$, $M$ is obtained as follows

$$M = T f_s \geq 2TW$$  \hspace{1cm} (4)$$

[14] Since $M$ is often very large, a Gaussian approximation on the energy statistic $\kappa(y)$ can be used so that the probabilities of false alarm and detection can be calculated easily by Quan et al. [2009]. The conditional means and variances of $\kappa(y)$ under $H_0$ and $H_1$ are, respectively, given as follows

$$E(\kappa(y)|H_0) = \sigma_n^2, \quad \text{Var}(\kappa(y)|H_0) = \frac{1}{M} \sigma_n^4$$  \hspace{1cm} (5)$$

$$E(\kappa(y)|H_1) = (1 + \gamma)\sigma_n^2, \quad \text{Var}(\kappa(y)|H_1) = \frac{1}{M} (1 + 2\gamma)\sigma_n^4$$  \hspace{1cm} (6)$$

where $\gamma = \sigma_n^2/\sigma_s^2$ is the received signal-to-noise ratio (SNR). By (5) and (6), it can be easily obtained that the probabilities
of false alarm and detection are, respectively, given as follows
\[
\begin{align*}
P_f &= Q\left(\frac{\lambda - \sigma^2_n}{\sqrt{W} \sigma^2_n}\right) = Q\left(\frac{\lambda}{\sigma^2_n} - 1\right)\sqrt{M} \\
P_d &= Q\left(\frac{\lambda - (1 + \gamma)\sigma^2_n}{\sqrt{W(1 + 2\gamma)}\sigma^2_n}\right) = Q\left(\frac{\lambda}{\sqrt{W(1 + 2\gamma)}\sigma^2_n} - 1\right)\sqrt{\frac{M}{2\gamma + 1}}
\end{align*}
\]
where the function \(Q(x)\) is defined as follows
\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2} dt
\]
(8)

[15] The miss detection probability is calculated by \(P_m = 1 - P_d\), and according to (7), the threshold \(\lambda\) is obtained from a target \(P_f\) or \(P_d\) as follows
\[
\lambda = \left\{\left(\frac{1}{\sqrt{M}} Q^{-1}(P_f) + 1\right)\sigma^2_n \right\}
\]
(9)

[16] The probabilities of false alarm and detection reflect the different characters of CR. Low false alarm probability improves the throughput of CR, while high detection probability decreases the interference to PU.

2.2. Wideband Cognitive Radio Model

[17] Similar to the communication systems based on orthogonal frequency division multiplexing, wideband CR can also operate over multiple subchannels synchronously. Consider a PU system, based on multicarrier modulation, operating over a wideband spectrum that is divided into \(L\) nonoverlapping narrowband subchannels. Since PU only partially selects the best subchannels for its communication, some of the subchannels might not be used and are available for the spectrum access of CR, as shown in Figure 2.

[18] Before transmitting data, the CR should sense all the \(L\) subchannels for identifying these idle subchannels. Supposing that \(P_{f,l}\) and \(P_{d,l}\) for \(l = 1,2,\ldots,L\) are the probabilities of false alarm and detection of subchannel \(l\), respectively, the CR can operate in the subchannel in the following two scenarios:

[19] 1. the CR makes an accurate decision on the absence of the PU with a probability of \(1 - P_{f,l}\) for using the subchannel without causing any interfering to the PU. In this scenario, the subchannel capacity of the CR is given by

\[
C_{0,l} = \Delta W \log \left(1 + \frac{P_h^2}{\sigma^2_{n,l}}\right)
\]
(10)

where \(\Delta W\) is the bandwidth of each subchannel, \(P_h\), \(h_l\), and \(\sigma_{n,l}^2\) are the transmitting power, the channel gain, and the noise variance of subchannel \(l\), respectively. Hence, the total throughput of the CR over all the \(L\) subchannels is given by

\[
R = \sum_{l=1}^{L} P_r(H_{0,l})(1 - P_{f,l}(\lambda_l))C_{0,l}
\]
(11)

where \(P_r(H_{0,l})\) is the probability of hypothesis \(H_{0,l}\) and \(P_{f,l}(\lambda_l)\) indicates the false alarm probability function about the threshold \(\lambda_l\).

[20] 2. the CR detects the absence of the PU falsely with a probability of \(1 - P_{d,l}\) for using the subchannel by causing harmful interference to the PU. In this scenario, the subchannel capacity of the CR is given by

\[
C_{1,l} = \Delta W \log \left(1 + \frac{P_h^2}{\sigma^2_{n,l}(1 + \gamma_l)}\right)
\]
(12)

where \(\gamma_l\) is the received SNR in subchannel \(l\). The total interference capacity to the PU from the CR over all the \(L\) subchannels is given as follows

\[
I = \sum_{l=1}^{L} P_r(H_{1,l})(1 - P_{d,l}(\lambda_l))C_{1,l}
\]
(13)

where \(P_r(H_{1,l})\) is the probability of hypothesis \(H_{1,l}\) and \(P_{d,l}(\lambda_l)\) indicates the detection probability function about the threshold \(\lambda_l\).
3. Joint Optimization Algorithm

[21] Let us maximize the total throughput of the CR subject to the constraint that keeps the total interference within the sufferance of PU or minimize the total interference to the PU subject to the constraint that guarantees the lowest throughput of the CR.

3.1. Maximizing Throughput

[22] A maximizing throughput problem is defined to increase the total throughput of the CR subject to the constraints on the total interference to the PU, the total transmission power of CR, and the probabilities of false alarm and detection of each subchannel. According to (10)–(13), the optimization problem, P1, is defined as follows

Problem P1

\[
\begin{align*}
\max_{P, \lambda} & \quad R(P, \lambda) = AW \sum_{i=1}^{L} P_i(H_{0,i}) \log \left( 1 + \frac{P_i h_i^2}{\sigma_{n,i}^2} \right) (1 - P_{f,i}(\lambda_i)) \\
\text{s.t.} & \quad \sum_{i=1}^{L} P_i \leq P_{\text{max}} \\
& \quad P_{f,i}(\lambda_i) \leq \alpha \quad \text{for } i = 1, 2, \ldots, L \\
& \quad P_{d,i}(\lambda_i) \geq \beta \quad \text{for } i = 1, 2, \ldots, L \\
& \quad P_i \geq 0
\end{align*}
\]

where \( \sigma_i \) is the standard deviation of interference, \( \alpha \) and \( \beta \) are the upper limits of false alarm probability and the lower limit of detection probability, respectively, and \( P_{\text{max}} \) is the maximized total power of the CR. \( R(P, \lambda) \) denotes the function related to the vectors \( P = \{P_1, P_2, \ldots, P_L\} \) and \( \lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_L\} \). Since CR needs high spectrum utilization and low interference to PU, we often set \( \alpha \leq 0.5 \) and \( \beta \geq 0.5 \).

[23] A single-variable optimization problem is formulated in Quan et al. [2009] for optimizing the sensing threshold. However, in this paper, problem P1 is a mixed-variable optimization problem, which is usually NP-hard to be solved directly [Boyd and Vandenberghe, 2003]. Instead of directly solving this problem, the bilevel optimization method is adopted to find the optimal solution to P1, which divides P1 into two single-variable convex optimization problems. Since each threshold has its own constraint compared with the aggregative constraint of the power, the optimization of sensing threshold is first solved as the upper level.

[24] When \( P \) is given, problem P1 is transformed into a single-variable optimization problem P2 as follows

Problem P2

\[
\begin{align*}
\max_{\lambda} & \quad R(\lambda) = \sum_{i=1}^{L} \eta_{0,i} (1 - P_{f,i}(\lambda_i)) \\
\text{s.t.} & \quad \sum_{i=1}^{L} \eta_{1,i} (1 - P_{d,i}(\lambda_i)) \leq \epsilon \\
& \quad P_{f,i}(\lambda_i) \leq \alpha \quad \text{for } i = 1, 2, \ldots, L \\
& \quad P_{d,i}(\lambda_i) \geq \beta \quad \text{for } i = 1, 2, \ldots, L
\end{align*}
\]

where \( \eta_{0,i} = \Delta WP_i(H_{0,i})C_{0,i} \) and \( \eta_{1,i} = \Delta WP_i(H_{1,i})C_{1,i} \).

According to (9), with the constraints \( P_{f,i}(\lambda_i) \leq \alpha \) and \( P_{d,i}(\lambda_i) \geq \beta \), we can obtain the bound of the threshold as follows

\[
\begin{align*}
\lambda_{\text{min},i} &= (Q^{-1}(\alpha) / \sqrt{M} + 1) \sigma_{n,i}^2 \\
\lambda_{\text{max},i} &= (Q^{-1}(\beta) / \sqrt{2\gamma_i + 1 / \sqrt{M} + \gamma_i + 1}) \sigma_{n,i}^2
\end{align*}
\]

where \( \lambda_{\text{min},i} \) and \( \lambda_{\text{max},i} \) are the minimum and maximum of the threshold \( \lambda_i \), respectively. Therefore, problem P2 is rewritten as follows

Problem P3

\[
\begin{align*}
\min_{\lambda} & \quad \sum_{i=1}^{L} \eta_{0,i} P_{f,i}(\lambda_i) \\
\text{s.t.} & \quad \sum_{i=1}^{L} \eta_{1,i} (1 - P_{d,i}(\lambda_i)) \leq \epsilon \\
& \quad \lambda_{\text{min},i} \leq \lambda_i \leq \lambda_{\text{max},i} \quad \text{for } i = 1, 2, \ldots, L
\end{align*}
\]

[25] To solve problem P3, we first prove that P3 is a convex optimization problem.

[26] **Lemma 1**: If \( \alpha \leq 0.5 \) and \( \beta \geq 0.5 \), \( P_{f,i}(\lambda_i) \) and \( P_{d,i}(\lambda_i) \) are convex and concave functions about \( \lambda_i \), respectively.

[27] **Proof**: Taking the second derivatives of \( P_{f,i}(\lambda_i) \) and \( P_{d,i}(\lambda_i) \), respectively, in (7), we have

\[
\begin{align*}
\frac{\partial^2 P_{f,i}}{\partial \lambda_i^2} &= \frac{M_i (\lambda_i - \sigma_{n,i}^2)}{\sigma_{n,i}^4 \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{\lambda_i}{\sigma_{n,i}} - 1 \right)^2 \right) \\
\frac{\partial^2 P_{d,i}}{\partial \lambda_i^2} &= \frac{M_i (\lambda_i - (\gamma_i + 1) \sigma_{n,i}^2)}{(2\gamma_i + 1) \sigma_{n,i}^4 \sqrt{4\gamma_i + 2\pi}} \exp \left( -\frac{1}{4\gamma_i + 2} \left( \frac{\lambda_i}{\sigma_{n,i}} - \gamma_i - 1 \right)^2 \right)
\end{align*}
\]

[28] Since \( \alpha \leq 0.5 \) and \( \beta \geq 0.5 \), we have the constraint from (16), as follows

\[
\sigma_{n,i}^2 \leq \lambda_i \leq \sigma_{n,i}^2 (\gamma_i + 1)
\]

[29] Substituting (20) into (18) and (19), we have

\[
\frac{\partial^2 P_{f,i}}{\partial \lambda_i^2} \geq 0, \quad \frac{\partial^2 P_{d,i}}{\partial \lambda_i^2} \leq 0
\]

which indicates that \( P_{f,i}(\lambda_i) \) and \( P_{d,i}(\lambda_i) \) are convex and concave functions about \( \lambda_i \), respectively.

[30] From (21), we also know that \( P_{m,i} = 1 - P_{d,i}(\lambda_i) \) is also convex. Note that the nonnegative weighted sum of a set of convex functions is also convex [Boyd and Vandenberghe, 2003], and therefore both of the objective and the first constraint of problem P3 are nonlinear and convex. Since the other constraints are linear, the problem P3 becomes a convex optimization problem. It is proposed in Quan et al. [2009] that the interior point method can be used to solve problem P3. However, the iteration complexity of the interior point method is often \( O(\sqrt{KL}) \) where \( K \) is the number of the constraint.
with less than solve the upper level, which can obtain the optimal solution algorithm based on Lagrangian Theorem is proposed to of (22) can be obtained by nonlinear constraint and 2 \( \mu \) where we obtain the solution as follows

where \( \mu \) is the Lagrangian coefficient. The optimal solution of (22) can be obtained by

\[
\frac{\partial U_1}{\partial \lambda_l} = 0, \quad l = 1, 2, \ldots, L
\]  

(23)

[32] By substituting (22) into (23), we further have

\[
\frac{\partial P_{d,l}}{\partial \lambda_l} |_{\lambda_l} = \frac{\eta_{0,l}}{\mu \eta_{1,l}} \frac{\partial P_{r,i}}{\partial \lambda_i} |_{\nu}
\]  

(24)

where we obtain the solution as follows

\[
\lambda_l = \sigma^2_{2,l} \left( \frac{1}{\mu \eta_{1,l}} \right)^{\frac{1}{2}} \left( \frac{1}{4} + \frac{\gamma_l}{2} + \frac{2\gamma_l + 1}{M_{\gamma_l}} \ln \left( \frac{\eta_{0,l}}{\mu \eta_{1,l}} \right)^{\frac{1}{2}} + 1 \right)
\]  

(25)

where \( \mu \) can be obtained by substituting (25) into the nonlinear constraint of P3. Since the linear constraint \( \lambda_{\min,l} \leq \lambda_l \leq \lambda_{\max,l} \) is needed, the optimal solution of P3 is given by

\[
\lambda_l^* = \begin{cases} 
\lambda_{\min,l}, & \text{if } \lambda_l < \lambda_{\min,l} \\
\lambda_l, & \text{if } \lambda_{\min,l} \leq \lambda_l \leq \lambda_{\max,l} \\
\lambda_{\max,l}, & \text{if } \lambda_l > \lambda_{\max,l}
\end{cases}
\]  

(26)

[33] If \( \lambda_l \) is outside the linear constraint, we have to change \( \lambda_l \) to \( \lambda_{\min,l} \) or \( \lambda_{\max,l} \) and the nonlinear constraint might not be satisfied. Hence, we have to update \( \mu \) in order to make \( \lambda_l \) for \( l = 1, 2, \ldots, L \) satisfy the nonlinear constraint again. This process is continued until all the \( \lambda_l \) for \( l = 1, 2, \ldots, L \) are within the linear constraint. The proposed threshold optimization algorithm is described in Algorithm 1. In Algorithm 1, the steps (5–6) are used to exclude the \( \lambda_l \) outside the linear constraint in order to update \( \mu \) in the next iteration. The worst case of Algorithm 1 is that there is only one \( \lambda_l \) outside the linear constraint in each iteration, and therefore at most \( L \) iterations are needed.

[34] When the optimal \( \lambda \) is obtained by Algorithm 1, the next step is to optimize the transmission power \( P \), which is seen as the lower level. If \( \lambda \) is given, problem P1 is transformed to a single-variable optimization problem as follows

**Problem P4**

\[
\max_P R(P) = \frac{1}{l} \sum_{i=1}^{L} \phi_{0,l} \log \left( 1 + \frac{P h_i^2}{\sigma_{\gamma_i}^2} \right)
\]

\[
\text{s.t.} \quad \frac{1}{L} \sum_{i=1}^{L} \phi_{1,l} \log \left( 1 + \frac{P h_i^2}{\sigma_{\gamma_i}^2 (1 + \gamma_l)} \right) \leq \epsilon
\]

\[
\sum_{l=1}^{L} P_l \leq P_{\text{max}}
\]

\[
P_l \geq 0, \quad l = 1, 2, \ldots, L
\]

where \( \phi_{0,l} = \Delta WP_i(H_{0,l})(1 - P_{r,l}(\lambda_{l})) \) and \( \phi_{1,l} = \Delta WP_i(H_{1,l})(1 - P_{d,l}(\lambda_{l})) \). In order to solve problem P4, we first prove that P4 is a convex optimization problem.
Lemma 2: \( P_4 \) is a convex optimization problem.

Proof: taking the second derivatives of \( C_0, l \) and \( C_1, l \), respectively, in \( P_l \) from (10) and (12), we have

\[
\frac{\partial^2 C_0, l}{\partial P_l^2} = - \frac{\Delta W l^4}{(\sigma_{n,l}^2 + h_l^2 P_l)^2}
\]

\[
\frac{\partial^2 C_1, l}{\partial P_l^2} = - \frac{\Delta W l^4}{(\sigma_{n,l}^2 + \sigma_i^2 P_l + h_l^2 P_l)^2}
\]

which indicates that \( C_0, l \) and \( C_1, l \) are both convex in \( P_l \). Since \( \phi_{0,l} > 0 \) and \( \phi_{1,l} > 0 \), the objective and the first constraint of \( P_4 \) are nonnegative weighed sum of a set of convex functions, and therefore both of them are nonlinear and convex.

Considering the objective and the first two constraints in (27), the Lagrangian function about \( P \) is given as follows

\[
U_2(P) = \sum_{l=1}^{L} \phi_{0,l} \log \left(1 + \frac{P_l h_l^2}{\sigma_{n,l}^2}ight)
\]

\[
- \mu_1 \left( \sum_{l=1}^{L} \phi_{1,l} \log \left(1 + \frac{P_l h_l^2}{\sigma_{n,l}^2(1+\gamma_l)}\right) - \epsilon \right)
\]

\[
- \mu_2 \left( \sum_{l=1}^{L} P_l - P_{\text{max}} \right)
\]

where \( \mu_1 \) and \( \mu_2 \) are both the Lagrangian coefficients. The optimal solution of (31) is obtained by

\[
\frac{\partial U_2(P)}{\partial P_l} = 0, \quad l = 1, 2, \ldots, L
\]

By substituting (31) into (32), we get the solution as follows

\[
P_l = \frac{\phi_{0,l} - \phi_{1,l}}{2\mu_2} - \frac{(2+\gamma_l)\sigma_{n,l}^4}{2h_l^2}
\]

[35] Lemma 2: \( P_4 \) is a convex optimization problem.

[36] Proof: taking the second derivatives of \( C_0, l \) and \( C_1, l \), respectively, in \( P_l \) from (10) and (12), we have

\[
\frac{\partial^2 C_0, l}{\partial P_l^2} = - \frac{\Delta W l^4}{(\sigma_{n,l}^2 + h_l^2 P_l)^2}
\]

\[
\frac{\partial^2 C_1, l}{\partial P_l^2} = - \frac{\Delta W l^4}{(\sigma_{n,l}^2 + \sigma_i^2 P_l + h_l^2 P_l)^2}
\]

[37] It is easily seen that

\[
\frac{\partial^2 C_0, l}{\partial P_l^2} < 0, \quad \frac{\partial^2 C_1, l}{\partial P_l^2} < 0
\]

[38] Considering the objective and the first two constraints in (27), the Lagrangian function about \( P \) is given as follows

\[
U_2(P) = \sum_{l=1}^{L} \phi_{0,l} \log \left(1 + \frac{P_l h_l^2}{\sigma_{n,l}^2}\right)
\]

\[
- \mu_1 \left( \sum_{l=1}^{L} \phi_{1,l} \log \left(1 + \frac{P_l h_l^2}{\sigma_{n,l}^2(1+\gamma_l)}\right) - \epsilon \right)
\]

\[
- \mu_2 \left( \sum_{l=1}^{L} P_l - P_{\text{max}} \right)
\]

which indicates that \( C_0, l \) and \( C_1, l \) are both convex in \( P_l \). Since \( \phi_{0,l} > 0 \) and \( \phi_{1,l} > 0 \), the objective and the first constraint of \( P_4 \) are nonnegative weighed sum of a set of convex functions, and therefore both of them are nonlinear and convex.

According to (33), especially if \( \gamma_l \approx 0 \) and \( P_{d,l} \approx 1 \) (there is no PU's signal, and detection probability is 1), the CR can use all the subchannels with the power of (33) that is given as follows

\[
P_l = \frac{\phi_{0,l}}{\mu_2} - \frac{\sigma_{n,l}^2}{h_l^2}
\]

[39] By substituting (31) into (32), we get the solution as follows

\[
P_l = \frac{1}{4\mu_2} \left( \phi_{0,l} - \phi_{1,l} \right)^2 + \gamma_l \sigma_{n,l}^4 + \mu_1 \phi_{1,l} + \frac{h_l^2 \gamma_l}{4}\]

[40] According to (33), especially if \( \gamma_l \approx 0 \) and \( P_{d,l} \approx 1 \) (there is no PU's signal, and detection probability is 1), the CR can use all the subchannels with the power of (33) that is given as follows

\[
P_l = \frac{\phi_{0,l}}{\mu_2} - \frac{\sigma_{n,l}^2}{h_l^2}
\]

[41] It is seen that (34) is the traditional water-filling formula where \( \phi_{0,l}/\mu_2 \) is the water level. Since the constraint
$P_l \geq 0$ is needed, the optimal solution to $P_4$ is given as follows:

$$P_l \begin{cases} P_l, & h_l > \sigma_{\delta j} \sqrt{\frac{\mu_2 (\gamma_l + 1)}{\phi_{0 l} (1 + \gamma_l) - \mu_1 \phi_{1 l}}, \quad l = 1, 2, \ldots, L} \quad (35) \\
0, & \text{otherwise}
\end{cases}$$

From (35), it is seen that the power is allocated to the subchannel only when the subchannel gain is above a specific value. This is because the subchannel with a low gain may achieve small throughput with a large power. Similar to Algorithm 1, we set $P_l = 0$ if $h_l$ is low. Therefore, the first two constraints of $P_4$ might not be satisfied. Hence, the updates of $\mu_1$ and $\mu_2$ are needed, which is continued until all the $P_l \geq 0$ for $l = 1, 2, \ldots, L$. The power optimization algorithm is described in Algorithm 2. Compared with the interior point method, the proposed algorithm can obtain the optimal solution with at most $L$ iterations.

The bilevel optimization method is applied to solve problem $P_1$, in which the upper-level problem $P_2$ (threshold optimization) and the lower-level problem $P_4$ (power optimization) are alternately optimized until the objective value is converged [Boyd and Vandenberghe, 2003]. The joint optimization of $\lambda$ and $P$ is defined in Algorithm 3. Noting that the objective function is nondecreasing, we have

$$R(P^{(k)}, \lambda^{(k)}) \leq R(P^{(k+1)}, \lambda^{(k+1)}) \leq R(P^{(k+1)}, \lambda^{(k+1)})$$

(36)

which indicates that if $R$ is convergent, both of $\lambda$ and $P$ are also convergent. Since we have proven that both of $P_2$ and $P_4$ are convex optimization problems, the convergence of $\lambda$ and $P$ can be achieved.

### 3.2. Minimizing Interference

Another important topic in CR communication is the minimization of the interference to PU. Hence, we can also formulate another optimization problem that minimizes the total interference subject to the constraints on the total throughput, the total power, and the probabilities of false alarm and detection of each subchannel. This optimization problem is defined as follows:

![Figure 3. Cooperative sensing model.](image-url)
where $t$ is lower limit of the total throughput, and $f(P, \lambda)$ denotes the function about $P$ and $\lambda$.

Similar to problem P1, the bilevel optimization method can also be adopted to solve problem P5. Let us decompose P5 into two convex single-variable optimization problems. The upper level problem for optimizing the sensing threshold is given by

**Problem P6**

\[
\begin{align*}
\max_{\lambda} & \quad \sum_{l=1}^{L} \eta_{l,j} P_{d,l}(\lambda_{l}) \\
\text{s.t.} & \quad \sum_{l=1}^{L} \eta_{l,j} (1 - P_{d,l}(\lambda_{l})) \geq \tau \\
& \quad \lambda_{\min,l} \leq \lambda_{l} \leq \lambda_{\max,l} \quad \text{for } l = 1, 2, ..., L
\end{align*}
\]

[46] The lower level problem for optimizing the transmission power is given as follows

**Problem P7**

\[
\begin{align*}
\min_{P} & \quad \sum_{l=1}^{L} \varphi_{l,j} \log \left(1 + \frac{P_{l} h_{l}^{2}}{\sigma_{n,l}^{2}(1 + \gamma_{l})} \right) \\
\text{s.t.} & \quad \sum_{l=1}^{L} \varphi_{l,j} \log \left(1 + \frac{P_{l} h_{l}^{2}}{\sigma_{n,l}^{2}} \right) \geq \tau \\
& \quad P_{l} \geq 0, \quad l = 1, 2, ..., L \\
& \quad \sum_{l=1}^{L} P_{l} \leq P_{\max}
\end{align*}
\]

(39)

[47] Problems P6 and P7 are similar to problems P2 and P4, and from (25) and (33), the solutions of Lagrangian functions in P6 and P7 are, respectively, given by

\[
\lambda_{l} = \frac{\sigma_{l,j}^{2}}{2} + \frac{2\gamma_{l}}{\sigma_{n,l}^{2}} + \frac{1}{\lambda_{l}} \ln \left(\frac{\eta_{l,j}}{\gamma_{l}}\sqrt{2\gamma_{l} + 1} \right)
\]

(40)

\[
P_{l} = \frac{1}{4\mu_{l}^{2}} (\mu_{l} \phi_{0,l,i} + \phi_{1,i})^{2} + \frac{\gamma_{l} \sigma_{n,l}^{2}}{2\mu_{l} h_{l}^{2}} (\mu_{l} \phi_{0,l,i} - \phi_{1,i}) + \frac{\gamma_{l} \sigma_{l,j}^{4}}{4h_{l}^{2}}
\]

(41)

[48] Hence, Algorithm 1 and Algorithm 2 can be used to solve problems P6 and P7 effectively. Then, Algorithm 3 can be used to obtain the joint optimal solution to problem P5.

\[\text{Figure 4.} \quad \text{Weighed cooperative sensing model.}\]

\[\text{Figure 5.} \quad \text{The throughput versus the constraint on interference.}\]
4.1. Weighed Cooperative Sensing

Because the received SNR of single CR is often different, the energy statistic of the CR with high SNR is more accurate than that with low SNR. The performance of cooperative sensing can be improved by increasing the combined proportion of the CR with high SNR while decreasing that of the CR with low SNR. In this paper, the weighed factor is used to represent the combined proportion of a single CR. Considering $N$ CRs participating in cooperative sensing, the weighed cooperative sensing model is shown in Figure 4.

The optimal weighed factors can be obtained through maximizing the modified deflection coefficient, and the optimization problem is solved by the interior point method [Quan et al., 2009]. In this paper, we obtain the optimal weighed factors through maximizing the detection probability based on the joint optimization algorithm mentioned above. In addition, the improvement on cooperative sensing is notable by jointly optimizing both the threshold and power.

According to Figure 4, we can derive the probabilities of false alarm and detection in weighed cooperative sensing from (7) as follows

$$Q_f = Q \left( Q^{-1} \left( \frac{\sum_{i=1}^{N} o_i \gamma_i}{\sqrt{M}} \right) \right)$$  \hspace{1cm} (42)

$$Q_d = Q \left( \frac{\sum_{i=1}^{N} o_i \gamma_i}{\sqrt{M}} \right)$$  \hspace{1cm} (43)

where the weighed vector $\phi = [\omega_1, \omega_2, ..., \omega_N]$ satisfies $\|\phi\|_2 = 1$. By (42) and (43), $Q_d$ is related with $Q_f$ as follows

$$Q_d = Q \left( Q^{-1} \left( Q_f \right) \right) \left( \frac{\sum_{i=1}^{N} o_i \gamma_i}{\sqrt{M}} \right)$$  \hspace{1cm} (44)

Our goal is to maximize the detection probability by selecting the optimal weighed factors under a specific $Q_f$. It is easily seen that (42) is not a convex function, and therefore a convex function that approximates to (44) is given as follows

$$Q_d \geq Q \left( Q^{-1} \left( Q_f \right) \right) \left( \frac{\sum_{i=1}^{N} o_i \gamma_i}{\sqrt{M}} \right)$$  \hspace{1cm} (45)

An efficient suboptimal method that maximizes $Q_d$ is to maximize its lower bound. Since $Q(x)$ is a monotone decreasing function, the optimization problem is given by

**Problem P7**

$$\max_{\phi} G(\phi) = \frac{\sum_{i=1}^{N} o_i \gamma_i}{\sqrt{\sum_{i=1}^{N} o_i^2}}$$  \hspace{1cm} (46)

s.t. $\|\phi\|_2 = 1$

![Figure 7. The interference versus the constraint on throughput.](image-url)
which indicates that \( \eta_1 \geq \eta_l \). Hence, the detection probability of weighed cooperative sensing is also larger than that of unweighed cooperative sensing.

### 4.2. Sensing Threshold Optimization

By substituting (49) into (14) and (37), we obtain the joint optimization problem of sensing threshold and transmission power based on weighed cooperative sensing. If the bilevel optimization method is used to divide the optimization problem into the upper and lower levels, only the upper level problem is altered compared with the case of single-user sensing. According to (49), if \( Q_{d, l} \leq \alpha \) and \( Q_{d, l} \geq \beta \), we obtain the bound of the cooperative sensing threshold as

\[
\begin{align*}
\lambda_{\text{min}, l} &= \left( Q^{-1}(\alpha) / \sqrt{M} + \sqrt{N} \right) \sigma_{n,l}^2 \\
\lambda_{\text{max}, l} &= \left( Q^{-1}(\beta) / \sqrt{M} + \sqrt{N} \left( \gamma_l + 1 \right) \right) \sigma_{n,l}^2
\end{align*}
\]

which is similar to (18). By substituting (49) into (22), the optimal solution to the Lagrangian function is obtained as follows

\[
\lambda_l = \frac{\sigma_{n,l}^2 \sqrt{N}}{2} + \frac{\sigma_{n,l}^2}{M} \sqrt{ \frac{N \sum_{i=1}^{N_1} \gamma_i}{2} + \frac{1}{2} \left( \frac{N_1}{2} + \frac{1}{2} \sqrt{\frac{2}{N_1}} \right) } \left( \frac{N_1}{2} + \frac{1}{2} \sqrt{\frac{2}{N_1}} \right)
\]

Hence, by substituting (51) and (52) into Algorithm 1, the optimal solution to the upper level problem is obtained. Accordingly, Algorithm 3 can be used to solve the joint optimization problem based on weighed cooperative sensing.

### 5. Simulation

In this section, we numerically evaluate the proposed joint optimization schemes. It is assumed that a wideband spectrum is divided into \( L = 10 \) subchannels whose bandwidth is \( \Delta W = 1 \text{ kHz} \) and the channel gains are \( h = [-2, -6, -10, -14, -18, -20, -16, -12, -8, -4] \text{dB} \). It is also assumed that the number of the sampling nodes is \( M = 128 \), the noise variance \( \sigma_n^2 = 0.01 \text{ mW} \), the received SNR \( \gamma = -10 \text{ dB} \), the upper limit of false alarm probability

\[
\begin{align*}
Q_{d,l} &= Q \left( \frac{\lambda_l}{\sigma_{n,l}} - \sqrt{N} \right) \\
Q_{d,l} &= Q \left( \frac{\lambda_l}{\sigma_{n,l}} - \sqrt{N} (\gamma_l + 1) \right)
\end{align*}
\]

where the square mean of SNRs \( \bar{\gamma}_l = \sqrt{\frac{\sum_{i=1}^{N_1} \gamma_i^2}{N}} \) where \( \gamma_i,l \) is the received SNR of CR in subchannel \( l \). Compared with the single-user sensing in (7), since \( \sqrt{N (\gamma_l + 1)} > \gamma_{l,1} + 1 \) and \( Q(\alpha) \) is a decreasing function, the cooperative detection probability of (49) is larger than that of the single-user sensing. In the traditional unweighed cooperative sensing, the average SNR is the mean \( \gamma = \frac{\sum_{i=1}^{N} \gamma_i l}{N} \). According to the Rayleigh-Ritz theorem, we have

\[
\sum_{i=1}^{N} \gamma_i l^2 \geq \left( \sum_{i=1}^{N} \gamma_i \right)^2
\]
and the lower limit of detection probability are $\alpha = 0.5$ and $\beta = 0.8$, respectively, and the total power is $P_{\text{max}} = 100\text{mW}$. It is noted that the optimization of transmission power is not sufficiently considered and only the sensing threshold is optimized in Quan et al. [2009]. For performance comparison, the uniform power is allocated to each subchannel in Quan et al. [2009].

5.1. Single-User Sensing

[62] Let us study the performance of single-user sensing in wideband CR. The proposed joint optimization algorithm of sensing threshold and transmission power (denoted by A1) is examined by comparing with two other algorithms: the threshold optimization algorithm of [Quan et al., 2009] with the uniform power of each subchannel (denoted by A2), and the conventional algorithm with the uniform threshold and power of each subchannel (denoted by A3).

[63] Figure 5 compares the throughput of the three algorithms mentioned above. It is seen that the throughput of the proposed joint optimization algorithm is larger than those of the other two. The throughput is improved with the increasing of the interference because the probabilities of false alarm and detection have the same monotony.

[64] Figure 6 shows the probabilities of false alarm and miss detection and the transmission power of each subchannel achieved by the three algorithms. Since $P_d \geq \beta = 0.8$, we have $P_m \leq 0.2$. It is seen that the sensing probabilities of the proposed joint optimization algorithm are kept within their limits, i.e., $P_f \leq 0.5$ and $P_m \leq 0.2$, because the bound of the threshold is constrained in problem P3. It is also seen that compared with the other two algorithms, the proposed one can allocate less power to the subchannel with a higher false alarm probability (i.e. lower spectrum utilization) for the improved throughput.

[65] An alternative optimization is presented in Figures 7 and 8 to show the numerical results of minimizing the interference (P5). From Figure 7, it is seen that the proposed joint optimization algorithm can produce lower interference to the PU compared with the other two. Figure 8 shows that the proposed algorithm allocates lower power to the subchannel with a higher missing detection probability (i.e., lower detection performance) for reducing the interference.

5.2. Weighed Cooperative Sensing

[66] This section studies the performance of weighed cooperative sensing for the case of five cooperative users in a wideband CR system. Figure 11 compares the detection probabilities of the three algorithms: the proposed weighed cooperative sensing, the traditional unweighed cooperative sensing, and the conventional algorithm with the uniform threshold and power of each subchannel.

[67] Figure 12 compares the throughput versus the constraint on interference. It is seen that the proposed joint optimization can cause lower interference to PU, and this is because the algorithm of Fan et al. [2011] includes the interference as a part of the throughput, which may increase the interference to PU while maximizing the throughput of CR.
sensing, and the single-user sensing. It is seen that the proposed one outperforms the other two.

[66] Figures 12 and 13, respectively, compare the throughput and interference of the proposed joint optimizations based on weighed cooperative sensing and the single-user sensing. It is seen that the joint optimization based on weighed cooperative sensing can achieve a larger throughput with lower interference because the weighed cooperative sensing has lower false alarm probability and higher detection probability.

6. Conclusion

[70] In this paper, we have proposed the joint optimization algorithm of sensing threshold and transmission power in wideband CR. The joint optimization algorithm has achieved the maximal throughput while keeping the total interference, the total transmission power, and the probabilities of false alarm and detection of each subchannel within their limits. The optimization is formulated as a mixed-variable optimization problem that is NP-hard to solve. The bilevel optimization method is used to obtain the optimal solution with a reduced iteration complexity by dividing the optimization problem into the upper level for optimizing the sensing threshold and the lower level for optimizing the transmission power. An alternative algorithm is also proposed to minimize the total interference under the constraint of the total throughput, etc. A weighed cooperative sensing is also applied to the joint optimization for maximizing the detection probability by selecting the optimal weighed factors. The simulations show that the proposed algorithm has achieved an increased throughput at the same interference level, and vice versa, by allocating the lower power to the subchannel with higher probabilities of false alarm and miss detection. The simulations also indicate that compared with the single-user sensing, the weighed cooperative sensing can obtain a larger throughput with smaller interference because of its higher detection performance.

[71] Acknowledgments. This work was supported by the National Natural Science Foundation of China (grants 61201143 and 61102069). This work was also supported by the College of Astronautics, Nanjing University of Aeronautics and Astronautics.

References


