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<td>Zhang, Mei; Cheng, Fang; Li, Guihua</td>
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Modeling CMM Dynamic Measuring Error Based on the 3B-Spline Orthogonal Projection Partial Least Squares

1Mei Zhang, 2Fang Cheng, 3Guihua Li

1College of Electrical Engineering and Automation, Anhui University, Hefei, 230601, China, hfren@126.com
2School of Mechanical & Aerospace Engineering, Nanyang Technological University, 639798, Singapore, CHENGFANG@ntu.edu.sg
3College of Electrical Engineering and Automation, Anhui University, Hefei, 230601, China, guihuali1@sina.com

Abstract

It's difficult to predict the dynamic error for coordinate measurement machine (CMM) by analyzing the error sources, because they are very complicated and have unknown interaction. In this paper an innovative modeling method is proposed by integrating 3B spline (3BS) transformation and orthogonal projection partial least squares (OPPLS). Three dimensional coordinates and direct computer control (DCC) parameters including positioning velocity, approaching distance and contacting velocity are used as the original independent variables of the model. The nonlinear relationship between the original independent variables and the CMM dynamic measurement errors is worked out by 3B spline transform. Then the orthogonal projection is used to build a new explanatory matrix by eliminating the components which are irrelative to the dependent variables. Finally, the operation of partial least-squares regression can be used to reduce the dimension and estimate the model parameters. With the proposed modeling method the nonlinear relationship between the independent variables and the dynamic measurement errors can be worked out without analyzing the error sources and their interactions. Besides, the problem of multi-collinearity caused by too many explanatory variables can also be overcome. The experimental results show that the mean square error of 3BS-OPPLS model is smaller than the 3B spline-partial least squares (3BS-PLS) model without the orthogonal projection, and the prediction accuracy of the model is notably improved.

Keywords: Coordinate Measuring Machine (CMM); Dynamic Measuring Error; Partial Least Squares (PLS); Multi-Collinearity

1. Introduction

Dynamic measurement errors of Coordinate Measuring Machine (CMM) is difficult to model[1,2,3,4] by analyzing error sources because they are very complicated and have unknown interactions. It's known that all the error sources contribute to the final measurement results, in the form of (x, y, z) coordinates. Besides, during the measurement the DCC (Direct Computer Control) parameters, including positioning velocity, approaching distance and contacting velocity, are easy to control and detect[2]. If the above factors are used as the impact variables, the generation mechanism of different kinds of errors can be ignored. Furthermore, it's also a challenging job to find a modeling method for not only expressing the function of dependent and independent variables, but also eliminating the multi-collinearity of different variables[5,6,7,8].

As mentioned above, the independent variables (including the coordinates of measurement position and DCC parameters) and dependent variables (the dynamic measurement errors) can be acquired from experiments, but the relationship between them remains unknown. The modeling will be very complicated especially in the case of high-dimension independents and nonlinear relationship, when both the transform function and the multi-collinearity influence should be considered.

In order to solve the problem of multi-collinearity and to find an accurate function to express the relationship between the dependents and independents[9,10], a novel method is proposed in this study by integrating 3B spline transform and improved PLS (Partial Least-Squares) regression[11,12,13], namely orthogonal projection PLS. This proposed 3B Spline-Orthogonal Projection Partial Least-Squares Regression can be used to analyze the dynamic error of CMM and its relationship with
(x, y, z) coordinates and DCC parameters. With this modeling method, the dynamic systematic errors of CMM can be analyzed and predicated.

2. Modeling based on 3B spline orthogonal projection partial least-squares regression

2.1 Modeling principle

In modeling practice, a series of data points are needed to derive the relationship between independent and dependent variables. A typical method is to configure a simple function by means of interpolation, to approach the actual transform function that is sometimes too complicated[14,15]. But high-order interpolation may cause runge phenomenon, which means tiny changing of input will cause unacceptable fluctuation of the fitting curve. In order to avoid runge phenomenon, instead of high-order polynomial, a piecewise interpolation called polynomial-spline method is thus widely used. The fitting range is divided into small sections, each of which has the individual fitting function and smooth connection with neighboring sections. The modeling parameters can be estimated by means of linear regression because the spline function is composed by polynomials. But the spline transform will notably increase the model dimension, causing the problem of multi-collinearity. To solve this problem, PLS can be used to estimate the model parameters after spline transform. This modeling idea of PLS regression based on spline transform has already had successful applications. In 1997, Jean-François Braive, Frédérique Duran proposed the regression model based on multivariate spline, which is effective to avoid Runge phenomenon and keep the continuity and smoothness so that it’s immune to the singular points or noises. This method has a limitation that the dimension of the model will be notably increased. When the number of variables exceeds that of sample points, the problem multi-collinearity will arise[16].

In the process of dimension reduction, when the explanatory matrix has much redundant information that is irreleative with response vectors, although the extracted components will have a big covariance it still has the limitation to explain the response variables. Some efforts[11, 14] are made to remove this kind of redundant information from the expository matrix by means of Orthogonal Projection. This method is called Orthogonal projection PLS (OPLS), which is proved to be effective to improve the modeling accuracy.

3B spline, with 2-order continuous derivativeness, can meet the requirement of most applications. In this study a 3BS-OPPLS regression algorithm is proposed for CMM dynamic error modeling by integrating 3B spline transform and Orthogonal projection PLS. In other words, the individual function of each independent variable and the CMM dynamic measurement errors is worked out by 3B spline transform. Then based on aggregating of individual functions about each independent variable, the analytic functions of CMM dynamic measurement errors is obtained by OPPLS mode.

2.2 Modeling process

Based on the idea in [12], the process of 3BS-OPPLS regression can be derived as following:

Assuming there is the dependent \{ y \}, independents \{ x_1, x_2, \ldots, x_p \} and n sample points, the matrix of independents and dependents can be configured as \( X = [x_1, x_2, \ldots, x_p]_{n \times p} \) and \( Y = \{ y \}_{n \times 1} \).

Step1: 3B spline transform is carried out for each dimension \( x_j (j = 1, 2, \ldots, p) \) of independent variable space \( X = [x_1, x_2, \ldots, x_p]_{n \times p} \), where every \( x_j \) represents the coordinates x, y, z of the measuring position, positioning velocity, approaching distance and contact velocity, respectively (1 \leq j \leq 6), with the basis function configured as equation (1):
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\[
\Omega \left( \frac{x_j - \xi_{j,l+1}}{h_j} \right) = \frac{1}{3! h_j^3} \sum_{p=0}^{4} (-1)^p \binom{4}{p} \left( x_j - \xi_{j,l+3p} \right)^3
\]  

(1)

where \( \xi_{j,l+1} \), \( h_j \), \( M_j \) are defined as dividing position in \( x_j \), section length and numbers of sections, respectively and can be expressed as below:

\[
\xi_{j,l+1} = \min \left( x_j \right) + (l - 1) h_j
\]  

(2)

where \( h_j = \frac{\max \{ x_j \} - \min \{ x_j \} }{M_j} \), \( l = 0, 1, \cdots, M_j + 2 \).

For any \( j = 1, 2, \cdots, 6 \), when \( M_j \) is determined, \( \xi_{j,l+1} \) can be calculated with equation (2).

For \( \{x_j\}_{n+1} \) the process of 3B spline transform can be expressed as equation (3) and(4):

\[
u_{j,0} = \Omega \left( \frac{x_j - \xi_{j,l+1}}{h_j} \right),
\]

\[
u_{j,1} = \Omega \left( \frac{x_j - \xi_{j,l+2}}{h_j} \right),
\]

\[\vdots\]

\[
u_{j,M_j+2} = \Omega \left( \frac{x_j - \xi_{j,M_j+1}}{h_j} \right)
\]

(3)

then,

\[
u_j = \left\{ \nu_{j,0}, \nu_{j,1}, \cdots, \nu_{j,M_j+2} \right\} = \Omega \left( \frac{x_j - \xi_{j,l+1}}{h_j} \right), l = 0, 1, \cdots, M_j + 2
\]

(4)

Step2: For dependent variables and new independent variables, the process of normalization is carried out as equation (5):

\[
u'_{j,l+1} = \frac{\nu_{j,l+1} - \overline{\nu}_{j,l+1}}{s_{j,l+1}}, \overline{\nu}_{j,l+1} = \frac{\nu_{j,l+1} - \overline{\nu}}{s_j}
\]

(5)

Where \( l = 0, 1, \cdots, M_j + 2; j = 1, 2, \cdots, 6; i = 1, 2, \cdots, n \); \( \overline{\nu}_{j,l+1}, \overline{\nu} \) are the mean values of \( u_{j,i}, y \) respectively; \( s_{j,l+1}, s_j \) represent the variances of \( u_{j,i}, y \).

Assuming \( \overline{U} \) and \( \overline{y} \) are the independents and dependents after the central standardization, the variable space can be transformed into:

\[
[X, Y] = [x_1, x_2, \cdots, x_n, y]_{n+1}
\]

\[
\downarrow
\]

\[
[\overline{U}, \overline{y}] = \left[ (\overline{U}_1)_{n(M_j+3)}, \cdots, (\overline{U}_n)_{n(M_j+3)}, \overline{y} \right] = \left[ \overline{U}_1, \cdots, \overline{U}_{1,M_j+2}, \cdots, \overline{U}_n, \cdots, \overline{U}_{n,M_j+2}, \overline{y} \right]_{n(M_j+3)+1}
\]

(6)

Step3: With equation (6) the explanatory matrix \( U \) can be worked out. When the eigenvalue of the matrix \( U' Y Y' U \) is set to 0, (p-1) eigenvectors \( h_1, h_2, \cdots, h_{p-1} \) can be calculated, which then forms a \( p \times (p-1) \) matrix \( B = (h_1, h_2, \cdots, h_{p-1}) \).
Step 4: For the matrix $B'U'UB$ the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_{p-1}$ and the corresponding eigenvectors $a_1, a_2, \ldots, a_{p-1}$ are calculated. Among these eigenvalues $s$ largest values are chosen, following the rule that their sum contributes the absolute majority of all eigenvalues. For example, $\sum_{i=1}^{s} \lambda_i / \sum_{i=1}^{p-1} \lambda_i \geq 99\%$ (in practice, typically $s=p-1$). Then $a_1, a_2, \ldots, a_s$ are defined as the $s$ largest eigenvalues.

Step 5: Let $H=UBA$, where $A = (a_1, a_2, \ldots, a_s)$, then $H$ represents the redundant information that is irrelative with the response vectors. By projecting $U$ in the orthogonal complement space of $U$, the projection matrix can be worked out as equation (7):

$$U_0 = (I_p - P_H)U = U - H(H'H)^{-1}H'U$$

$$U_0 = U(I_p - BA(H'H)^{-1}H'U) = UD$$

Equation (7)

Where $I_p$ is an identity matrix. Then $U_0$ is applicable for the linear square regression of $Y$.

Step 6: Regression of $Y$ as equation (8):

$$\hat{Y} = UD\hat{a} = U\hat{\beta}$$

Equation (8)

Then with 3BS-OPPLS arithmetic, the regression coefficient of explanatory matrix $U$ is $\hat{\beta}$.

3. Acquisition of experimental data

Some earlier researches focus on the positioning error of a typical moving bridge CMM without actual touching operation[1,2,3,8]. In this study the proposed method was testified at different measurement positions and with different DCC parameters. The whole process of an actual measurement is included. The experimental setup, as figure 1, therefore, corresponds to the definition of the dynamic errors. In this experiment, all errors caused by the mainframe, guide way, environment and the touch probe can be considered. So the proposed method is applicable for analyzing the composite spatial dynamic error of a moving bridge CMM.

Figure 1. The experimental setup for error sampling
Table 1. Values of independents

<table>
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<th>Values</th>
<th>Number of combinations</th>
<th>Number of data</th>
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<tr>
<td>$x/\text{mm}$</td>
<td>0,150,300,450,600,750</td>
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<td></td>
</tr>
<tr>
<td>$y/\text{mm}$</td>
<td>150,300,450,550</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>$z/\text{mm}$</td>
<td>-581,-473,-324</td>
<td></td>
<td>3456</td>
</tr>
<tr>
<td>$v_1/\text{mm/s}$</td>
<td>20,60,100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a/\text{mm}$</td>
<td>1,2,5,8</td>
<td></td>
<td>48</td>
</tr>
<tr>
<td>$v_2/\text{mm/s}$</td>
<td>2,4,6,8</td>
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Figure 2. The measurement plan for error sampling

A moving bridge CMM MC850 (equipped with the probe RenishawTP20, stylus length: 20mm, tip ball diameter: 4mm) is used to testify the proposed modeling method. The experiment for error sampling is arranged as following.

There are 6 independents: spatial coordinates $x/\text{mm}$, $y/\text{mm}$, $z/\text{mm}$, and DCC parameters: positioning velocity $v_1/\text{mm/s}$, approaching distance $a/\text{mm}$, and contacting velocity $v_2/\text{mm/s}$. The dependent is the composite spatial dynamic error $e/\mu\text{m}$. Different values of each independent are used and in every individual experiment only one variable is changed, as figure 2. Table 1 shows the combinations of the independents. There are totally 3456 combinations of the variables.

In order to testify the proposed method, about 5% randomly acquired data among these 3456 groups, say 167 data were used for the model evaluation, while the rest were used for modeling.

4. 3BS-OPPLS modeling for CMM dynamic errors

As mentioned in section 2, first of all the original variables of the CMM dynamic errors $x$, $y$, $z$, $v_1$, $v_2$ and $a$ are separated into 3 sections averagely, so $M_j=3$. With equation (2) the section length $h_j$ and dividing points $\xi_{j+1}$ were determined, where $j=1,2,\ldots,6$. With equation (3) 6 new variables can be derived from the every original independent, totally 36 new variables, which forms an explanatory matrix $U$. After Orthogonal Projection $U_0$ was worked out. Then with PLS regression to $\hat{Y} = U\hat{\beta}$, the regression coefficient $\hat{\beta}$ can be found. In this study MATLAB programming was employed to get matrix $U_0$, then the software tool SIMCA-P was used for PLS regression analysis.
In order to evaluate the accuracy of the regression equation, the 167 randomly acquired data were taken into the 3BS-OPPLS model. Figure 3 shows an acceptable fitting result: with this 3BS-OPPLS modeling process, the mean square error, which can be used as evaluation index, is 1.2649 μm. As comparison, the conventional 3BS-PLS without Orthogonal Projection was also calculated, which turns out to have a larger mean square error of 1.3479 μm. This improvement can also be apparently shown in Fig.3.

![Figure 3. Effect of 3BS-OPPLS modeling](image)

5. Conclusions

Since the CMM error sources have complexity and interaction, it’s difficult to use them for error analysis and prediction. In this study the authors tried to use 3BS-OPPLS method by integrating conventional 3BS-PLS and Orthogonal Projection, in order to avoid the analysis on error sources. The six original variables, including position coordinates x, y, z, positioning velocity, approaching distance and contact velocity, are used as the independent variables for the process of dynamic error modeling. The experimental data show that although the error sources have unknown interaction, this model is able to express the nonlinear influence of each independent variables. Besides, with the proposed method, the multi-collinearity influence can be avoided effectively. Compared with conventional 3BS-PLS modeling, the proposed modeling arithmetic shows better results.

In the future work, we will study the more effective method to improve the regression results, mainly focus on the improvement PLS research for multi-component analysis.

6. Acknowledgment

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7. References


