Title	An analysis of a viscous dissipation flow
Author(s)	Sheela-Francisca, J.; Tso, Chih Ping.
Citation	Sheela-Francisca, J., & Tso, C. P. (2012). An analysis of a viscous dissipation flow. The 4th International Meeting of Advances in Thermofluids (IMAT 2011), 1440, 392-399.
Date	2012
URL	http://hdl.handle.net/10220/12456
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Citation: AIP Conference Proceedings **1440**, 392 (2012); doi: 10.1063/1.4704241
View online: http://dx.doi.org/10.1063/1.4704241
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1440?ver=pdfcov
Published by the AIP Publishing

An Analysis of a Viscous Dissipation Flow

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Abstract. A laminar Couette-Poiseullie flow of a Newtonian fluid is considered and heat transfer characteristics are analyzed, attention being given to the effect of viscous dissipation for the thermal boundary condition that both the plates being kept at specified and at different constant heat fluxes. The momentum equation is solved to obtain the velocity profile in such a way that it consists of the velocity of the upper moving plate and in turn the energy equation is solved to yield temperature distribution and Nusselt number. Interesting results are observed based on the influence of various parameters which are in terms of Brinkman number, dimensionless velocity and heat flux ratio.

Key-words: Viscous dissipation, laminar flow, Newtonian fluid, Nusselt number, Brinkman number
PACS: 83.50.Lh

INTRODUCTION

The heat transfer characteristics with effect of viscous dissipation of Newtonian fluid in small devices and in micro-channels may vary substantially from that of large objects. The important consequence of viscous dissipation in regard to temperature profile and Nusselt number through geometry of infinitely long fixed parallel plates, both plates having specified constant heat flux have been analyzed [1-5].

Couette-Poiseuille flow of nonlinear visco-elastic fluids and with the simplified Phan-Thien-Tanner fluid between parallel plates was analytically solved where the fixed plate is kept at constant heat flux and the moving plate was insulated [6]. For the geometry of Couette flow with one plate kept at Constant heat flux and the other insulated, numerical solution was obtained for power-law non-Newtonian fluid [7] and analytical solution was derived for Newtonian fluid [8].Analytical investigation had been done for Couette-Poiseuille flow, with slip effect at the porous wall, assuming that Bingham fluid is flowing in between two porous parallel plates [9].

The study on internal heat generation due to effect of viscous dissipation is not found in the literature for the Couette-Poiseuille flow with both the plates being kept at specified but different constant heat fluxes. The heat transfer analysis with one plate moving is a different fundamental problem worth pursuing. This study is necessary because of the high demand for the increasing degree of miniaturization in designing of devices. Hence, the case of lower plate being fixed and the upper plate moving with constant velocity, both being imposed to different but constant heat fluxes is considered. The energy equation is solved leading to expressions in temperature profiles and Nusselt number.

THE ANALYSIS

FIGURE 1 shows two flat infinitely long parallel plates distanced W or $2w$ apart, where the upper plate is moving with constant velocity U and the lower plate is fixed, with the x-y coordinate system chosen as shown.

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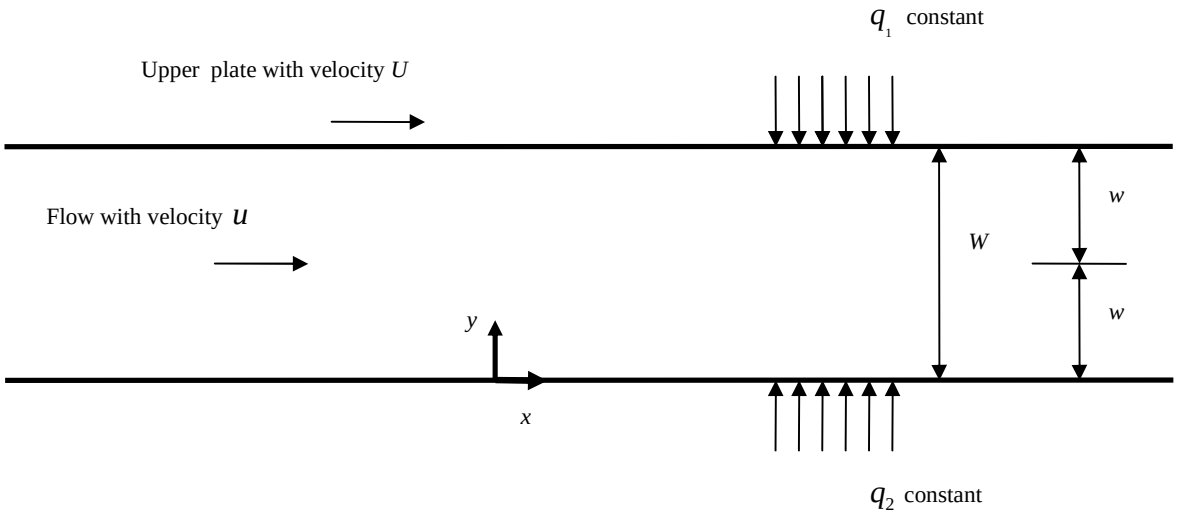


FIGURE 1. The problem.

The flow through the plates is considered at a sufficient distance from the entrance such that it is both hydro-dynamically and thermally fully developed. The axial heat conduction in the fluid and through the wall is assumed to be negligible. The fluid is assumed to be Newtonian and with constant properties. The thermal boundary conditions are the upper plate is kept at constant heat flux q_1 while the lower plate at different constant heat flux q_2.

The momentum equation in the x-direction is

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx}, \tag{1}$$

where u is the velocity of the fluid, μ is the dynamic viscosity, P is the pressure.

The velocity boundary conditions are $u = 0$ when $y = 0$ and $u = U$ when $y = W$. Using the following dimensionless parameters:

$$u^* = u/u_m, \quad U^* = U/u_m, \quad \mathrm{Y} = y/W, \tag{2}$$

the well-known velocity-distribution is [8]

$$u^* = (3U^* - 6)\left(Y^2 - Y\right) + U^*Y, \tag{3}$$

where the mean velocity is (u_m) is given by

$$u_m = \frac{1}{W}\int_0^W u\,dy. \tag{4}$$

For the above equation, expression for u is obtained by solving the momentum equation, Eq. (1). The energy equation, including the effect of viscous dissipation, is given by

$$u\frac{\partial T}{\partial x} = \frac{\gamma}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{5}$$

where the second term on the right-hand side is the viscous-dissipative term. In accordance to the assumption of a thermally fully developed flow with uniformly heated boundary walls, the longitudinal conduction term is neglected in the energy equation [10]. Following this, the temperature gradient along the axial direction is independent of the transverse direction and given as

$$\frac{\partial T}{\partial x} = \frac{dT_1}{dx} = \frac{dT_2}{dx}, \tag{6}$$

where T_1 and T_2 are the upper and lower wall temperatures, respectively. By taking $\alpha = k/\rho c_p$, introducing the non-dimensional quantity

$$\theta = \frac{T - T_1}{q_1 W/k}, \tag{7}$$

and defining a dimensionless constant β,

$$\beta = \frac{Pru_m kW}{\gamma q_1}\frac{dT_1}{dx}, \tag{8}$$

and modified Brinkman number Br_{q_1} as

$$\mathrm{Br}_{q_1} = \frac{\mu u_m{}^2}{2Wq_1}, \tag{9}$$

Eq. (5) can be written as

$$\frac{d^2\theta}{dY^2} = \beta\left[\left(3U^* - 6\right)\left(Y^2 - Y\right) + U^* Y\right] - 2Br_{q_1}\left[\left(3U^* - 6\right)\left(2Y - 1\right) + U^*\right]^2. \tag{10}$$

The thermal boundary conditions are

$$k\frac{\partial T}{\partial y} = q_1 \text{ at } y = W, \text{ or } \frac{\partial \theta}{\partial Y} = 1 \text{ at } Y = 1,$$
$$T = T_1 \text{ at } y = W, \text{ or } \theta = 0 \text{ at } Y = 1. \tag{11}$$

The solution of Eq. (10) under the above thermal boundary conditions can be obtained as

$$\begin{aligned}\theta(Y) = &\left(\frac{1}{4}\beta U^* - \frac{1}{2}\beta - 6U^{*2}Br_{q_1} + 24U^* Br_{q_1} - 24Br_{q_1}\right)Y^4 \\ &+\left(\frac{-1}{3}\beta U^* + \beta + 8U^{*2}Br_{q_1} - 40U^* Br_{q_1} + 48Br_{q_1}\right)Y^3 \\ &+\left(-4U^{*2}Br_{q_1} + 24U^* Br_{q_1} - 36Br_{q_1}\right)Y^2 \\ &+\left(1 - \beta + 8U^{*2}Br_{q_1} - 24U^* Br_{q_1} + 24Br_{q_1}\right)Y \\ &+\frac{1}{12}U^*\beta + \frac{1}{2}\beta - 6U^{*2}Br_{q_1} + 16U^* Br_{q_1} - 12Br_{q_1} - 1.\end{aligned} \tag{12}$$

To evaluate β in the above equation, a third boundary condition is required:

$$-k\frac{\partial T}{\partial y} = q_2 \text{ at } y = 0, \text{ or } \frac{\partial \theta}{\partial Y} = -\frac{q_2}{q_1} \text{ at } Y = 0. \tag{13}$$

By substituting Eq. (13) into Eq. (12), β can be expressed as a function of heat flux ratio, U^* and Br_{q1}

Therefore, the solution, Eq. (12), can be written as a function of Y, as well as heat flux ratio, U^* and Br_{q1}

In fully developed flow, it is usual to utilize the mean fluid-temperature, T_m , rather than the centerline temperature, when defining the Nusselt number. This mean or bulk temperature is given by

$$T_m = \frac{\int_{A_c} \rho u T dA_c}{\int_{A_c} \rho u dA_c}, \tag{14}$$

with A_c the cross-sectional area of the channel and the denominator on the right-hand side of Eq. (14) can be written as

$$\rho\int_0^1\left((3U^*-6)\left(Y^2-Y\right)+U^*Y\right)dA_c=\rho LW\,. \tag{15}$$

The dimensionless mean temperature is given by

$$\theta_m=\frac{k}{q_1W}\left(T_m-T_1\right). \tag{16}$$

At this point, the convective heat transfer coefficient can be evaluated by the equation

$$q_1=h(T_1-T_m)\,. \tag{17}$$

Defining Nusselt number to be

$$Nu=\frac{hD_h}{k}=\frac{q_1 2W}{k\left(T_1-T_m\right)}=-\frac{2}{\theta_m}, \tag{18}$$

where D_h is the hydraulic diameter defined by D_h =2W, the expression for Nusselt number can be expressed as a function of heat flux ratio, U^*, and $Br_{q1.}$ When q_2=0, it can be shown that Nu becomes

$$Nu=\frac{210}{-522U^*Br_{q_1}-94U^{*3}Br_{q_1}+366U^{*2}Br_{q_1}-11U^*+U^{*2}+8U^{*4}Br_{q_1}+162Br_{q_1}+39}, \tag{19}$$

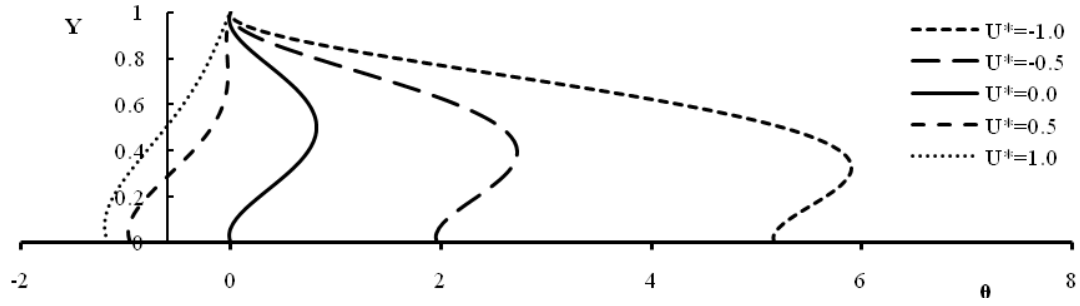
agreeing with reference [8].

RESULTS AND DISCUSSIONS

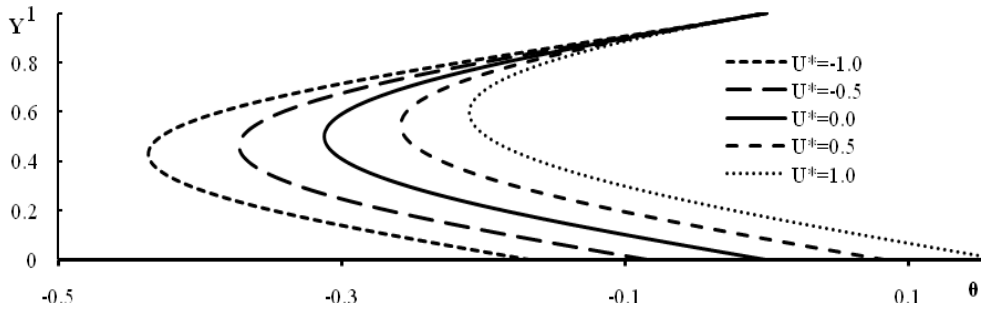
The characteristics of the flow and the heated region can be observed through various graphical representations. In the following discussion, the temperature profiles and the Nusselt number variations are plotted.

Temperature Profiles

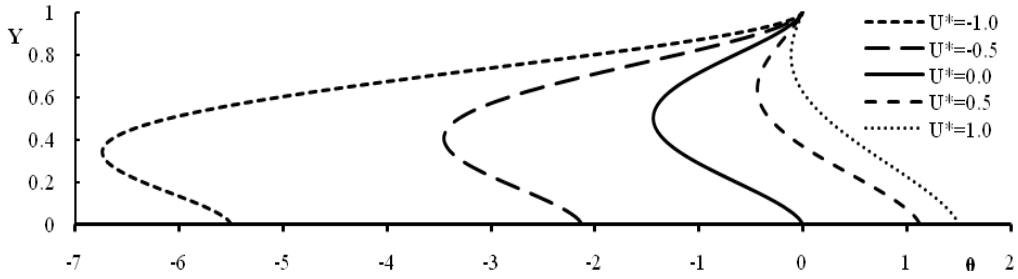
From the expression for the temperature profile in terms of various parameters such as moving plate velocity, constant heat flux ratio, modified Brinkman number, it is interesting to observe the behavior of various temperature profiles while keeping any two parameters fixed and vary the third parameter with different values.



(a) Br_{q1}=-0.5



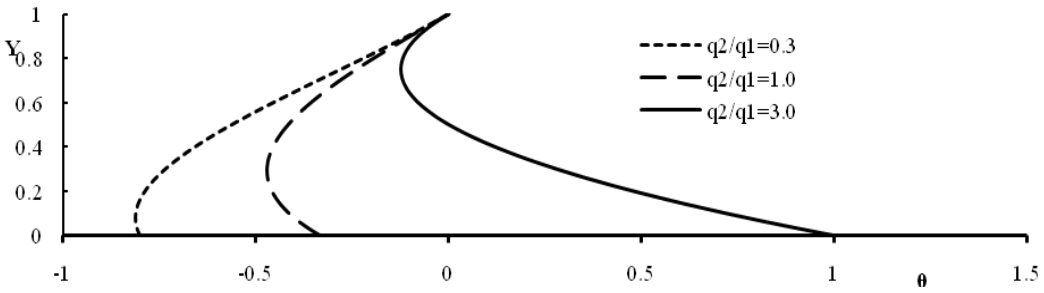
(b) Br_{q1}=0.0



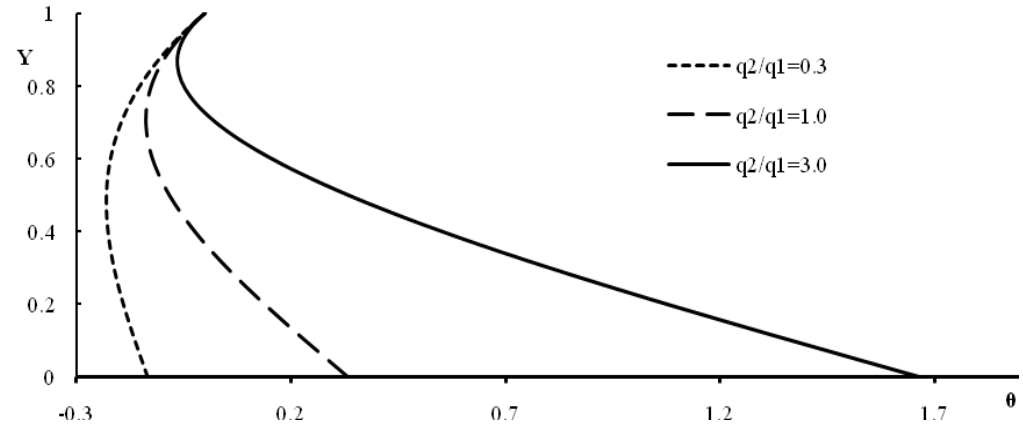
(c) Br_{q1}=0.5

FIGURE 2. Temperature profiles at U^*=-1.0, -0.5, 0.0, 0.5 and 1.0 at various Br_{q_1} for the case of q_2/q_1=1. (a) Br_{q1} = -0.5 (b) Br_{q1} = 0.0 (c) Br_{q1} = 0.5.

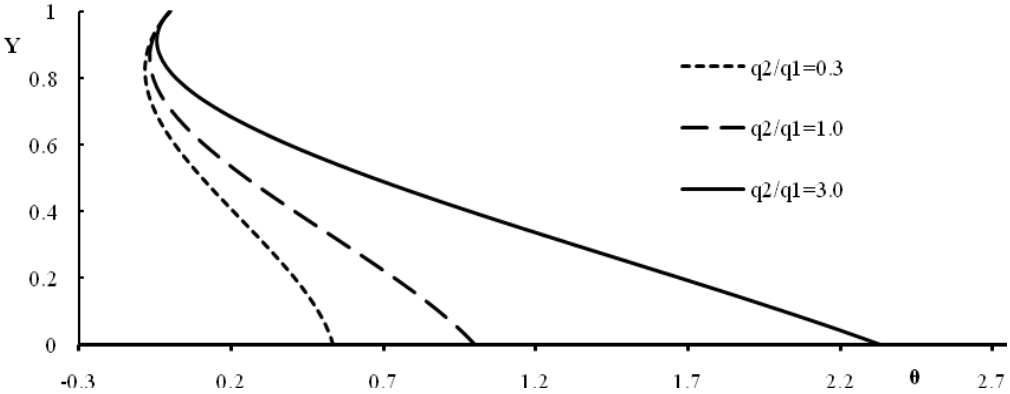
FIGURE 2 shows the dimensionless temperature profiles of θ versus Y, where both upper and lower plate are kept at specified equal constant heat flux at five dimensionless velocities U^* = -1.0, -0.5, 0.0, 0.5 and 1.0, and at three selected Br_{q_1} -0.5, 0.0 and 0.5, as shown in (a) to (c). It is observed that in FIGURE 2(a), when Br_{q_1} = -0.5, the temperature takes only positive values for U*=-1.0, -0.5 and 0.0 which implies there is an increase in heat transfer whereas when U*=0.5 and 1.0 the temperature takes minimum negative values. In FIGURE 2(b), in the absence of viscous dissipation, when U*=0.0, 0.5 and 1.0 the temperature distributions are purely negative which implies there is decrease in heat transfer and when U*=-0.5 and –1.0, the temperature takes very minimum positive values. All the curves in this case decreases, reaches minimum and then increases to 1. In FIGURE 2(c), when Br_{q_1} = 0.5, the temperature takes only negative values for U*=-1.0, -0.5 and 0.0 which implies there is a decrease in heat transfer whereas when U*=0.5 and 1.0 the temperature takes minimum positive values. All the curves converge at Y=1, at zero θ , by definition.



(a) Br_{q1}= - 0.5



(b) Br_{q1}= 0.0



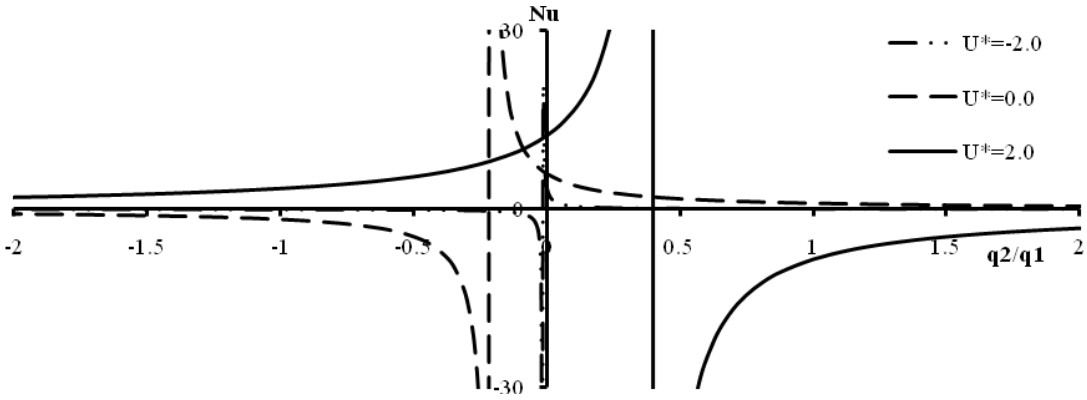
(c) Br_{q1}= 0.5

FIGURE 3. Temperature profiles at q_2/q_1 =0.3, 1.0 and 3.0 at various Br_{q_1}, for the case of U^*=2.0. (a) Br_{q1} = -0.5 (b) Br_{q1}= 0.0 (c) Br_{q1} = 0.5.

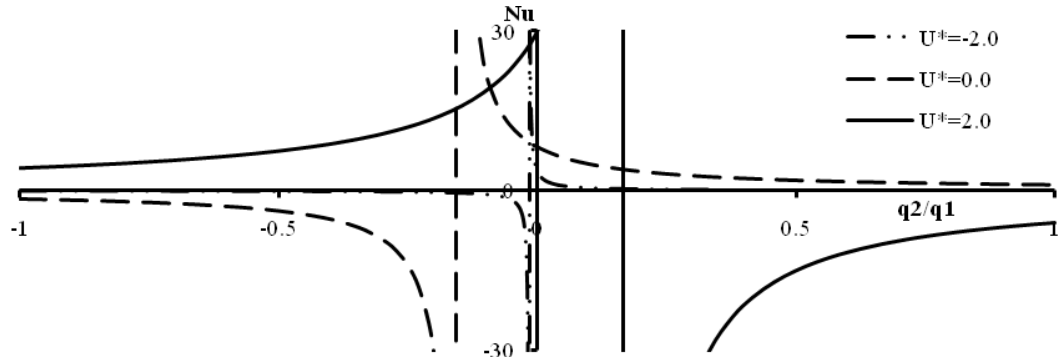
In FIGURE 3, the magnitude of the velocity of the moving plate is fixed as U^* = 2.0 and the pattern of temperature distribution is studied at q_2/q_1 = 0.3, 1.0 and 3.0, at various Br_{q_1} values. When Br_{q_1} =-0.5 and at q_2/q_1 = 0.3, 1.0, the temperature profiles are on negative side whereas when q_2/q_1 = 3.0, the values for the temperature distribution are both positive as well as negative. When Br_{q_1} =0.0 and at q_2/q_1 = 0.3, the values of the temperature are negative whereas when q_2/q_1 = 1.0 and 3.0, the values for the temperature distribution are both positive as well as negative and is observed that there is s decrease in the temperature values. When Br_{q_1} =0.5, the manifestation is different in such a way that the values of the temperature are positive and there is decrease in the values for all the specified constant heat flux ratios. All the curves converge at Y=1, at zero θ, by definition.

Nusselt Number Variations

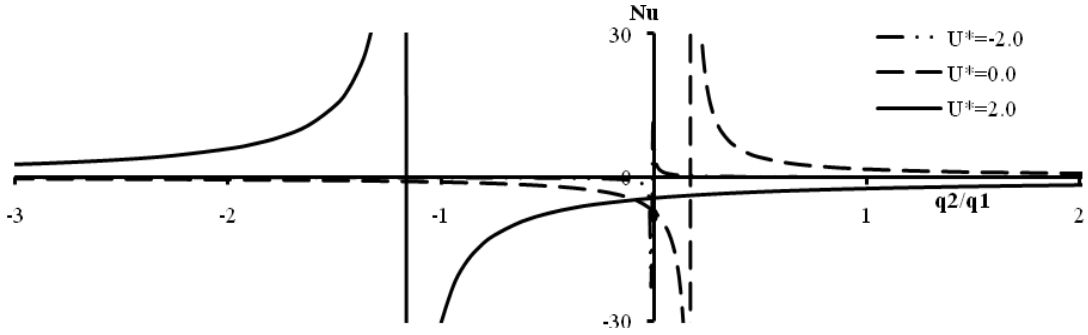
The following results show the effects of various parameters on the Nusselt number.



(a) q_2/q_1=0.3



(b) q_2/q_1=1.0



(c) q_2/q_1=5.0

FIGURE 4. Nusselt number versus Br_{q_1}, at U^*=-2.0, 0.0 and 2.0, at various q_2/q_1. Vertical lines are asymptotes.

(a) q_2/q_1 = 0.3 (b) q_2/q_1 = 1.0 (c) q_2/q_1 = 5.0.

FIGURE 4 illustrates the variation of the Nusselt number against the modified Brinkman number at U^* = -2.0, 0.0 and 2.0 at various constant heat flux ratios of 0.3, 1.0 and 5.0, as shown in (a) to (c). All the curves display rectangular hyperbolic shapes with singularities appearing along the Br_{q_1} axis, The singularities obtained are given in TABLE 1.

TABLE (1). Values of Br_{q1} at various q_2/q_1 and U* .

	q_2/q_1		
	0.3	1.0	5.0
U^*			
-2	-0.0179	-0.0169	-0.0113
0	-0.2157	-0.1574	0.1759
2	0.3999	0.1667	-1.1667

It is useful to know these singularities, since there will be change in the direction of heat transfer across these Br_{q_1} points for the above specified constant heat flux ratios. As the heat flux ratio varies the singularities will fall in different Br_{q_1} axis. This is a fundamental phenomena in the heat transfer analysis.

CONCLUSIONS

Hydro-dynamically and thermally fully developed laminar flow of Newtonian fluid through infinitely long parallel plates is considered, where the moving upper plate is kept under a specified constant heat flux and the lower fixed plate is kept under different but specified constant heat flux. Heat transfer analysis is done for this thermal boundary condition. The momentum equation and then the energy equation are solved and results obtained have various parameters such as moving plate velocity, modified Brinkman number and the constant heat flux ratio. A number of discussions and observations have been carried out which can play an important role in the designing of micro scale devices.

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