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Simulations of the linear plasma synthetic jet actuator utilizing a modified Suzen-Huang model

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The linear plasma synthetic jet actuator (L-PSJA) is a unique form of flow control device which harnesses the interaction of induced flows from two linear plasma actuators to form an upward jet. Since each injection can be manipulated in intensity, the synthetic jet has thrust vectoring properties. Our study simulates the L-PSJA by utilizing a modified Suzen-Huang (S-H) model that accounts for drift and diffusive properties in the surface charge. The results of the present model show that the centreline velocity is closer to the experimental values found in literature as compared to the default form of S-H modelling. Thrust vectoring simulations were also performed to demonstrate the feasibility of flow directional variation in the L-PSJA.

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I. INTRODUCTION

A. Flow control by synthetic jets

The main motivation for the study of synthetic jets stems from the potential utility in flow control applications. Previous research on flow control has had a great impact on many technological developments leading to enhanced performance. However, a review study of flow control by Gad el-Hak1 shows that energy wastage due to drag resistance results in losses amounting to billions of dollars in the aerospace industry. The ability to control the dynamics of fluid flow over a body allows engineers to incorporate designs that will drastically save fuel and energy. Recent research efforts with regards to flow control are being directed towards reducing drag, enhancing lift, and augmenting the mixture of mass, momentum, and energy. In order to achieve any of these, three issues have to be resolved; (1) transition from laminar to turbulent flow has to be delayed or advanced, (2) flow separation prevented or controlled, and (3) turbulence levels have to be suppressed or enhanced. Many methods have already been formulated to tackle the mentioned aerodynamics issues but the key is in discovering a control device or mechanism that is inexpensive to manufacture as well as to operate and has greater savings than penalties involved. The methods are most effective when applied near the transition or separation points. In other words, near the critical flow regimes where the instabilities magnify quickly.

A recent flow control review by Braun et al.2 categorises three main forms of flow control actuators: Fluidic, moving object/surface, and plasma, as shown in Figure 1. For completeness, we add one group (others), which constitutes the electromagnetic and magnetohydrodynamic forms of flow control.

Fluidic actuators result in suction or an ejection of fluid near separation points. This is the main working mechanism of a synthetic jet, where a zero net mass flux is added to the aerodynamic system. The second group—moving object or surface—alters wall effects of the flow, similarly reducing boundary layer separation and/or turbulence levels. Plasma actuators constitute the third group in Figure 1 and have evolved in the last two decades. Reviews concerning the plasma actuator can be

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found in Corke et al.\textsuperscript{3} and Moreau.\textsuperscript{4} The fourth and last group is made up of the electromagnetic or magnetohydrodynamic form. In this study we focus on the third group, more specifically the single dielectric barrier discharge (SDBD), highlighted in Figure 1. The specific geometry we consider here can also be considered belonging to the fluidic group since the working principle of the linear plasma synthetic jet actuator (L-PSJA) stems from the addition of a zero net mass flux.

The advantages of using plasma actuators (especially, the SDBD) for controlling airflow are that these actuators have no moving parts and are light. The plasma actuator also results in real-time changes to the aerodynamic system because the fundamental working mechanism of plasma stems from the motion of charge particles that are governed by a varying electric field. These fast changes are appealing when designing a flow control device.

Another review by Cattfesta and Sheplak\textsuperscript{5} concludes that flow control with electric fields is an exciting topic due to two reasons; its multidisciplinary nature, and more importantly, long-term research is being conducted which eventually will result in new sources of atomic energy which can produce extremely high power. These new sources of energy would greatly benefit control devices based on electric fields since atomic energy can provide high power to a low cost during long time periods without recharging or maintenance.

Plasma actuators have been tested in various applications, including separation control on a delta wing,\textsuperscript{6} turbine blades,\textsuperscript{7,8} and airfoils.\textsuperscript{9–11} The wide range of applications in the aerospace industry highlights the potential of the plasma actuator as a separation control device.

The latest research on plasma actuators aims at geometrical modification and optimization. For example, the work in the University of Manchester augments multiple encapsulated electrodes to the plasma actuator,\textsuperscript{12} while in the University of Florida, research on multi-barrier plasma actuators are conducted.\textsuperscript{13} Studies at ONERA describe experimental analysis of pulsed plasma actuators.\textsuperscript{14} The group from Technical University of Darmstadt, Germany, experimentally investigates the cancelling of the Tollmien-Schlichting waves using plasma actuators.\textsuperscript{15} In the University of Poitiers a group studies the surface corona discharges and their applications in aerodynamic surfaces.\textsuperscript{4}

In this study, we attempt to modify an inherent flaw present in a physics-based modelling technique of the plasma actuator. This point to a more fundamental research compared to the works mentioned above. Details of the physics-based model will be substantiated in Sec. II.

B. Modelling of the L-PSJA

The L-PSJA is made up of two exposed and one encapsulated electrode as shown in Figure 2. This is a modification to the design of the SDBD plasma actuator by a second exposed electrode placed downstream from the first. The position of the electrodes produces a net upward directional body force, inducing flow in a similar upward direction. Note that Figure 2 is not drawn to scale for clarity (however, the correct scales are shown later in Figure 8).

The design resembles a synthetic jet discharging from a rectangular opening, which can be considered to be nominally two-dimensional. The interaction between the synthetic jet and a cross
flow over a solid surface can lead to local displacement of the cross flow, inducing a modification of flow boundary and alter local pressure and vorticity distributions.

The study by Porter et al.\textsuperscript{16} provides a brief history of the research on the L-PSJA. The first investigations were conducted by Sherman,\textsuperscript{17} who created jets at 0°, 90°, and 180° measured from the surface. Subsequently, further investigations into the L-PSJA were conducted by various groups.\textsuperscript{18, 19} In Santhanakrishnan et al.,\textsuperscript{20} results of the Suzen-Huang (S-H) model utilizing the in-house code, UNCLE, have been compared with experimental data only in the 90° direction measured from the surface.

In the present work, we extend the investigations made of the L-PSJA by Santhanakrishnan et al.\textsuperscript{20} in a quiescent medium by utilizing a modified form of the S-H model initially investigated by Ibrahim and Skote.\textsuperscript{21} This modified form utilizes a time-varying boundary condition that enables the model to account for drift and diffusion in the electric field and surface charge density. We also performed thrust vectoring simulations which were absent in the work by Santhanakrishnan et al.\textsuperscript{20} Although time-dependent velocity streamlines and cross-stream distributions at various downstream stations were compared with pulsed experimental results in the study by Santhanakrishnan et al.,\textsuperscript{20} the present work will focus on the comparisons of steady state centreline velocities. However, in order to facilitate the utilization of the boundary condition modifications, a transient simulation over the first half-cycle of the applied ac voltage is performed. A pseudo-steady state is then assumed from the non-dimensional time of unity (at the peak of the first half-cycle) and onward. Details of this methodology are given in Sec. II. The reason behind assuming a pseudo-steady state condition is that the working principle of the plasma actuator, when applied to a flow control scenario, is essentially steady state. This has been demonstrated by earlier researchers into the plasma actuator and verified in the review by Corke et al.\textsuperscript{22}

The original and modified models are described in Secs. II A and II B, respectively. Simulations are conducted in a multiphysics software, COMSOL Multiphysics\textsuperscript{23} and details of the software will be mentioned in Sec. II D. Results will be presented in Sec. III. We compare contour plots of the variables that constitute the body force from the original S-H model with the modified form to verify the proper implementation of the boundary conditions. The resultant body forces
and induced velocities are also presented. Furthermore, centreline velocities of the L-PSJA from experiment, UNCLE and COMSOL are compared. In addition to the modified boundary condition and comparison with experimental and earlier simulation results, we introduce thrust vectoring results by presenting velocity vector plots from the L-PSJA with different maximum voltage settings applied to the electrodes. Even though Santhanakrishnan et al. did not perform simulations of thrust vectoring, an experimental study by Porter et al. provided a set of data, albeit limited, for comparison with present computations.

II. MODEL DESCRIPTION AND COMPUTATIONAL METHODOLOGY

The electrode configuration used in our simulation is similar to that by Santhanakrishnan et al. and is shown in Figure 2. The two exposed electrodes in the linear PSJA are 10 mm wide and 100 μm thick. The encapsulated electrode is 12.7 cm wide with a thickness of 100 μm. The electrodes are separated by an alumina ceramic dielectric of 125 μm, which has a relative permittivity of 9.4. A horizontal gap of 500 μm exists between the exposed and encapsulated electrodes.

The working principle of the S-H model stem from the splitting of the electric potential into two governing modes; one describing the effects of electric field and the other characterizing the effects of surface charge density. Results from simulations utilizing the boundary condition modifications have shown an improvement of about 50% when maximum induced velocities of a linear plasma actuator were compared with experimental data.

Details of the derivation of the S-H model can be found in Suzen and co-workers. We proceed in the details of the non-dimensionalised form of the S-H model in Sec. II A. This section is crucial for the application of time-varying surface charge boundary condition which will be introduced in Sec. II B. The simulation procedure is presented in Sec. II C. In Sec. II D, the computational methodology is described.

A. The S-H model

The plasma actuator model designed by Suzen and Huang studied the implication of splitting the total electric potential term (Φ) into two parts: one being influenced by external electric field (φ), and the other potential affected by the net charge density (ρc). This modelling technique has been utilized for the numerical analysis of the plasma actuator applied on turbine blades to achieve reduction in flow separation. The equations governing the plasma actuator are

\[ \nabla \cdot (\varepsilon_r \nabla \phi) = 0, \]

\[ \nabla \cdot (\varepsilon_r \nabla \varphi) = \rho_c / \varepsilon_0, \]

where \( \varepsilon_0 \) is the vacuum permittivity, \( \varepsilon_r \) is the relative permittivity, and \( \rho_c \) is the surface charge potential.

The net charge density potential (\( \varphi \)) can be eliminated by utilizing the Debye length (\( \lambda_D \)) which relates \( \varphi \) to \( \rho_c \) through

\[ \rho_c / \varepsilon_0 = (-1/\lambda_D^2) \varphi, \]

which combined with (2) yields

\[ \nabla \cdot (\varepsilon_r \nabla \rho_c) = (\rho_c / \lambda_D^2). \]

Thus, the model consists of solving Eqs. (1) and (4) with suitable boundary conditions. Finally, the Lorentz body force which is used in the Navier-Stokes equations is obtained through

\[ F_b = \rho_c (\nabla \varphi). \]
respectively. The term $\phi_o$ is set as a boundary condition and refers to the applied ac voltage at upper electrode,

$$\phi_0 (\tau) = \phi_{\text{max}} \times f(\tau),$$

where $\phi_{\text{max}}$ (V) refers to the maximum amplitude of the ac voltage supplied.

The term $\rho_{c,0}$ is set as a boundary condition at the lower electrode as

$$\rho_{c,0} (\tau) = \rho_{c,\text{max}} \times f(\tau),$$

where $\rho_{c,\text{max}}$ (C/m$^3$) refers to the maximum charge density of the ac voltage supplied.

The function $f(\tau)$ for the ac voltage source appearing in Eqs. (6) and (7) is

$$f(\tau) = \sin \left( \frac{\pi}{2} \tau \right),$$

where $\tau$ refers to a non-dimensionalised time quantity which is related to the frequency $\omega$ as

$$\tau = \omega t,$$

where $\omega$ (Hz) refers to the frequency of the ac voltage supply and is equal to 4.5 kHz in experiments.\(^5\), 8\)

We also relate the frequency to the characteristic time, $t_c$ as

$$\omega = \frac{1}{t_c},$$

where $t_c = 2.22 \times 10^{-4}$ s. Note that this characteristic time is the time taken to complete one sinusoidal cycle and has the same order of $10^{-4}$ s as other studies mentioned in the review by Corke et al.\(^3\) The simulations are run in the non-dimensionalised time quantity, $\tau$, from a value of 0 to 1. A time step of 0.1 is chosen to adequately observe the evolution of the four variables of interests: electric field, charge density, body force, and induced velocity.

The resulting non-dimensionalised quantities are

$$\phi^* = \phi / \phi_0(\tau),$$

$$\rho_c^* = \rho_c / \rho_{c,0}(\tau),$$

$$F_b^* = F_b / (\rho_{c,\text{max}} \times \phi_{\text{max}}).$$

The non-dimensionalised form of Eqs. (1) and (4) become

$$\nabla \cdot (\varepsilon_r \nabla \phi^*) = 0,$$

$$\nabla \cdot (\varepsilon_r \nabla \rho_c^*) = \rho_c^*/\lambda_D.$$

The boundary condition for the upper electrode for $\phi^*$ is set to unity so that once $\phi^*$ is determined, the dimensional value $\phi$ can be obtained at any given time by multiplying the distribution with the corresponding value $\phi_o(\tau)$ given by (6). Similarly, the boundary condition for the lower electrode for $\rho_c^*$ is set to unity. This allows the dimensional value $\rho_c$ to be obtained by multiplying the non-dimensionalised distribution $\rho_c^*$ with the corresponding value $\rho_{c,0}(\tau)$ given by (7).

The maximum amplitude of the ac voltage supplied, $\phi_{\text{max}}$, is set as 5 kV. Equations (14) and (15) are the governing equations used to model the non-dimensionalised body force in COMSOL and are solved separately from the Navier-Stokes equations. The dimensionalised body force can be obtained from (13) and is inserted into the Navier-Stokes computations.

Finally, Eqs. (14) and (15), and their boundary equations are shown together with the geometries in Figures 3 and 4, respectively. The terms GE and BC refer to governing equation and boundary conditions, respectively. Next we introduce the modification to the boundary condition.
B. Boundary condition modifications

Two new BCs are introduced for the Kapton surface above the encapsulated electrode. BC3 for Eq. (14) (shown in Figure 3) and BC1 for Eq. (15) (shown in Figure 4) are replaced as described below.

1. Dielectric shielding

The “dielectric shielding” condition for the non-dimensionalised electric potential shown as BC3N in Figure 5 describes a thin layer of thickness $\lambda_D$ and relative bulk permittivity $\varepsilon_{r,m}$. The terms $\nabla_n$ and $\nabla_t$ describe the normal and tangential derivatives of the non-dimensionalised electric potential variable. The condition equates the normal and tangential derivatives of the dimensionless electric potential, to produce a thin layer across the boundary that shields the electric field formed by the two electrodes. This results in a spreading of the electric potential and electric field magnitude across the boundary. Results from the modification will be shown in Sec. III A. We proceed to the modification boundary condition governing the surface charge density.

FIG. 3. Governing equation (14) for the S-H model. The illustration is not drawn in scale with simulation.

FIG. 4. Governing equation (15) for the S-H model. The illustration is not drawn in scale with simulation.
2. Fokker-Planck characteristics

For the equation governing the non-dimensionalised charge density, the original boundary condition (BC1 in Figure 4) used in the S-H model resulted in an instantaneous charge density growth along the boundary of the lower electrode that propagates in the normal direction (upward) of the dielectric surface. This propagation direction is different when compared to the results obtained by charge transport models, where propagation towards the right-side of the exposed electrode was obtained.\textsuperscript{28, 29} This motion of the surface charge corresponds to the physics of the plasma actuator, in which streamers originate from the exposed electrode and travel along the dielectric surface.\textsuperscript{7} These streamers dissipate and propagate from the exposed electrode and have not been shown in previous S-H models.

The dissipation and propagation characteristics can be modelled by the solution to the one-dimensional Fokker-Planck equation, which is written as

\[
\frac{\partial f}{\partial t} = -D_1 \frac{\partial f}{\partial x} + D_2 \frac{\partial^2 f}{\partial x^2},
\]

where $D_1$ and $D_2$ corresponds to drift and diffusion constants.

Figure 6 shows a one-dimensional representation of a normal distribution function exhibiting characteristics of Equation (16). The value of the constants $D_1$ and $D_2$ are set to unity. The Dirichlet boundary conditions are inserted at both ends of the one-dimensional analysis. The figure shows that the function is initially (at $t_1$) at a maximum, dissipating and propagating in time. The dissipation is manifested by the decreasing peaks of each successive time step. Similarly, propagation can be observed by the increasing $x$-coordinates of the dashed lines, which indicate the horizontal locations of the peaks.

We imposed the boundary modifications as described by Ibrahim and Skote\textsuperscript{21} at the interface above the encapsulated electrode as shown in Figure 7. To replicate Fokker-Planck characteristics in our new boundary condition, two boundary conditions are inserted. They form a new boundary condition, which is obtained by multiplying BC1 and BC2 or BC1 and BC3. These boundary conditions are written as

\[
\text{BC1: } \exp \left( -\left( \frac{x-x_1}{2\sigma^2} \right)^2 \right),
\]
FIG. 6. A graphical representation of the one-dimensional Fokker-Planck equation, with increasing time (t1) to (t5).

\[ BC_2 : \exp \left( -\left( x - x_1 \right) - \tau \right) x > x_1, \quad (18) \]

\[ BC_3 : \exp \left( -\left( x - x_2 \right) - \tau \right) x < x_2, \quad (19) \]

where \( x_1 \) and \( x_2 \) are the x-coordinates of the left and right edges of the encapsulated electrode, which were \(-6.85 \times 10^{-3} \) m and \(6.85 \times 10^{-3} \) m, respectively. The terms \( x_3 \) and \( \sigma \) are determined by the width of the normal distribution function and are taken as \(2 \times 10^{-3} \) m and 0.3, respectively, as used previously by Ibrahim and Skote.\(^{21}\)

C. Simulation procedure

The values for \( \lambda_D \) and \( \rho_{c,\text{max}} \) are chosen as \(2 \times 10^{-4} \) m and \(3 \times 10^{-3} \) C/m\(^3\), respectively, based from the previous simulation used in Santhanakrishnan et al.\(^{20}\) The maximum voltage applied to the

FIG. 7. Governing equations for the \( \rho_\lambda \) equation in S-H model. The modified boundary condition is placed at the lower boundary as shown.
exposed electrode was 5 kV. The Reynolds number based on the induced jet diameter (or equivalently the spacing between the exposed electrodes) is of the order of 1000. The simulation was performed for the non-dimensionalised time quantity $\tau$, from 0 to 1, similar to the previous study done on the linear actuator.\textsuperscript{21} At $\tau = 1$, the peak voltage of first half cycle is reached. The results at this instant are assumed as the pseudo-steady state of the actuator.

The system is taken as pseudo-steady because of the nature of plasma formation and its overall effect to the fluid system. The validity of the assumption is based on the three factors. First, the plasma discharge has a characteristic time scale that is several orders of magnitude lower than the electric field. The ac period required to power the actuator is long ($10^{-4}$ s) compared to the time needed for the charges in the plasma to redistribute ($10^{-8}$ s-$10^{-9}$ s), hence the system is quasi-steady in this respect. Similar physics-based modelling assumptions have been made in Orlov \textit{et al.}\textsuperscript{30,31} Second, the response of the fluid to the body force is much slower than the time duration of an ac period, at least for the low-speed applications (incompressible flow) that are considered here, and hence the time scale of the fluid is much larger ($10^{-2}$ s) than the applied plasma dynamics. In the present study we will therefore focus on one time instant (at the peak of the first ac cycle) in the same way as was done in the studies by Jayaraman \textit{et al.}\textsuperscript{32,33} Third, the discharge process is predominantly characterized in the first cycle, when the exposed electrodes are acting as cathodes. This has been verified by the results obtained in various experiments.\textsuperscript{34,35} Hence, in order to capture the formation of the induced body force, the first half cycle of the applied ac should be modelled accurately.

The assumptions above results in a fast solution technique of the equations governing the plasma, which hence leads to a model well suited for implementation in a design tool. Furthermore, in applications where the fluid flow experiences rapidly changing flow conditions the plasma model can easily be employed. However, there is of course a penalty for the simplifications and the drawbacks will be discussed in Sec. III D.

D. Computational method and grid independence

The simulations were solved using the finite element computational package, COMSOL Multiphysics\textsuperscript{TM} 4.2a.\textsuperscript{23} Since the study of plasma actuators is multiphysics in nature, the electrostatics application in COMSOL was used to model the potentials in the S-H model. The fluid domain is governed by the incompressible Navier-Stokes application, which is the ruling application mode for the investigation. The solver settings were based on the ruling application model. The application mode also used Lagrange p2-p1 elements to stabilize the pressure. Thus, 2nd-order Lagrange elements modelled the velocity components while linear elements modelled the pressure.

We have performed a grid convergence index study as described in Celik \textit{et al.}\textsuperscript{36} on the linear plasma actuator by Ibrahim and Skote.\textsuperscript{21} The results of the verification analysis conducted shows that the choice of grid settings in the order of $10^{-4}$ m was adequate in producing numerically acceptable results in the grid convergence analysis. To verify the grid independence for the present geometry, a simulation with a grid consisting of twice the amount of elements was performed. The coarse grid was defined as a setting where regions near the electrodes had a maximum grid size of $10^{-4}$ m. Fine grid setting had a maximum size of $5 \times 10^{-5}$ m. The two settings are shown in Figure 8. Note that only a part of the computational region is show. Furthermore, the true dimensions of the encapsulated electrode are revealed in Figure 8.

Figure 9 shows the comparison of centreline velocity results for the two grid settings. We focus on the region where the maximum velocities are recorded. The fine grid setting resulted in a maximum velocity of 0.4259 m/s, compared to the coarse grid result of 0.4257 m/s. As the results show almost negligible variations, further studies will utilize the coarse grid setting mentioned above.

III. RESULTS

The domain of the computational grid extends from $-8$ to 8 cm in the $x$-axis and from $-2$ to 15 cm in the $y$-axis. The simulations were conducted for non-dimensionalised time scales of $\tau = 0.1$ to $\tau = 1.0$. Result comparisons were done at $\tau = 1.0$, which is taken as the pseudo-steady state in COMSOL and steady state for experiment.
FIG. 8. Zoomed image of the grid region (left). Coarse grid (right, top) and fine grid (right, bottom) settings.

FIG. 9. Centreline velocity comparison for the two grid settings.
As the comparisons of variables ($\varphi$ and $\rho_c$) that constitute the body force in the S-H model have already been done for the linear actuator, and for the L-PSJA, we present data for the contour plots of the electric potential $\varphi$ and charge density $\rho_c$ for the modified boundary condition and make comparisons with the original BCs (Sec. III A). The centreline velocity and streamlines will be shown in Sec. III B. The body force and induced velocity profiles will be presented in Sec. III C. Finally, in Sec. III D, we introduce the thrust vectoring of the L-PSJA. All the results in Secs. III B–III D are from simulations using the modified BCs.

A. Non-dimensionalised electric potential and charge density

Figure 10 shows the non-dimensionalised electric potential contours in a zoomed image of the electrodes for the default and “dielectric shielding” boundary conditions. Both contour results show...
symmetry about the y-axis. The region of interest is above the dielectric, where the body force is present.

The default boundary condition used in Santhanakrishnan et al. results in the merging of the electric potential contours at the edges of the exposed electrode (near the dielectric surface above encapsulated electrode). This results in a highly localised body force intensity focused only at the exposed electrode edges. The “dielectric shielding” boundary condition as utilized in the present COMSOL simulation spreads the electric potential contours across the dielectric surface, hence resulting in a more gradual distribution of the body force.

The resultant electric field distribution for the default and dielectric shielding BCs are shown in Figure 11. The electric field magnitude is extracted across the dotted cut line as illustrated in the smaller inserted diagram in the figure.

The results in Figure 11 show that electric field strength is consistently larger for the dielectric shielding BC. The difference between the highest and lowest electric field magnitude is also smaller for the dielectric shielding condition, leading to a more even spread field distribution across the boundary. These two features are the desired characteristics of our “dielectric shielding” boundary condition.

Figure 12 shows the non-dimensionalised charge density distribution for both the default (as used in Santhanakrishnan et al.) and the modified boundary conditions. The default boundary condition sets a time-independent function which results in an instantaneously appearing surface charge density which then remains constant throughout the simulation, as shown in Figure 12(a). In this case, the governing equation describing the surface charge density is also solved at the dielectric domain, resulting in its unphysical presence in the region.

In contrast, the modified boundary condition used in our simulations results in an evolution of the non-dimensionalised charge density from $\tau = 0.1$ to $\tau = 1.0$, as shown in Figure 12(b). The modified boundary condition results in a diffusive motion towards the centre of the dielectric domain. Furthermore, the peaks of non-dimensionalised charge density are decreasing when approaching to the centre of the dielectric surface as seen in Figure 12(b). The modifications hence show both drift and diffusive characteristics as imposed by the boundary condition.
B. Centreline velocity and streamlines

In the following, we describe simulations with the original BCs (results taken from Ref. 20), denoted as UNCLE, and our simulations with the modified BCs which are referred to as COMSOL.

The streamline experimental result shows an asymmetry character in the flow field in Figure 13. Santhanakrishnan et al. 20 attributed this feature to uncertainties in the experimental settings. The streamlines show symmetry in COMSOL (Figure 13, right) and UNCLE (not shown in the above figure). Flow is sucked into the exposed electrode, before jetting upward at the centre of the L-PSJA. Vortices are also seen at the top end of the region.
The general flow features for centreline velocity (shown in Figure 14) are similar for both simulations (UNCLE and COMSOL) and the experimental result by Santhanakrishnan et al. The velocity peaks near 0.005 m, before decreasing further away from the dielectric surface. However, there are some differences between the three results. The maximum velocity induced for both COMSOL and the experiment was about 0.43 m/s, but was closer to 0.45 m/s for UNCLE. The simulation by COMSOL records that the flow reaches a maximum velocity slightly at about $2.5 \times 10^{-4}$ m earlier than the experiment. Furthermore, the decrease in velocity, at the higher positions, is also smaller in COMSOL (~0.24 m/s at 0.035 m) when compared to the experimental result (~0.13 m/s at 0.035 m). However, the agreement between COMSOL and experiment was better than that recorded by UNCLE, which obtained an even larger value of 0.32 m/s at 0.035 m.
C. Non-dimensionalised body force and induced velocity

Next, the evolution of non-dimensionalised body force and induced velocity are shown in Figure 15 at $\tau = 0.4$, 0.7, and 0.9, respectively. The non-dimensionalised body force figures in the first column show symmetrical expansion across the dielectric surface, characterizing both drift and diffusive properties. The second column shows the resulting induced velocity at the same non-dimensionalised time quantities. The length of the non-dimensionalised body force expansion is proportional to the non-dimensionalised time quantity; at $\tau = 0.4$, about 40% of the dielectric is covered by the non-dimensionalised body force, while at $\tau = 0.9$, about 90% is covered. Similar length correlations can be observed in both the charge densities and body force when comparing Figures 12 and 15. This linear relationship can be seen in Eq. (5) as well. Figure 15 also shows that the non-dimensionalised body force peaks at the initiating point (leading edge) of the dielectric surface on top of the encapsulate electrode. The non-dimensionalised body force then reduces in intensity during the expansion towards the centre of the L-PSJA.

Two vortices can be seen above the induced velocity formation at $\tau = 0.4$. The fluid induced by both electrodes merges to a core flow at the centre at $\tau = 0.5$. Subsequently, the core flow continues to build up in intensity. A lower maximum induced velocity of about 0.5 m/s is recorded when compared to the 1.3 m/s recorded in the linear actuator. This is because the fluid is not fully...
entained along the dielectric surface, in line with results seen previously in studies\textsuperscript{21} of the linear actuator model. In that study, the maximum velocity was obtained at positions further downstream of the exposed electrode (for the linear actuator).

D. Thrust vectoring

In this section, we briefly illustrate the potential of the L-PSJA to direct flow at different angles. The working principle behind this characteristic is the implementation of different peak voltage ($\Delta \Phi_{\text{max}}$) at each electrode. This produces asymmetric body forces, which in turn induces directional velocity. Figure 16 shows the implementation of difference voltage magnitudes on the exposed electrodes.

The results are tabulated in Figure 17. The voltage setting of only one exposed electrode was varied due to the symmetry in results which have been obtained. At $\Delta \Phi_{\text{max}} = 5$ kV, the velocity is pointing at the rightwards direction. The direction remains similar but with decreasing intensity, until $\Delta \Phi_{\text{max}} = 2$ kV. The induced velocity starts tilting upwards and becomes more pronounced at $\Delta \Phi_{\text{max}} = 1$ kV. Finally, as seen in the Figure 17, the induced jet points entirely upward when $\Delta \Phi_{\text{max}} = 0$.

It can be seen that the change in direction is most pronounce when $\Delta \Phi_{\text{max}}$ is varied from 0 V to 2 kV. This is due to the increased presence of electric field strength at the vicinity of the second electrode, resulting in an upward directional body force. Similarly, the induced velocity will also be pushed upward.

Porter \textit{et al.}\textsuperscript{16} conducted experimental thrust vectoring investigations and compiled results for the jet angle against the root-mean-squared voltage differential. The jet angle is measured relative to the left exposed electrode as shown in Figure 16. We converted their voltage differential into peak voltage differential and compared them with the results obtained by COMSOL as shown in Figure 18.

The experimental data for the jet angle were asymmetrical about the y-axis. The jet angles are not mirror-imaged when the voltage settings in the exposed electrodes are reversed. Furthermore, the experimental result showed that the jet angle deviated more to the left (at angles lower than 90\degree). When the jet angle is lower than 90\degree, the left exposed electrode has a larger voltage value compared to the right electrode. No explanation was made to this asymmetricity, but it is also noted that this feature was also seen in the streamlines results from the experimental work by Santhanakrishnan \textit{et al.}\textsuperscript{20}, as shown in Figure 13.

In our simulation, the jet angle would be symmetrical since the boundary conditions are mirrored in the two exposed electrode. The results for both simulation and experiment are in good agreement until 1 kV, as seen in Figure 18. Porter \textit{et al.}\textsuperscript{16} mentioned that “the endpoints tend to provide a larger
ΔΦ_{\text{max}} = 5000 \text{ V}

ΔΦ_{\text{max}} = 4000 \text{ V}

ΔΦ_{\text{max}} = 3000 \text{ V}

ΔΦ_{\text{max}} = 2000 \text{ V}

ΔΦ_{\text{max}} = 1000 \text{ V}

ΔΦ_{\text{max}} = 0 \text{ V}

FIG. 17. The velocity arrows for thrust vectoring investigations.

FIG. 18. Jet angle against voltage differential of the L-PSIA from Porter et al.\textsuperscript{16} and our simulations with COMSOL.
IV. CONCLUSION

The L-PSJA is simulated using a modified form of the Suzen-Huang plasma actuator model. The modifications introduce a “dielectric shielding” boundary condition to the potential governing the electric field, and takes into account Fokker-Planck characteristics of the boundary condition describing the surface charge density. These modifications result in a spreading of the electric field across the dielectric surface and a drift and diffusive surface charge motion. The motion of the surface charge corresponds to the physics of the plasma actuator, in which streamers originate from the exposed electrode and travel along the dielectric surface, and has previously not been captured in the Suzen-Huang modelling framework.

The centreline velocity of the L-PSJA was compared with the default form of Suzen-Huang modelling and experimental data. Results showed that the modified Suzen-Huang model was closer to the experimental results in terms of peak velocity magnitude as compared to the default form. The reduction of velocity at regions above the dielectric surface of the L-PSJA was also more pronounced for the modified Suzen-Huang model. The results also indicate that the oppositely directed induced fluid merges to a core flow at $\tau = 0.5$. Thrust vectoring investigations show that the change in jet direction is most apparent when $\Delta\Phi_{\text{max}}$ is varied between zero and 2 kV. Good agreement in jet angle prediction is obtained for $\Delta\Phi_{\text{max}}$ up to 1 kV above which the Coanda effects result in larger jet angles in experiments as compared to the simulations presented here.