Spacecraft Relative Attitude Determination

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Abstract—A method to determine the relative attitudes between three spacecraft is developed. [1 2] The method requires four direction measurements between the three spacecraft. The simulation results and covariance analysis show that the method's error falls within a three sigma boundary without exhibiting any singularity issues. A study of the accuracy of the proposed method with respect to the shape of the spacecraft formation is also presented.

TABLE OF CONTENTS

1. INTRODUCTION

Relative Attitude Determination (RAD) continues to receive a great deal of interest from researchers. Missions that require RAD include the spacecraft docking missions, chaser and target space missions, and a group of clusters that perform their tasks in a specific formation, such as Proba-3 [1] and LISA Pathfinder's mission [2; 3]. In these missions, the spacecraft are often required to maintain a specific orientation with respect to each other. Thus, the relative attitude determination between spacecraft is a fundamental task in these types of missions.

Relative attitudes between spacecraft can be calculated if the absolute attitude of each spacecraft is known. However, the determination of the absolute attitude for each spacecraft requires a complete set of attitude sensors onboard each spacecraft [4]. On the other hand, the determination of relative attitudes require less hardware on each spacecraft. So, if absolute attitudes are not needed, then measuring relative attitudes directly offers a lower cost advantage. Also, the determination of relative attitudes between spacecraft can be used to calculate the absolute attitudes, if the absolute attitude for one spacecraft is measured.

Several sensors that are capable of providing measurements for relative attitude and position have been developed. The Autonomous Formation Flying sensor of the Deep Space program [5] works in a similar way to an active GPS [6]. The Vision Based Navigation System [7] has been introduced as a candidate for relative attitude and position estimation [8].

A relative attitude determination (RAD) method for a three vehicle formation has been proposed in reference [9]. Given two directions and an angle measurement, the relative attitude between two spacecraft is determined. However, there are a few limitations: (1) the angle measurement is required to fall within a certain boundary; and (2) the covariance analysis shows that the Fisher information matrix can be singular [9; 10].

A three-axis attitude determination (TRIAD) method was developed by Shuster [11]. This method can be used for RAD between two spacecraft. Consider that two measurements, for the same relative direction between two spacecraft together are called a pair of measurements. The TRIAD method requires two "pairs" of measurements between two spacecraft at two different times to construct the attitude matrix between the two spacecraft.

In this paper, we propose an extension of the TRIAD method to determine the relative attitude between the spacecraft using measurements for the relative directions between all spacecraft in the formation. Unlike the standard TRIAD method, this method is able to determine the relative attitude between spacecraft at any time without multiple observations. We consider the case of a three spacecraft formation, where each spacecraft measures the direction of the other two. Similar to the TRIAD method, this method requires four measurements. The covariance analysis that corresponds to the constraint functions of the method is derived and studied in the paper.

Section II presents the derivation for relative attitude determination. Section III presents the covariance analysis of the proposed method. The simulation is set up and discussed in Section IV.

2. RELATIVE ATTITUDE DETERMINATION

In this section, the algorithm to determine the relative attitude will be presented. Figure 1 shows three spacecraft flying in a formation. Each spacecraft measures the

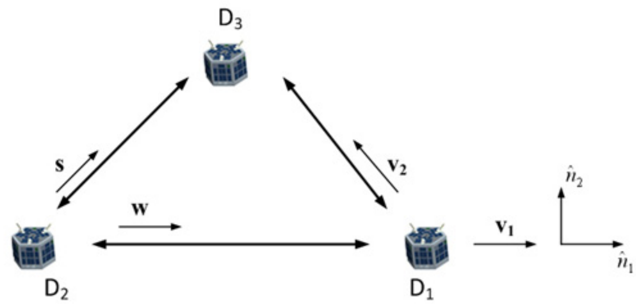

Figure 1. Measurements in three spacecraft formation.

direction of the other two in its respective coordinate reference frame. We define the relative position of the j^{th} spacecraft observed by the i^{th} spacecraft as the vector, $\mathbf{r}_{\mathrm{ij}}^{k}$; where the superscript k designates that the vector compenent are in the k^{th} frame.

The direction of the j^{th} spacecraft observed by i^{th} spacecraft is represented by the unit vector in the direction of $\mathbf{r}_{\mathrm{ij}}^{k}$.

$$\mathbf{p}_{\mathrm{ij}}^{\mathrm{k}} = \frac{\mathbf{r}_{\mathrm{ij}}^{k}}{\parallel \mathbf{r}_{\mathrm{ij}}^{k} \parallel} \tag{1}$$

where the subscript of ij represents the observation of D_j spacecraft by D_i spacecraft, and the superscript k represents the observation vector is expressed in D_k reference frame.

Define the following unit vectors:

$$\mathbf{w} = \mathbf{p}_{21}^{2} \tag{2}$$
$$\mathbf{v}_1 = \mathbf{p}_{21}^{1} \tag{3}$$
$$\mathbf{v}_2 = \mathbf{p}_{13}^{1} \tag{4}$$
$$\mathbf{s} = \mathbf{p}_{23}^{2} \tag{5}$$

Next, we consider the relative attitude determination between the D_1 and D_2 spacecraft. As seen in Figure 1, an intermediate frame (N) is introduced, which can be defined by three unit vectors $\hat{\mathbf{n}}_1$, $\hat{\mathbf{n}}_2$ and $\hat{\mathbf{n}}_3$. $\hat{\mathbf{n}}_1$ is directed from D_2 to D_1, and $\hat{\mathbf{n}}_2$ is perpendicular to $\hat{\mathbf{n}}_1$ and lies in the plane defined by D_1, D_2 and D_3. Finally, $\hat{\mathbf{n}}_3$ is the vector that is normal to the plane, which is $\hat{\mathbf{n}}_3 = \hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2$.

Using the definition of $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_3$, we may write that:

$$\mathbf{v}_1 = A_1 \hat{\mathbf{n}}_1 \tag{6}$$
$$\frac{\mathbf{v}_1 \times \mathbf{v}_2}{\parallel \mathbf{v}_1 \times \mathbf{v}_2 \parallel} = A_1 \hat{\mathbf{n}}_3 \tag{7}$$

where A_1 is the relative attitude matrix, which is also known as the Direction Cosine Matrix [12; 13] between the intermediate (N) frame and D_1. The $\hat{\mathbf{n}}_2$ can be obtained using the cross product between $\mathbf{v}_1$ and $(\mathbf{v}_1 \times \mathbf{v}_2)$, which can be written as:

$$\frac{\mathbf{v}_2 - (\mathbf{v}_2^T \mathbf{v}_1)\mathbf{v}_1}{\parallel \mathbf{v}_2 - (\mathbf{v}_2^T \mathbf{v}_1)\mathbf{v}_1 \parallel} = A_1 \hat{\mathbf{n}}_2 \tag{8}$$

Then, using equations (6) - (8), the relative attitude matrix between the intermediate (N) frame and D_1 is:

$$A_1 = \left[\mathbf{v}_1 \quad \frac{\mathbf{v}_2 - (\mathbf{v}_2^T \mathbf{v}_1)\mathbf{v}_1}{\parallel \mathbf{v}_2 - (\mathbf{v}_2^T \mathbf{v}_1)\mathbf{v}_1 \parallel} \quad \frac{\mathbf{v}_1 \times \mathbf{v}_2}{\parallel \mathbf{v}_1 \times \mathbf{v}_2 \parallel} \right] \tag{9}$$

The relative attitude between the intermediate (N) frame and D_2 may be constructed in a similar way as (9), which is:

$$A_2 = \left[\mathbf{w} \quad \frac{\mathbf{s} - (\mathbf{s}^T \mathbf{w})\mathbf{w}}{\parallel \mathbf{s} - (\mathbf{s}^T \mathbf{w})\mathbf{w} \parallel} \quad \frac{\mathbf{w} \times \mathbf{s}}{\parallel \mathbf{w} \times \mathbf{s} \parallel} \right] \tag{10}$$

The relative attitude matrix between D_2 and D_1 can now be constructed from A_1 and A_2 as given below.

$$A_1^2 = A_2 A_1^T \tag{11}$$

The relative attitude matrices between D_1 and D_3, D_2 and D_3 can be obtained in a similar fashion.

3. Covariance Analysis

In this section, the covariance of relative attitude between two spacecraft is derived. The covariance represents the error boundary for the RAD method. The RAD errors are expected to be within the three sigma covariance boundary.

The relative attitude matrix between two spacecraft have been derived in (9) - (11). Combining both equations (9) - (10), we may write that:

$$\mathbf{w} = A_1^2 \mathbf{v}_1 \tag{12}$$
$$\frac{\mathbf{s} - (\mathbf{s}^T \mathbf{w})\mathbf{w}}{\parallel \mathbf{s} - (\mathbf{s}^T \mathbf{w})\mathbf{w} \parallel} = A_1^2 \frac{\mathbf{v}_2 - (\mathbf{v}_2^T \mathbf{v}_1)\mathbf{v}_1}{\parallel \mathbf{v}_2 - (\mathbf{v}_2^T \mathbf{v}_1)\mathbf{v}_1 \parallel} \tag{13}$$
$$\frac{\mathbf{w} \times \mathbf{s}}{\parallel \mathbf{w} \times \mathbf{s} \parallel} = A_1^2 \frac{\mathbf{v}_1 \times \mathbf{v}_2}{\parallel \mathbf{v}_1 \times \mathbf{v}_2 \parallel} \tag{14}$$

The left hand sides of equations (12) to (14) represent the three virtual body axes attached on spacecraft, D_2. Similarly, the right hand sides of equations (12) to (14) represent the three virtual body axes attached on spacecraft, D_1. Here, we may consider equations (12) to (14) to be the constraint functions. To optimize the solution obtained using equations (12) to (14), we intend to minimize the errors in equations (12) to (14); which are the errors between the left hand side and the right hand side together with the relative attitude obtained. Thus, the cost function that we intend to minimize is:

$$\begin{aligned} J \quad &= \frac{1}{2}(\mathbf{w} - A_1^2 \mathbf{v}_1)^T R_1^{-1} (\mathbf{w} - A_1^2 \mathbf{v}_1) \\ &+ \frac{1}{2}(\mathbf{w}_3 - A_1^2 \mathbf{v}_3)^T R_2^{-1} (\mathbf{w}_3 - A_1^2 \mathbf{v}_3) \\ &+ \frac{1}{2}(\mathbf{w}_4 - A_1^2 \mathbf{v}_4)^T R_3^{-1} (\mathbf{w}_4 - A_1^2 \mathbf{v}_4) \end{aligned} \tag{15}$$

with,

$$\mathbf{w}_3 = \frac{\mathbf{s} - (\mathbf{s}^T\mathbf{w})\mathbf{w}}{\| \mathbf{s} - (\mathbf{s}^T\mathbf{w})\mathbf{w} \|} \quad (16)$$

$$\mathbf{w}_4 = \frac{\mathbf{w} \times \mathbf{s}}{\| \mathbf{w} \times \mathbf{s} \|} \quad (17)$$

$$\mathbf{v}_3 = \frac{\mathbf{v}_2 - (\mathbf{v}_2^T\mathbf{v}_1)\mathbf{v}_1}{\| \mathbf{v}_2 - (\mathbf{v}_2^T\mathbf{v}_1)\mathbf{v}_1 \|} \quad (18)$$

$$\mathbf{v}_4 = \frac{\mathbf{v}_1 \times \mathbf{v}_2}{\| \mathbf{v}_1 \times \mathbf{v}_2 \|} \quad (19)$$

where R_1, R_2 and R_3 are the measurement covariances corresponding to the cost functions in (12) - (14) respectively. R_2 and R_3 in (15) are the nonlinear combination of the four observation vectors.

Assume that the attitude errors are small error angles. Define the $\boldsymbol{\delta\alpha}$ to be the small error angle vector. The estimated attitude that is expressed in terms of the true attitude and $\boldsymbol{\delta\alpha}$ is [9]:

$$A_1^2 = (I_{3\times3} - [\boldsymbol{\delta\alpha}_1^2 \times])A_{1\,true}^2 \quad (20)$$

Substituting (20) into (15), and taking the partial derivative with respect to $\boldsymbol{\delta\alpha}$, the error covariance matrix of (15) corresponds to:

$$\begin{aligned} P_1^2 &= E\{\boldsymbol{\delta\alpha}_1^2 \boldsymbol{\delta\alpha}_1^{2^{\mathrm{T}}}\} \\ &= ([A_1^2\mathbf{v}_1 \times]R_1^{-1}[A_1^2\mathbf{v}_1 \times]^T \\ &+ [A_1^2\mathbf{v}_3 \times]R_2^{-1}[A_1^2\mathbf{v}_3 \times]^T \\ &+ [A_1^2\mathbf{v}_4 \times]R_3^{-1}[A_1^2\mathbf{v}_4 \times]^T)^{-1} \end{aligned} \quad (21)$$

where $E\{.\}$ denotes the expectation, and the operator, $\mathbf{a} \times$ denotes the cross product matrix [3] of vector $\mathbf{a} = [a_1 \quad a_2 \quad a_3]^T$, which is:

$$[\mathbf{a} \times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad (22)$$

Here, the vector $\mathbf{v}_4$ is perpendicular to both vectors $\mathbf{v}_1$ and $\mathbf{v}_3$. Thus the covariance matrix derived in (21) is always non singular. In addition, the covariance analysis shown in (21) is also applicable to the relative attitude determination between D_1 and D_3, and D_2 and D_3.

4. Measurement Covariance

As stated previously, the measurement covariance matrices, R_2 and R_3 in (15) and (21) consist of nonlinear combination of the observation vectors. In addition, both left and right hand side of the observation vectors in (12) - (14) consists of measurement noises. In this section, the measurement covariances associated with (12) - (14) are derived.

First, we define the measurement of the observation vector as:

$$\tilde{\mathbf{p}}_i = \mathbf{p}_i + \boldsymbol{\nu} \quad (23)$$

where $\boldsymbol{\nu}$ denotes the measurement noises.

We assume that there are no cross-correlation between the measurement noises in (23). Thus if the measurement noise is added to the observation vector as shown in (23), the corresponding measurement noise covariance, $\Omega(\tilde{\mathbf{p}}_i)$ is:

$$\Omega(\tilde{\mathbf{p}}_i) = E\{\tilde{\mathbf{p}}_i\tilde{\mathbf{p}}_i^T\} = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \quad (24)$$

where σ_x, σ_y and σ_z are the standard deviation of the measurement noise, $\boldsymbol{\nu}$ for each axes.

The measurement noise covariances for equations (12) - (14) can be written in matrix form as:

$$R = E\{(\mathbf{p}_i - A\mathbf{p}_j)(\mathbf{p}_i - A\mathbf{p}_j)^T\} = \Omega_i + A\Omega_j A^T \quad (25)$$

Equation (25) can be used to find R_1. The observation vectors in both (13) and (14) are the norm of the resultant cross product. The respective covariance of the norm has been derived in the literature [9]. Let $\mathbf{b}$ be a unit vector in the direction of $\mathbf{r}$. Then its covariance is:

$$\Omega_{\mathbf{b}} = \left(\frac{\partial \mathbf{b}}{\partial \mathbf{r}}\right)\Omega_{\mathbf{r}}\left(\frac{\partial \mathbf{b}}{\partial \mathbf{r}}\right)^T \quad (26)$$

where,

$$\frac{\partial \mathbf{b}}{\partial \mathbf{r}} = -\| \mathbf{r} \|^{-3} \, [\mathbf{r} \times]^2 \quad (27)$$

Next, we define that $A \odot B$ represents the elements-by-elements multiplication between matrix A and B, and $M(\mathbf{b})$ represents the matrix of vector $\mathbf{b}$ as:

$$M(\mathbf{b}) = [\mathbf{b} \quad \mathbf{b} \quad \mathbf{b}]^T \quad (28)$$

Then, the covariance of (13) is:

$$\begin{aligned} R_2 &= \Omega_2(\mathbf{w}, \mathbf{s}) \\ &+ A\left(\frac{\partial \mathbf{v}_3}{\partial \mathbf{r}}\right)\Omega_2(\mathbf{v}_1, \mathbf{v}_2)\left(\frac{\partial \mathbf{v}_3}{\partial \mathbf{r}}\right)^T A^T \end{aligned} \quad (29)$$

where the $\mathbf{r}$ represents $\mathbf{v}_2 - \mathbf{v}_2^T\mathbf{v}_1\mathbf{v}_1$, the matrix A represents the relative attitude matrix between D_1 and D_2 that is A_1^2, and the respective covariance of $(\mathbf{b}_j - (\mathbf{b}^T j\mathbf{b}_i)\mathbf{b}_i)$, $\Omega_2(\mathbf{b}_i, \mathbf{b}_j)$ is:

$$\begin{aligned} \Omega_2(\mathbf{b}_i, \mathbf{b}_j) = \; & \Re(\mathbf{b}_j) + \mathbf{b}_j^T\mathbf{b}_i\Re(\mathbf{b}_i) \\ & -\mathbf{b}_j\mathbf{b}_j^T \odot [C(\mathbf{b}_i) + \Re(\mathbf{b}_i)] \\ & -\mathbf{b}_i\mathbf{b}_i^T \odot [C(\mathbf{b}_j) + \Re(\mathbf{b}_j)] \\ & +\mathbf{b}_i\mathbf{b}_i^T \left[(\mathbf{b}_j \odot \mathbf{b}_j)^T \boldsymbol{\sigma}_{\mathbf{b}_i}^2 + (\mathbf{b}_i \odot)^T \boldsymbol{\sigma}_{\mathbf{b}_j}^2\right] \\ & +2\mathbf{b}_j^T\mathbf{b}_i\left(\left[\mathbf{b}_i\mathbf{b}_j^T + \mathbf{b}_j\mathbf{b}_i^T\right] \odot M(\mathbf{b}_i)\right) \end{aligned} \quad (30)$$

with,

$$C(\mathbf{b}_i) = |2[\sigma_{\mathbf{b}_i}^2 \times]| \quad (31)$$

The vector, $\boldsymbol{\sigma}_{\mathbf{b}_i}^2$ represents the diagonal vector of the measurement covariance, $\Re(\mathbf{b}_i)$ in (24).

In a similar manner, R_3 can be calculated as:

$$R_3 = \Omega_3(\mathbf{w}, \mathbf{s}) + A\left(\frac{\partial \mathbf{v}_4}{\partial \mathbf{r}}\right)\Omega_3(\mathbf{v}_1, \mathbf{v}_2)\left(\frac{\partial \mathbf{v}_4}{\partial \mathbf{r}}\right)^T A^T \tag{32}$$

where the $\mathbf{r}$ represents $\mathbf{v}_1 \times \mathbf{v}_2$, the matrix A represents the relative attitude matrix between D_2 and D_1 that is A_1^2, and the covariance between the two cross product vectors, $\mathbf{b}_i \times \mathbf{b}_j$, $\Omega_3(\mathbf{b}_i, \mathbf{b}_j)$ is:

$$\Omega_3(\mathbf{b}_i, \mathbf{b}_j) = S(\mathbf{b}_i)S(\mathbf{b}_i)^T + S(\mathbf{b}_j)S(\mathbf{b}_j)^T \tag{33}$$

with,

$$S(\mathbf{b}_i) = [\mathbf{b}_i \times]\mathrm{e}M(\sigma_{\mathbf{b}_i}) \tag{34}$$

$$\boldsymbol{\sigma}_{\mathbf{b}_i} = [(\sigma_x)_{\mathbf{b}_i} \quad (\sigma_y)_{\mathbf{b}_i} \quad (\sigma_z)_{\mathbf{b}_i}]^T \tag{35}$$

Both R_2 and R_3 that derived in (29) and (32) are singular in this case. However, Shuster has shown that these matrices can be modified to become nonsingular matrices [11; 14]. For example, the matrix R_2 may be replaced by $R_2^{new} = R_2 + \frac{1}{2}\mathbf{b}\mathbf{b}^T trace(R_2)$, where $\mathbf{b} = (\mathbf{s} - \mathbf{s}^T\mathbf{w}\mathbf{w})/(\| \mathbf{s} - \mathbf{s}^T\mathbf{w}\mathbf{w} \|)$. Then, both R_2^{new} and R_3^{new} are invertible.

5. SIMULATION

We consider an isosceles triangle shape of three spacecraft formation, such that $\theta_1 = \theta_2$ in Figure 2. Each spacecraft has its respective absolute attitude matrices with respect to the Earth Center Inertial (ECI) frame. Assume that all spacecraft perform the 3-1-3 orientation [13], where the orientation of D_1 is $A_1(30°, 15°, -20°)$, D_2 is $A_2(25°, 15°, 30°)$ and D_3 is $A_3(30°, 0°, 10°)$. Given the spacecraft absolute orietation is $A_\mathrm{i}(\phi_1, \phi_2, \phi_3)$, the respective 3-1-3 attitude matrix is:

$$A_\mathrm{i}(\phi_1, \phi_2, \phi_3) = \begin{bmatrix} c\phi_3 c\phi_1 - s\phi_3 c\phi_2 s\phi_1 & c\phi_3 s\phi_1 + s\phi_3 c\phi_2 c\phi_1 & s\phi_3 s\phi_2 \\ -s\phi_3 c\phi_1 - c\phi_3 c\phi_2 s\phi_1 & -s\phi_3 s\phi_1 + c\phi_3 c\phi_2 c\phi_1 & c\phi_3 s\phi_2 \\ s\phi_2 s\phi_1 & -s\phi_2 c\phi_1 & c\phi_2 \end{bmatrix} \tag{36}$$

where cϕ denotes cosine and sϕ denotes sine.

For simplicity, we assume that the plane of the formation is parallel to the Equatorial plane. Thus, the observation vector between D_1 and D_3 spacecraft expressed in ECI frame is:

$$\mathbf{p}_{13}^N = [-\cos\theta_1 \quad \sin\theta_1 \quad 0]^T \tag{37}$$

and the observation vector between D_2 and D_3 spacecraft expressed in ECI frame is:

$$\mathbf{p}_{23}^N = [\cos\theta_2 \quad \sin\theta_2 \quad 0]^T \tag{38}$$

The observation vector expressed in the k^{th} spacecraft reference frame is given by $\mathbf{p}_{\mathrm{ij}}^k = A_k \mathbf{p}_{\mathrm{ij}}^N$.

The standard deviation of measurement noise is assumed to be 1 arcsec in each axis. In order to study the effect of shape of the spacecraft formation on the relative attitude determination error, we vary the value of θ_1 from 1 to 89 degrees in increments of 1 degree. θ_3 can be obtained easily since $\theta_2 = \theta_1$, and sum of the inner angles of the triangle is 180 degrees. For each increment of θ_1 , the simulation is run for 1000 iterations. The average of Root Mean Square Error (RMSE) over the 1000 iterations is plotted to study the performance study in the simulation.

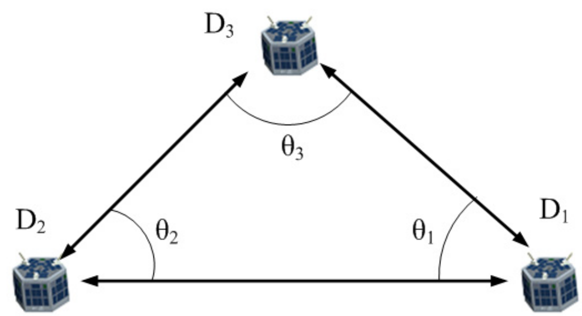

Figure 2. Three Spacecraft Formation Configuration.

Next, the performance of the RAD method is studied with respect to measurement noise. The equilateral formation is considered. The measurement noise varied from 1μrad to 100μrad in increments of 1μrad. The simulation is run over 1000 iterations for each increment of measurement noise. The mean of the absolute error is compared with the analytical error, which is the three sigma boundary of the covariance.

6. DISCUSSION

Figures 3, 4 and 5 show the three sigma boundary of relative attitude error in roll, pitch and yaw angle. The figures show that for 1000 iterations of determination, the relative attitude determination error fall within a three sigma boundary. Thus, the simulation results confirm that the relative attitude between spacecraft can be determined using equations (9) to (11). The simulation trials in Figures 3, 4 and 5 that do not fall within a three sigma boundary are caused by the modification of the measurement covariance discussed in Section 4. The measurement covariance derived in (29) and (32) are singular in nature. However, these matrices can be replaced by other nonsingular matrices as discussed in the section.

Figure 6 shows the change of RMSE of relative attitude determination between the D_2 and D_3 spacecraft when the shape of the spacecraft formation is changed. When θ_1 is very small or θ_3 is very large, that is the D_3 spacecraft is almost colinear with the D_1 and D_2 spacecraft, the RMSE is higher. In addition, the RMSE is also high when θ_1 becomes very large, or θ_3 becomes very small as when D_1 and D_2 spacecraft become very close to each other. In both of these cases, the shape of the formation comes close to a straight line, which is known as colinear. So, two out of the four measurements required by this method become redundant. The proposed method is capable of providing accurate results that are below 100 μrad when the three spacecraft are not colinear.

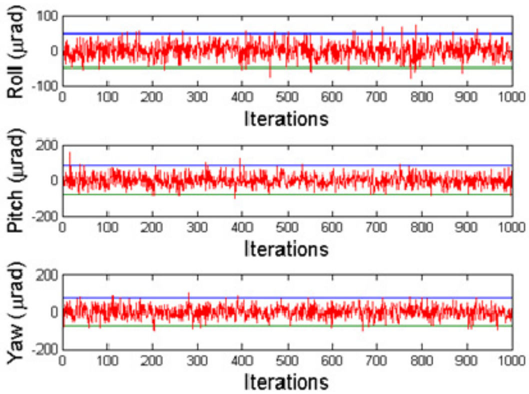

Figure 6. Three axes relative attitude error of A_3^1.

Figure 7 compares the analytical and simulation error with respect to measurement noises. The measurement noises in the figure are expressed in terms of standard deviation. The analytical error is obtained through the determination of the three sigma boundary using the covariance of relative attitude between two spacecraft (derived in (21)). As expected, both analytical and simulation errors increase as the measurement noise increase. In Figure 7, only the roll axis of relative attitude between spacecraft 2 and spacecraft 3 is considered. The pitch and yaw axes provide the same characteristic as the roll axis. The three sigma boundary represents the boundary that the RAD error would fall within. The average simulated error for each increment is lower than the analytical error, which is expected.

In the literature, the accuracy of estimated absolute attitude is 1×10^{-5} degrees of error in each axis [15]. The theoretical relative attitude errors using the estimated absolute attitude is determined using the quaternion multiplication properties, which are $\hat{q} = q \otimes \delta q$ and $q_{12} = q_2 \otimes q_1^{-1}$ where δq is the attitude error in quaternion [12; 16].

The accuracy performance between the proposed RAD method and relative attitude determination using the

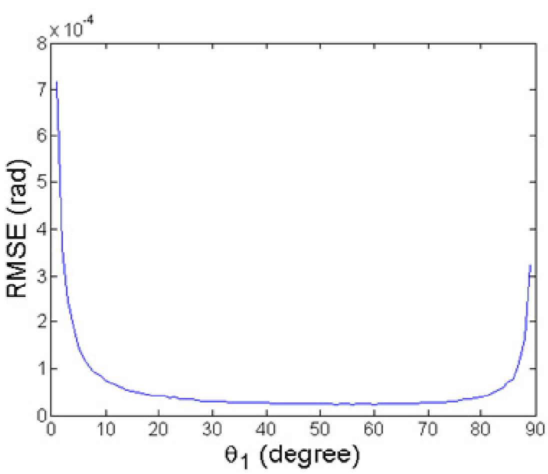

Figure 5. The RMSE of A_3^2 with respect to change of θ_1.

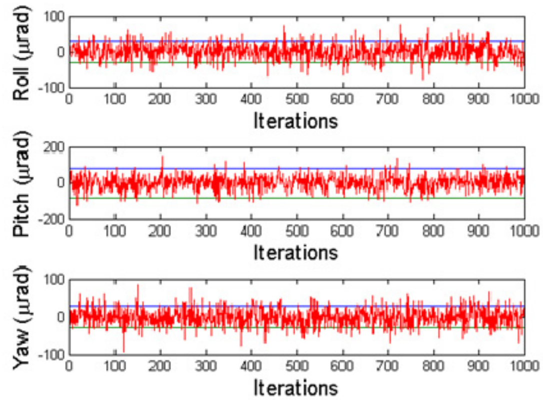

Figure 3. Three axes relative attitude error of A_3^2.

estimated absolute attitude is compared. Figures 3 to 5 show the accuracy of proposed RAD method falls between 100 and 200 μrad (0.005 to 0.11 degrees). However, if the accuracy of absolute attitude is 1×10^{-5} degrees of error, the relative attitude errors are in between 1×10^{-4} degrees, in each axis. This may increase the cost in terms of the spacecraft structure. Also, the proposed RAD method requires 15 multiplication steps to calculate the relative attitude matrix. In most of the cases, the absolute attitude is expressed in term of the quaternion vectors. This requires 12 multiplication steps to calculate the relative quaternion vectors, and another 12 multiplication steps to calculate the relative attitude using the relative quaternion vector; a total of 24. Therefore, determining the relative attitude directly from the relative measures would cost less computation that in absolute attitude were used.

7. Conclusion

A method to determine the relative attitude between spacecraft in a three spacecraft formation has been introduced in this paper. A covariance analysis was derived. The covariance analysis and its comparison shows that the

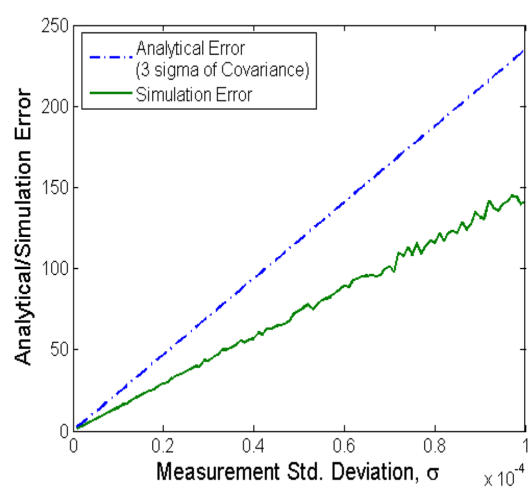

Figure 4. Comparison between Analytical and Simulation Error with respect to measurement noise standard deviation.

relative attitude determination errors fall within a three sigma error boundary. As expected, the error increases significantly when the formation becomes close to a straight line. The proposed method is capable of providing an accurate and consistent result if the three spacecraft are not close to being colinear.

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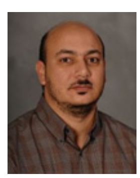

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