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<td>Author(s)</td>
<td>Cheng, Z. Y.; Leong, Eng Choon</td>
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Estimation of P-wave velocities for unsaturated geo-materials

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ABSTRACT: Using P-wave velocity as a proxy for degree of saturation is useful in many applications. The effect of saturation on P-wave velocity of geo-materials has been frequently modeled using Biot-Gassmann equation. However Biot-Gassmann equation is not able to fully account for the effect of partial saturation. In this study, the effects of degree of saturation on P-wave velocity of rocks are investigated. Modifications to the Biot-Gassmann equation are then suggested to improve the estimation of P-wave velocities for unsaturated rocks. The modifications are evaluated with several data sets and were found to provide an improved estimate of P-wave velocity for unsaturated rocks to within ±10%.

KEYWORDS: P-wave; S-wave; velocity; degree of saturation; unsaturated.

1 INTRODUCTION

Primary (P) and secondary (S) waves have long been used in subsurface exploration due to their ability to reflect properties like material types, structure, porosity and degree of fluid saturation. For unsaturated geo-materials, P-wave velocity ($V_p$) can be used as a proxy for degree of saturation if a good correlation between $V_p$ and degree of saturation is obtained. To date, most correlations are either purely empirical or based on a modification of the famous Biot-Gassmann equation. An empirical correlation like that described in Han et al. (1986) is limited to a narrow range of geo-materials. On the other hand, some suggested modifications of the Biot-Gassmann equation are too complicated to be easily employed. For example, Naesgaard et al. (2007) and Yang (2005) suggested the use of Skempton pore-pressure parameter B to estimate $V_p$ at various degrees of saturation. However, evaluations by Naesgaard et al. (2007) have shown that $V_p$ is independent of Skempton pore-pressure parameter B at non-homogeneous partial saturation as the P-wave will route through the stiffer saturated zones leading to an inaccurate representation of the bulk modulus. Conte et al. (2009) introduced the soil-water characteristic curve (SWCC) into the Biot-Gassmann equation. However, SWCC varies greatly depending on the material type and structure of the geo-material. Extensive laboratory measurement will also need to be undertaken to derive the SWCC for the specific type of geo-material hence making attempts to use this equation tedious and time consuming.

Biot-Gassmann equation provides a good basis for development of $V_p$ equation for unsaturated geo-materials. Such an equation will not only be beneficial in site investigation and mineral prospecting (e.g. coal and oil) but also in laboratory tests for unsaturated geo-materials where $V_p$ can be used as a proxy for degree of saturation. In this paper, a brief review of the Biot-Gassmann equation is first given followed by development of a $V_p$ equation to account for degree of saturation. The derivation of the various parameters of the $V_p$ equation are illustrated for rocks followed by evaluation of the $V_p$ equation using independent data sets.

2 REVIEW OF BIOT-GASSMANN EQUATION

From elastic theory, P-wave velocity ($V_p$) and S-wave velocity ($V_s$) of a material are:

$$V_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}}$$

(1)

$$V_s = \sqrt{\frac{G}{\rho}}$$

(2)
where $K$ is the bulk modulus of the material, $G$ is the shear modulus of the material and $\rho$ is the bulk density of the material. For an unsaturated three-phase (air, liquid, solid) geo-material, $\rho$ is given by the following equation:

$$\rho = (1-n)\rho_a + S_r \rho_w + (1-S_r)\rho_s$$  \hspace{1cm} (3)$$

where $n$ is the porosity, $S_r$ is the degree of saturation, $\rho_a$ is the density of the mineral solid, $\rho_w$ is the density of the liquid phase (liquid is usually water in most applications i.e. $\rho_w = 1000 \text{ kg/m}^3$) and $\rho_s (=1.2 \text{ kg/m}^3)$ is the density of the gaseous phase.

The shear modulus of the geo-material $G$ can be easily obtained using Equation 2 if $V_s$ is measured and bulk density is obtained using Equation 3. Therefore to calculate $V_p$ using Equation 1, bulk modulus of the geo-material $K$ is needed. Bulk modulus $K$ is a function of various variables like porosity, composition, mineralogy, matrix stiffness and the bulk moduli of the different mineral grains (Santamarina, 2001). In addition, $V_p$ is highly dependent on the degree of saturation.

In Biot-Gassmann equation, $K$ consists of bulk modulus of the matrix, $K_m$, and a pore-space modulus $M$ as shown below:

$$K = K_m + \alpha^2 M$$  \hspace{1cm} (4)$$

$$\frac{1}{M} = \frac{(\alpha - n)}{K_g} + \frac{n}{K_f}$$  \hspace{1cm} (5)$$

$$\alpha = 1 - \frac{K_m}{K_g}$$  \hspace{1cm} (6)$$

where $K_g$ is the bulk modulus of the mineral grains, $K_f$ is the bulk modulus of the interstitial fluids, and $\alpha$ is the Biot-Willis coefficient. In the derivation of $K$, Gassmann (1951) assumed that: (i) the medium is homogeneous and isotropic; (ii) the pores are interconnected (i.e. no occluded pores) and pressure is in equilibrium; (iii) the pore fluid is frictionless (i.e. low-viscosity fluid); (iv) relative motion between fluid and solid during propagation of the wave is negligible (applicable for low frequencies wave only); and (v) the pore fluid does not interact with the solid matrix mineral (i.e. shear modulus remains constant).

Many of these assumptions are not valid in reality. For example, most geo-materials are anisotropic due to preferential alignment of minerals or cracks as seen in Winkler et al. (1995). Also, geo-materials that have undergone repeated drying and wetting processes would not be able to produce homogeneous saturation. Moreover, Mavko (1998) have shown that the Biot-Gassmann equation works best at low seismic frequency preferably no more than 100 Hz. However, this limitation is frequently violated as seismic surveys are usually performed using frequencies in the kHz range. Finally, the Biot-Gassmann equation is only applicable for a two-phase medium. Unsaturated geo-materials are three phase materials which consist of solid, liquid and air phases. Violations of assumptions will cause theoretical predictions of $V_p$ to deviate from experimental results.

3 DEVELOPMENT OF $V_P$ EQUATION

Starting from Biot-Gassmann equation and assuming $K_g$ is much greater than $K_m$ (i.e. $\alpha \approx 1$), $V_p$ equations for a geo-material at zero degree of saturation and at full saturation are, respectively:

$$V_p = \sqrt{\frac{K_m + \frac{4}{3}G + \frac{K_g}{n}}{\rho}}$$  \hspace{1cm} (7a)$$

$$V_p = \sqrt{\frac{K_m + \frac{4}{3}G + \frac{K_w}{n}}{\rho}}$$  \hspace{1cm} (7b)$$

where $K_g$ is bulk modulus of air (=142 kPa) and $K_w$ is bulk modulus of water (=2.18 GPa). However, $\alpha \approx 1$ is too restrictive and any violation of the assumptions of Biot-Gassmann equation may invalidate the expression for $\alpha$. In view of this, a factor $I_F$ is introduced and Equation 7 can be rewritten in the following general form:

$$V_p = \sqrt{\frac{K_m + \frac{4}{3}G + I_F K_f}{\rho}}$$  \hspace{1cm} (8)$$

where $K_f$ is the bulk modulus of the fluid(s) in the pore space.

Values of $K_m$ and $G$ can be obtained if $V_p$ and $V_s$ of the geo-material at zero degree of saturation are
available or if $V_s$ of the geo-material at zero degree of saturation and Poisson’s ratio $\nu$ are available. If $V_p$ and $V_s$ of the geo-material at zero degree of saturation are available, $G$ can be obtained using Equation 2 and $K_m$ can be obtained as

$$K_m = V_p^2 \rho - \frac{4}{3} G$$

(9)

If $V_s$ of the geo-material at zero degree of saturation and Poisson’s ratio $\nu$ are available, $G$ can be obtained using Equation 2 and $K_m$ can be obtained as

$$K_m = \frac{2G(1+\nu)}{3(1-2\nu)}$$

(10)

For degree of saturation other than zero and full, $K_f$ is the bulk modulus of the fluid constituents occupying the pore space. To estimate $K_f$, Reuss and Voigt models as shown in Equations 11 and 12, respectively, are often used.

$$\frac{1}{K_f} = \frac{1-S_r}{K_a} + \frac{S_r}{K_w}$$

$$K_f = S_t K_w + (1-S_t) K_a$$

(11)

(12)

Modeling $K_f$ according to Ruess model (Equation 11) is analogous to treating both the liquid and gaseous phases as springs connected in series while modeling $K_f$ according to Voigt model (Equation 12) is analogous to treating both the liquid and gaseous phases as springs connected in parallel. According to Avseth et al. (2005), the Ruess model gives the lower bound and the Voigt model gives the upper bound as there is no way that nature can put together a mixture elastically softer or stiffer than the Ruess model and Voigt model, respectively.

While Equations 11 and 12 are useful upper and lower bounds for $K_f$, in reality the variation of $K_f$ with $S_t$ for a geo-material is more complicated as it depends on their matrix as well as their mineral type. Therefore it is proposed that $K_f$ is given by the following equation:

$$K_f = (1-\beta)K_a + \beta K_w$$

(13)

where $\beta$ is a function of degree of saturation and varies between zero and unity. One possible form for $\beta$ is

$$\beta = \left(\frac{1}{1 + [a(1-S_t)]^c}\right)^b$$

(14)

where $a$, $b$, and $c$ are coefficients dependent on material type and properties.

For the derivation of $I_F$ and $\beta$, several sets of rock data are collated from the literature as summarized in Table 1. The data sets consist of sedimentary (limestone, sandstone, travertine), igneous (granite) and metamorphic (marble, serpentinite) rocks. The information available from these data sets include $V_p$ at various degrees of saturation, $V_s$ at various degrees of saturation or shear modulus $G$, porosity $n$, bulk modulus of mineral grain $K_g$, specific gravity $G_s$ and bulk density $\rho$.

<table>
<thead>
<tr>
<th>No. of Data Sets</th>
<th>Description</th>
<th>Reference</th>
</tr>
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<tbody>
<tr>
<td>2 Fort Union Sandstone, Schuler-Cotton Valley Sandstone</td>
<td>Murphy (1982a)</td>
<td></td>
</tr>
<tr>
<td>1 Massilon Sandstone</td>
<td>Murphy (1982b)</td>
<td></td>
</tr>
<tr>
<td>1 Spirit River Sandstone</td>
<td>Knight et al. (1990)</td>
<td></td>
</tr>
<tr>
<td>1 Estailades Limestone</td>
<td>Cadoret et al. (1995)</td>
<td></td>
</tr>
<tr>
<td>6 Granite, Travertine, Marble, Serpentinite</td>
<td>Kahraman (2007)</td>
<td></td>
</tr>
</tbody>
</table>

### 3.1 Derivation of $I_F$

Saturation has been known to stiffen the bulk modulus of geo-materials and hence enhance the propagation of $P$-waves. To obtain $I_F$, $V_p$ of dry ($S_t = 0$) and saturated ($S_t = 1$) rocks are examined. The values of $I_F$ are back-calculated and plotted against porosity $n$ in Figure 1. Figure 1 shows that generally the higher the porosity of the rock the larger is the value of $I_F$ with the exception of two high porosity sedimentary rocks. Differences in $V_p$ between high and low porosity sedimentary rocks were also noted by Murphy (1982a) however no porosity value on the demarcation of low and high porosity sedimentary rocks was given and no mention was made on how to account for the differences. The data in Figure 1 indicate that the increase in $I_F$ with $n$ forms....
two parallel lines with the $I_F$ values for metamorphic, igneous and low porosity sedimentary rocks lying on one line to the left while the $I_F$ values for high porosity sedimentary rocks lying on the line to the right (Figure 1). Equations of the respective lines are shown in Equations 15 and 16.

$$I_F = 9.596n + 0.0272$$  \hspace{1cm} (15)  

$$I_F = 9.596n - 2.3$$  \hspace{1cm} (16)

---

Figure 1. Correlation between $I_F$ and porosity $n$.

3.2 Derivation of $\beta$

To obtain the coefficients $a$, $b$ and $c$ for $\beta$, the values of $\beta$ according to Equations 8 and 13 were back-calculated for the rock data sets. The values of the coefficients $a$, $b$ and $c$ were then determined according to Equation 14. Cross-correlations of coefficients $a$, $b$ and $c$ and correlations of coefficients $a$, $b$ and $c$ with rock properties ($K_g$, $G_s$, $n$, $\rho$ and combinations) were examined. The highest correlations are illustrated in Figures 2 to 4 and the best-fit equations for the correlations are shown in Equations (17) to (19), respectively.

$$c = 20.855e^{-15.385n}$$  \hspace{1cm} (17)  

$$b = 29.031c^{-0.4634}$$  \hspace{1cm} (18)  

$$a = 8.8085c^{-0.5821}$$  \hspace{1cm} (19)  

---

Figure 2. Correlation between coefficient $c$ and porosity $n$.

Figure 3. Correlation between coefficients $c$ and $a$.

Figure 4. Correlation between coefficients $c$ and $b$.  

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4 EVALUATION OF PROPOSED $V_p$ EQUATION

The proposed $V_p$ equation is evaluated with two independent data sets as summarized in Table 2. The coefficients $a$, $b$, and $c$ were obtained from Equations 16 to 18, respectively, and the value of $I_F$ was obtained using either Equation 15 or Equation 16. Figures 5 and 6 show that the predicted values of $V_p$ is very close to the experimental values of $V_p$ with errors much less than 10% most of the time.

Table 2. Data for evaluation of $V_p$ equation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sandstone from Chen et al. (1991)</th>
<th>Sierra White Granite from Murphy (1985)</th>
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<tbody>
<tr>
<td>Porosity, $n$</td>
<td>0.33</td>
<td>0.008</td>
</tr>
<tr>
<td>Mineral density, $\rho_s$</td>
<td>2650 kg/m$^3$</td>
<td>2661 kg/m$^3$</td>
</tr>
<tr>
<td>Shear modulus, $G$</td>
<td>3.29 GPa</td>
<td>26.40 GPa</td>
</tr>
<tr>
<td>Mineral bulk modulus, $K_g$</td>
<td>40 GPa</td>
<td>56 GPa</td>
</tr>
<tr>
<td>Coefficient $a$</td>
<td>28.87</td>
<td>1.62</td>
</tr>
<tr>
<td>Coefficient $b$</td>
<td>74.72</td>
<td>7.53</td>
</tr>
<tr>
<td>Coefficient $c$</td>
<td>0.13</td>
<td>18.44</td>
</tr>
<tr>
<td>$I_F$</td>
<td>0.867</td>
<td>0.10</td>
</tr>
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</table>

Figure 5. Predicted $V_p$ vs $S_r$ for Sandstone by Chen et al. (1991).

Figure 6. Predicted $V_p$ vs $S_r$ for Sierra White Granite by Murphy (1985).

5 CONCLUSION

An equation to estimate $V_p$ for geo-materials at various degrees of saturation is proposed. Method to obtain the parameters in the $V_p$ equation was illustrated for rocks. The proposed $V_p$ equation was evaluated with two independent rock data sets. Very good agreement between estimated and experimental values of $V_p$ was obtained, with errors much less than 10% most of the time. The robustness of the proposed $V_p$ equation need to be evaluated with more extensive rock data sets. Further research is also needed to examine the validity of the proposed $V_p$ equation for unsaturated soils.

ACKNOWLEDGMENTS

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