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Affine Nonmagnetic Transformation Design and its Application

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Abstract: Based on the transformation optical design which possesses simultaneously unitary permeability and piece-wise homogeneity, transformation-optics devices can practically avoid magnetism and inhomogeneous parameters, and be implemented directly using existing dielectric materials with ideal performance.

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Transformation optics is applicable to invisibility cloaking [1, 2] and a broad variety of electromagnetic wave converters [3-5], and is moving from theoretical study to practical use. However, the original implementation of transformation-optics device necessitates the utilization of media that are both inhomogeneous and anisotropic. To address this challenge, homogeneous transformation methods [6, 7] were recently proposed and experimentally demonstrated [8-10]. A second limitation to date has still remained: The transformation typically results in nonunit values for the magnetic permeability \( \mu \), which, except being renormalization with scattering trade-off [11], is not known generally how to implement in optical wavelengths with conventional optical materials.

To tackle this problem, we introduce a transformation optical design which possesses simultaneously a unitary permeability (i.e. nonmagnetism) and homogeneous permittivity. Our design, admitting both macroscale and nanoscale fabrication, can be achieved using natural birefringence materials such as Calcite [8, 9], or via subwavelength anisotropic patterning [12, 13]. Moreover, our geometrical approach simplifies the problem to one of graphical design that in many cases can be carried out analytically.

Consider a general two-dimensional (2D) affine coordinate transformation \( x' = ax + by + c \), \( y' = dx + ey + f \), from triangle \( AOB \) to \( AOB' \) in the \( xy \) plane, as shown in Fig. 1. The associated Jacobian matrix is \( \mathbf{J} = [a, b, d, e] \). For transverse magnetic (TM) modes where the \( \mathbf{H} \) field is perpendicular to the \( xy \) plane, we obtain the transformed relative dielectric permittivity and permeability as \( \varepsilon' = \frac{\mathbf{J} \cdot \mathbf{J}'}{\det(\mathbf{J})}, \mu' = \frac{1}{\det(\mathbf{J})} \). We can achieve unitary permeability \( \mu' \) by imposing a unitary Jacobian \( \det(\mathbf{J}) = 1 \) [14]. The geometric interpretation is that the transformation is area-preserving, such that the area of the triangle \( AOB \) is always equal to that of \( AOB' \).

As a special case (\( \angle AOB = 90^\circ \)) of the horizontal shear area-preserving (No transformation imposed along \( y \) direction) transformation (APT), we introduce the application of a 2D surface plasmonic (SP) resonator based on transverse magnetic (TM) bending adapters. Its APT method is illustrated in Fig. 2(a). The rectangular region \( AOBC \) is a part of a rectangular planar waveguide. To form one arm of the bending adapter, we transform \( \Delta AOB \) to the new triangle \( \Delta AOB' \) where the angle \( \alpha \) is the half-bending angle of the adapter. The final bending angle is \( 2\alpha \), formed by mirroring the structure with regard to axis \( BO' \), as shown in the inset.

Figure 1. (Color online) the area-preserving transformation from triangle \( AOB \) (blue) to \( AOB' \) (green) along both \( x \) and \( y \) directions.

![Figure 1](https://example.com/image1.png)
figure. Without loss of generality, we set length $|OBI|=1$, and fix the point $A$ on the horizontal axis at location $(-L, 0)$. To impose the nonmagnetic condition we require that the area of triangle $AOB$ should equal that of $AO'B$. Thus, $O'$ should be placed such that $OO'\parallel AB$, i.e. $x_r+y_r=0$. The Jacobian is thus unitary, leading to $\mu'=1$. An equilateral polygonal resonator can be formed by using multiple bends under the condition $p2\alpha=\pi$ [Fig. 2(b)], where $p$ is an integer. Furthermore if metallic material such as silver is placed in the center, SP resonance can be excited at the interface between metal and dielectrics [15-17], as shown in Fig. 2(c).

![Figure 2](JTh2A.79.pdf)

**Figure 2.** (Color online) (a) APT (from $AOB$ to $AB'O'$) for one arm of the bending adapter. (b) A transformation optics based polygonal resonator formed by multiple bending adapters. (c) The distribution of magnetic field of a transformation optics based SP hexagonal resonator.

In summary, based on the transformation optical design which possesses simultaneously unitary permeability and piece-wise homogeneity, transformation-optics devices can practically avoid magnetism and inhomogeneous parameters, and be implemented directly using existing dielectric materials with ideal performance.

**References**