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<td>Fang, Cheng; Butler, David Lee</td>
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An innovative method for coordinate measuring machine one-dimensional self-calibration with simplified experimental process

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Citation: Rev. Sci. Instrum. 84, 055103 (2013); doi: 10.1063/1.4801953
View online: http://dx.doi.org/10.1063/1.4801953
View Table of Contents: http://rsi.aip.org/resource/1/RSINAK/v84/i5
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An innovative method for coordinate measuring machine one-dimensional self-calibration with simplified experimental process

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(Received 22 January 2013; accepted 1 April 2013; published online 3 May 2013)

In this paper, an innovative method for CMM (Coordinate Measuring Machine) self-calibration is proposed. In contrast to conventional CMM calibration that relies heavily on a high precision reference standard such as a laser interferometer, the proposed calibration method is based on a low-cost artefact which is fabricated with commercially available precision ball bearings. By optimizing the mathematical model and rearranging the data sampling positions, the experimental process and data analysis can be simplified. In mathematical expression, the samples can be minimized by eliminating the redundant equations among those configured by the experimental data array. The section lengths of the artefact are measured at arranged positions, with which an equation set can be configured to determine the measurement errors at the corresponding positions. With the proposed method, the equation set is short of one equation, which can be supplemented by either measuring the total length of the artefact with a higher-precision CMM or calibrating the single point error at the extreme position with a laser interferometer. In this paper, the latter is selected. With spline interpolation, the error compensation curve can be determined. To verify the proposed method, a simple calibration system was set up on a commercial CMM. Experimental results showed that with the error compensation curve uncertainty of the measurement can be reduced to 50%. © 2013 AIP Publishing LLC.

[http://dx.doi.org/10.1063/1.4801953]

\textbf{I. INTRODUCTION}

Software-based error compensation, with its high performance-cost ratio, has become one of the key technologies to improve the accuracy for commercially available CMMs (Coordinate Measuring Machines).\textsuperscript{1,2} The basic idea of error compensation is to calibrate the measuring errors with a reliable benchmark. The laser due to its stable wavelength has become one of the benchmarks and accordingly laser interferometry has become one of the more popular standards in this field.\textsuperscript{3–5} However, the volume size and the sensitivity to the environment have limited its application in high-precision CMMs. Especially for those with long-duration measurements, the reading drift caused by temperature or airflow cannot be neglected.\textsuperscript{6} Another method is to calibrate the errors with standard parts, such as a ballbar,\textsuperscript{7} hole-ball plate,\textsuperscript{8,9} double ring string (DRS),\textsuperscript{10} and other non-standardized shapes.\textsuperscript{11,12} In these applications, the performance of the calibration relies heavily on the fabrication accuracy or the declared dimension of the standard parts themselves. In order to overcome the above disadvantages, the concept of self-calibration, once used in micro-lithography industry, where no higher precision standard is available,\textsuperscript{13} has been introduced in the area of CMM performance enhancement.\textsuperscript{14}

The principle of self-calibration, which has long been used in forms of reversal measurement,\textsuperscript{15} rotation method,\textsuperscript{16} or permutation technique,\textsuperscript{17} is based on the rule that a particular geometric parameter should have the same measurement results regardless of its position or direction in the measuring space.\textsuperscript{18,19} Specifically, the main work of CMM self-calibration focuses on the separation of errors of the calibrated machine with those from artefact itself.\textsuperscript{14,20}

The word self-calibration, however, has some misleading connotation that the whole process of calibration can be done without any external information. In fact, unlike angular parameter measurement, which has an absolute reference of 360°, linear parameter measurement must introduce some external information as the benchmark. Sometimes this process appears to be pure self-calibration because some external information is seen to be believable.\textsuperscript{14,19} However, it should be recognized that to reduce the dependence on external instrumentation is a challenging job for self-calibration.

For the artefact-based calibration, the step-by-step shifting mode is seen to be a conventional method,\textsuperscript{20} but it introduces too many redundant measurements, which renders the process to be time-consuming especially when extended to 2D or 3D applications. In this study, with a 1D artefact design and calibration, the authors intend to demonstrate through a theoretical approach that measurement uncertainty can be reduced using a calibrated machine and artefact. While internal self-calibration involves the machine and artefact and is important in relative measurement, the regular offset which increases with the measuring range can be reduced by an external measurement. Furthermore, by optimising the sampling positions both the internal and external measurements can be significantly simplified.

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II. PRINCIPLE OF 1D SELF-CALIBRATION AND DESIGN OF THE 1D ARTEFACT

With a mature fabrication process and fitting algorithm, spherical surfaces are often used as standard units in the design of artefacts for CMM calibration. In this study, the mathematic model is built based on a 1D ball bar fabricated with commercial ball bearings.

A. Design of 1D artefact

The 1D artefact as shown in Fig. 1 has 5 individual section lengths \( L_1, L_2, L_3, L_4, \) and \( L_5 \), defined by the centres of the 5 stainless steel ball bearings with 1 mm diameter and 10 mm distances in between. The ball bearings are fixed into hemispherical holes. The section lengths cannot be used as the benchmark because the location error of each sphere centre needs to be separated or compensated.

B. Shifting method and redundancy analysis

Figure 2 shows the ordinary step-by-step shifting mode.20 The same geometric parameters, the section lengths, are measured on different positions in the calibration range. The measurement results will include both the errors from the machine and those from the artefact. By compensating the artefact errors in the mathematic operation, the machine errors can be separated.

The redundancy can be easily seen in this method. For example, the points \( P_0 \) and \( P_1 \) have been measured only once by length \( L_1 \), but those between \( P_2 \) and \( P_5 \) have been measured by different sections for 5 times, which means they have different “weightage” in the whole process. If the information is sufficient for the calibration of \( P_0 \) and \( P_1 \), there must be some redundancy for other calibrated points. The theoretical analysis of the redundancy helps to find out the best rearrangement of the data sampling positions. In the theoretical analysis, the parameters are defined as follows:

- \( L_i \): The real length of the \( i \)th section on the artefact, with the unknown fabrication error.
- \( L_{i,j} \): The measured length of the \( i \)th section on the artefact in the experiment of \( j \)th shift.
- \( E_k \): The machine error on the \( k \)th sampling point. On the original point, \( E_0 = 0 \).
- \( R_k \): The reading of uni-axis coordinate on the \( k \)th sampling point.

So on \( j \)th shift when \( L_i \) is located between \( P_k \) and \( P_{k+1} \), the error equation can be given as

\[
L_i = (R_{k+1} - E_{k+1} - R_k + E_k) = L_{i,j} + E_k - E_{k+1}.
\]

Accordingly, five individual equations for \( L_1, L_2, L_3, L_4 \), and \( L_5 \) can be derived from shift 0, and another five from shift 1. Among these ten independent equations, there are a total of 11 unknown parameters: five section lengths \( L_1-L_5 \) and six single point errors \( E_1-E_6 \). It is apparent that if any one of the unknown parameters is seen as a constant, this equation set has the sole solution, which means that the first two shifts are only one equation short. After that each shift will introduce only one more unknown parameter but five more equations. Thus, the redundancy increases with an increasing number of shifts.

From the above, the equation set configured with this step-by-step mode has some over determined conditions although it still has at least one equation short.

C. Rearrangement of the sampling positions

From Sec. II B, in order to avoid the redundancy, the number of equations should increase with that of the unknown parameters synchronously. Figure 3 shows the improved arrangement of the sampling positions, in which after shift 1 the artefact will move a total length of itself and accordingly each shift introduces five new equations with five unknown parameters.

Generally, if the artefact has \( m \) sections and the calibration has in total \( n \) shift steps, the number of machine errors that should be calibrated (exclusive of the original point) is

\[
(n-1)m + 1.
\]

Considering the \( m \) unknown section lengths, the total number of unknown parameters is

\[
(n-1)m + 1 + m = nm + 1.
\]

![FIG. 1. 1D artefact.](image1)

![FIG. 2. Step-by-step shifting mode.](image2)

![FIG. 3. Rearrangement of the sampling positions.](image3)
Each shift step introduces \( m \) equations, so the total equation number is \( nm \), which means the error equation set is short of one equation. A supplemental equation can be configured with a dimensional parameter which can be calibrated using a higher precision CMM, or a single point error of the machine which can be calibrated by a laser interferometer. The following analysis is based on the latter.

### III. SOLUTION OF THE ERROR EQUATION SET

For mathematical brevity, the case of Fig. 3 is taken for instance. The error equations for each shift step are listed as below

**Shift 0:**

\[
L_{0,1} = L_1 + E_1 - E_0 = L_1 + E_1, \quad (4)
\]

\[
L_{0,2} = L_2 + E_2 - E_1, \quad (5)
\]

\[
L_{0,3} = L_3 + E_3 - E_2, \quad (6)
\]

\[
L_{0,4} = L_4 + E_4 - E_3, \quad (7)
\]

\[
L_{0,5} = L_5 + E_5 - E_4. \quad (8)
\]

**Shift 1:**

\[
L_{1,1} = L_1 + E_2 - E_1, \quad (9)
\]

\[
L_{1,2} = L_2 + E_3 - E_2, \quad (10)
\]

\[
L_{1,3} = L_3 + E_4 - E_3, \quad (11)
\]

\[
L_{1,4} = L_4 + E_5 - E_4, \quad (12)
\]

\[
L_{1,5} = L_5 + E_6 - E_5. \quad (13)
\]

**Shift 2:**

\[
L_{2,1} = L_1 + E_7 - E_6, \quad (14)
\]

\[
L_{2,2} = L_2 + E_8 - E_7, \quad (15)
\]

**Shift 3:**

\[
L_{3,1} = L_1 + E_{12} - E_{11}, \quad (19)
\]

\[
L_{3,2} = L_2 + E_{13} - E_{12}, \quad (20)
\]

\[
L_{3,3} = L_3 + E_{14} - E_{13}, \quad (21)
\]

\[
L_{3,4} = L_4 + E_{15} - E_{14}, \quad (22)
\]

\[
L_{3,5} = L_5 + E_{16} - E_{15}. \quad (23)
\]

Theoretically, when any one of \( \{E_i\} \) is given, the equation set composed by (4)–(23) has the sole solution. But considering the error curve normally has a notable linear part, the point for external calibration should be selected close to the farthest point. In the following analysis, \( E_{15} \) is given constant value \( E \), and then a repeatable and programmable mathematical process is derived

\[
(4) + (5) + (6) + (7) + (8):
\]

\[
L_{0,1} + L_{0,2} + L_{0,3} + L_{0,4} + L_{0,5} = L + E_5 - 0, \quad (24)
\]

\[
(13) + (14) + (15) + (16) + (17):
\]

\[
L_{1,5} + L_{2,1} + L_{2,2} + L_{2,3} + L_{2,4} = L + E_{10} - E_5, \quad (25)
\]

\[
(18) + (19) + (20) + (21) + (22):
\]

\[
L_{2,5} + L_{3,1} + L_{3,2} + L_{3,3} + L_{3,4} = L + E_{15} - E_{10}. \quad (26)
\]

Where \( L \) is the total length of the artefact.

With the given value \( E_{15} = E \), the unknown parameter \( L \), \( E_5 \), and \( E_{10} \) can be worked out

\[
L = \frac{L_{0,1} + L_{0,2} + L_{0,3} + L_{0,4} + L_{0,5} + L_{1,5} + L_{2,1} + L_{2,2} + L_{2,3} + L_{2,4} + L_{2,5} + L_{3,1} + L_{3,2} + L_{3,3} + L_{3,4} - E}{3}, \quad (27)
\]

\[
E_5 = \frac{2L_{0,1} + 2L_{0,2} + 2L_{0,3} + 2L_{0,4} + 2L_{0,5} - L_{1,5} - L_{2,1} - L_{2,2} - L_{2,3} - L_{2,4} - L_{2,5} - L_{3,1} - L_{3,2} - L_{3,3} - L_{3,4}}{3}, \quad (28)
\]

\[
E_{10} = \frac{2E_{0,1} + 2L_{0,2} + 2L_{0,3} + 2L_{0,4} + 2L_{0,5} + L_{1,5} - L_{2,1} + L_{2,2} + L_{2,3} + L_{2,4} + 2L_{2,5} - 2L_{3,1} - 2L_{3,2} - 2L_{3,3} - 2L_{3,4}}{3}. \quad (29)
\]
Then with (4) \( L_1 \) can be expressed by \( E_1 \): 
\[
L_1 = L_{0,1} - E_1.
\]
Substitute into (9), \( E_2 \) can be expressed with \( E_1 \):
\[
E_2 = L_{1,1} + E_1 - L_1 = L_{1,1} + E_1 - L_{0,1} + E_1
= L_{1,1} - L_{0,1} + 2E_1.
\] (30)
Substitute into (5), \( L_2 \) can be expressed with \( E_1 \)...
With this recurrent operation following the sequence (4)\( \rightarrow \) (9)\( \rightarrow \) (5)\( \rightarrow \) (10)\( \rightarrow \) (11)\( \rightarrow \) (7)\( \rightarrow \) (12), \( E_5 \) can be expressed with \( E_1 \):
\[
E_5 = L_{1,4} + 2L_{1,3} + 3L_{1,2} + 4L_{1,1} - 4L_{0,1} - 3L_{0,2}
- 2L_{0,3} - L_{0,4} + 5E_1.
\] (31)
Then with (28) \( E_1 \) can be expressed in the form of \( \tilde{f}(E) \). Similarly, other unknown parameters can be found. As a result, all the single point machine errors can be expressed with the following format:
\[
E_i = \frac{iE}{15} + C_i,
\] (32)
where \( C_i \) is a constant expressed by measured values. Generally, if the artefact has \( m \) sections and the whole calibration has \( n \) shift steps, the machine errors can be expressed in the form of
\[
E_i = \frac{iE}{(n-1)m} + C_i.
\] (33)
From (32) and (33), it is clearly seen that the expression of machine errors includes a linear part which can be calibrated by a laser interferometer and a random part which can be expressed by the measured data. It can be deduced that even without external calibration the uncertainty can be notably reduced by the pure self-calibration, because for a particular length, the linear part will cause a fixed error that has no contribution to the measuring uncertainty.

In this process, the individual section lengths \( L_1, L_2, L_3, L_4 \), and \( L_5 \) can also be calculated, which can be used for interpolation, as shown in Fig. 4. This technique can be used in the applications where finer shift steps are needed.

IV. EXPERIMENTAL TEST

As discussed above, the complete error compensation curve includes both a linear part and a high-order part composed of residual errors which will be verified by experiments and data analysis in Secs. IV A–IV C.

A. Experimental data analysis for residual errors

The proposed method for CMM self-calibration has been applied with a simple setup on a commercial CMM Mitutoyo Crystal Apex-C. A vernier caliper was used to locate the artefact on each shift step. This low-cost shifting method will inevitably introduce positioning error. However, compared with the measurement error variation over the minimum sampling distance of 10 mm, the ideal position and actual position with around 10 \( \mu \)m positioning error can be considered to be the same point. If a finer sampling interval is used, say 0.1 mm, then more accurate positioning methods should be employed.

To measure the ball bearings, a probe with 0.3 mm diameter tip ball was used to contact 25 points on each hemispherical surface, as shown in Fig. 5. With the coordinates of the centres, the section length \( \ell_{ij} \) can be calculated, as shown in Table I.
TABLE I. Measured section lengths.

<table>
<thead>
<tr>
<th>Section lengths</th>
<th>Measured values (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_{0,1}</td>
<td>10.000</td>
</tr>
<tr>
<td>L_{0,2}</td>
<td>10.022</td>
</tr>
<tr>
<td>L_{0,3}</td>
<td>9.985</td>
</tr>
<tr>
<td>L_{0,4}</td>
<td>9.998</td>
</tr>
<tr>
<td>L_{0,5}</td>
<td>10.021</td>
</tr>
<tr>
<td>L_{1,1}</td>
<td>10.002</td>
</tr>
<tr>
<td>L_{1,2}</td>
<td>10.020</td>
</tr>
<tr>
<td>L_{1,3}</td>
<td>9.989</td>
</tr>
<tr>
<td>L_{1,4}</td>
<td>10.000</td>
</tr>
<tr>
<td>L_{1,5}</td>
<td>10.020</td>
</tr>
</tbody>
</table>

Then with the algorithm demonstrated in Sec. III, the machine errors at different sampling points can be calculated, as shown in Table II.

It can be seen that the error curve has a linear part $iE/15$ and the residual errors can be expressed by a smooth curve as shown in Fig. 6.

In order to evaluate the performance of the error compensation curve in Fig. 6, $L_3$ and $L_5$ were measured five times again at the positions P1-P2, P3-P4, P5-P6, P9-P10, and P13-P14. The data before and after error compensation are listed in Table III.

$L_3$ and $L_5$ are used as the constants to verify the error compensation curve. In an ideal measuring system, a constant length should have same readings over the whole measurement range. However, because of the high-order systematic errors along the measuring axis, $L_3$ and $L_5$ have instable measurement results with the standard deviation of 5.1 $\mu$m and 4.2 $\mu$m, respectively. Data correction with the error compensation curve is used to reduce this type of high-order systematic errors. The experimental data in Table III have shown that the standard deviations of $L_3$ and $L_5$ have been reduced to 2.9 $\mu$m and 2.3 $\mu$m, respectively. It can be calculated that the measurement uncertainty is reduced to 50% or so.

B. Interferometric calibration for $E$

With a laser interferometer HP5530A, the single point error at P15, which has been seen as a constant $E$ in the above, was calibrated to be 7.6 $\mu$m. Then the complete error compensation curve, as shown in Fig. 7, can be worked out by combining the linear part of $E$ and the high-order part shown in Fig. 6.

C. Error analysis

From the above, the repeatability of the measurement can be notably improved after error compensation, but there are still about 50% of the errors that cannot be corrected, for which the error sources can be analysed as below:

1. The probing errors: For the touch-trigger probe, there is an inevitable error caused by pretravel, the distance between the positions of contact and being triggered.

![FIG. 6. Residual error curve.](image)

![FIG. 7. Complete error compensation curve.](image)
TABLE III. Measured length for L3 and L5 before and after error compensation.

<table>
<thead>
<tr>
<th>Positions (Pk-Pk+1)</th>
<th>Measured length of L3 (mm)</th>
<th>Length of L3 after error compensation (mm)</th>
<th>Measured length of L5 (mm)</th>
<th>Length of L5 after error compensation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P4-P5</td>
<td>9.976</td>
<td>9.977</td>
<td>10.007</td>
<td>10.008</td>
</tr>
<tr>
<td>P7-P8</td>
<td>9.984</td>
<td>9.982</td>
<td>10.011</td>
<td>10.006</td>
</tr>
<tr>
<td>P13-P14</td>
<td>9.982</td>
<td>9.984</td>
<td>4.2</td>
<td>2.3</td>
</tr>
<tr>
<td>Standard deviation (μm)</td>
<td>5.1</td>
<td>2.9</td>
<td>4.2</td>
<td>2.3</td>
</tr>
</tbody>
</table>

The probing signal is generated by the embedded sensors only when the probe has a sufficient deformation. The pretravel may cause random errors because it varies with different contacting forces, moving speed, and directions.22, 23

2. The roundness error of the ball bearings, which may cause random errors for the spherical surface fitting: The roundness error of the ball bearings will affect the fitting results of the centres, which then accordingly introduce some random errors for the section lengths.

3. Errors caused by the environment: The measurements will also be influenced by the environmental factors, including temperature, vibration, and air flow.


For most of the commercial CMMs, the measuring line can hardly be arranged coaxially with the embedded scale of the machine. The random angular errors during the positioning will accordingly cause the Abbe errors.24

V. CONCLUSIONS

In this paper, a new method for CMM self-calibration is proposed. By optimizing the mathematical model and re-arranging the sampling positions, a simplified experimental process is designed. The proposed method provides for the fabrication of a relatively low-cost artefact using conventional machining methods. The approach has the benefit of minimising the number of sampling positions as well as the amount of external calibration required. Furthermore, it allows for the establishment of a clear physical definition for error separation and for self-calibration to focus on determining the uncertainty of measurement.

ACKNOWLEDGMENTS

The authors would like to thank Mr. Sean Paul de Rozario for his help in fabricating the artefacts.