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Author(s)	Wu, Hao; Tang, Lihua; Avvari, Panduranga Vittal; Yang, Yaowen; Soh, Chee Kiong
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# Broadband Energy Harvesting Using Nonlinear 2-DOF Configuration

Hao Wu, Lihua Tang, Panduranga Vittal Avvari, Yaowen Yang\*, Chee Kiong Soh  
School of Civil and Environmental Engineering,  
Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798  
\* Corresponding author, yyw@pmail.ntu.edu.sg

## ABSTRACT

Vibration energy harvesting using piezoelectric material has received great research interest in the recent years. To enhance the performance of piezoelectric energy harvesters, one important concern is to increase their operating bandwidth. Various techniques have been proposed for broadband energy harvesting, such as the resonance tuning approach, the frequency up-conversion technique, the multi-modal harvesting and the nonlinear technique. Usually, a nonlinear piezoelectric energy harvester can be easily developed by introducing a magnetic field. Either mono-stable or bi-stable response can be achieved using different magnetic configurations. However, most of the research work for nonlinear piezoelectric energy harvesting has focused on the SDOF cantilever beam. A recently reported linear 2-DOF harvester can achieve two close resonant frequencies with significant power outputs. However, for this linear configuration, although a broader bandwidth can be achieved, there exists a deep valley in-between the two response peaks. The presence of the valley will greatly deteriorate the performance of the energy harvester. To overcome this limitation, a nonlinear 2-DOF piezoelectric energy harvester is proposed in this article. This nonlinear harvester is developed from its linear counterpart by incorporating a magnetic field using a pair of magnets. Experimental parametric study is carried out to investigate the behavior of such harvester. With different configurations, both mono-stable and bi-stable behaviors are observed and studied. An optimal configuration of the nonlinear harvester is thus obtained, which can achieve significantly wider bandwidth than the linear 2-DOF harvester and at the same time overcome its limitation.

**Keywords:** energy harvesting; broadband; 2-DOF system; nonlinear vibration

## 1. INTRODUCTION

Energy harvesting from environment is a promising technology for powering electronics such as wireless sensing system. The ultimate goal is achieving self-sustaining devices for various applications such as environmental sensing or structural health monitoring. As mechanical vibration source is the most ubiquitous energy source in our daily life, vibration based energy harvesting technique has attracted great research interest in recent years. Typically, a vibration energy harvester comprises of a mass-spring system with a transducer working in different mechanisms: electrostatic, electromagnetic or piezoelectric. However, such harvester often works as a linear resonator. Its performance greatly relies on the matching between its resonance frequency and the vibration frequency. Its operating bandwidth is thus quite narrow. With a slight shift from the resonance, the performance will decrease drastically. However, the vibration source in the ambient environment is usually frequency-variant for different states or covers a large frequency range. Hence, one major challenge for developing an applicable energy harvester is to broaden its operating bandwidth, or to make its resonance tunable to match with the variant excitation frequency [1].

To achieve broader bandwidth, one promising method is to use the multi-modal technique to design an energy harvester. Shahruz [2] and Ferrari et al. [3] proposed a similar system using an array of cantilever beams with different resonant frequencies, so that the whole system can achieve greater bandwidth covered by the frequency range of all cantilever harvesters. Lien and Shu [4] also investigated the performance of array energy harvesting system connected with different harvesting circuits.

Another way to achieve multi-modal response is to harvest energy using more than one mode from a single cantilever beam. Tadesse et al. [5] presented a design of multi-modal hybrid energy harvesting beam employing both electromagnetic and piezoelectric transduction mechanisms, with each efficient for a specific mode. Ou et al. [6] derived a 2-DOF theoretical model for a two-mass cantilever beam. Ertuk [7] developed an L-shaped harvester, with the second natural frequency approximately doubled its first natural frequency. However, the problem for such proposed devices is that the frequencies for different modes are separated far away from each other, while the excitation energy source from

the environment cannot cover so wide frequency range. In these designs, the responses for higher-order modes are usually much smaller than the first one, rendering much lower power output at higher frequency. Hence, to increase the applicability of a multi-modal energy harvester, its multiple vibration modes should be designed very close to each other. Jang et al. [8] and Kim et al. [9] developed similar energy harvesting devices using both translational and rotational vibration motions to achieve two close resonant frequencies. Tang and Yang [10] derived a general lumped mass model for multi-modal energy harvesting. Wu et al. [11] proposed a “cut-out” 2-DOF harvester with a secondary beam enclosed within the main beam, which achieves two close resonances with significantly large amplitudes. Besides, this design can fully utilize the material of a cantilever beam. However, one major drawback of this design is the existence of a deep valley in the response between the two resonances, which may dramatically deteriorate the performance of the harvester.

Other than the multi-modal energy harvesting, nonlinear technique is also a promising approach to provide a much broader bandwidth as compared to its linear counterpart. Generally, most proposed nonlinear harvester is a Duffing-type oscillator developed from a single-degree-of-freedom (SDOF) system. Its response curve will cover a large frequency range by “folding” the resonance peaks toward either left or right. Depending on the value of the linear stiffness of the Duffing oscillator, the nonlinear harvester can be operated in a mono-stable or bi-stable configuration. Stanton et al. [12] proposed a mono-stable harvester in which both the hardening and softening responses can be observed by tuning the magnets. Cottone et al. [13] developed an inverted pendulum piezoelectric energy harvester using the bi-stable configuration. Erturk et al. [14] investigated the mechanism of a broadband bi-stable Duffing oscillator. However, for nonlinear vibrations, different oscillation states co-exist at the same frequency range, while the high amplitude oscillation cannot be always guaranteed. Tang et al. [15] conducted a parametric study of both mono-stable and bi-stable configurations, and concluded that: to achieve the maximum power output under random excitation, an optimal nonlinear configuration should be near the mono-stable to bi-stable transition point. Researchers are also seeking to capture the high energy orbit of nonlinear harvester by implement self-excitation controlling circuit [16].

In this paper, a nonlinear two-degree-of-freedom (2-DOF) piezoelectric energy harvester (PEH) is proposed to achieve a wider bandwidth based on the linear 2-DOF harvester proposed by Wu et al [11]. Experimental parametric study is carried out and a significantly enhanced operating bandwidth is obtained.

## 2. PREVIOUS LINEAR 2-DOF PIEZOELECTRIC ENERGY HARVESTER

Our previous study [11] proposed a 2-DOF “cut-out” PEH shown in Fig. 1(a). The harvester was fabricated from a SDOF cantilever beam by cutting out a secondary beam inside it. Piezoelectric elements were attached on the root of the main beam as well as the secondary beam. Thus, both areas can be used to generate energy. Compared to the conventional SDOF cantilever harvester, this design was demonstrated to be more efficient and compact.

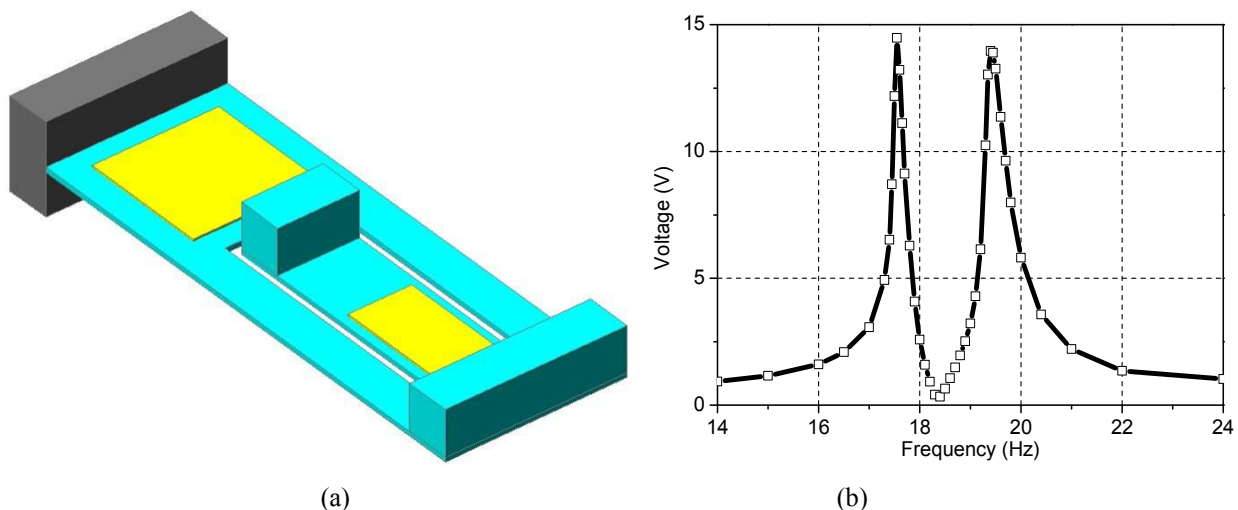


Figure 1. (a) A “cut-out” 2-DOF energy harvester, (b) frequency response with two close and equal peaks

By carefully designing the geometric parameters of the 2-DOF harvester, the first two natural frequencies can be tuned close to each other. The result in Fig. 1(b) shows that two close response peaks with significant voltage output were achieved. However, there is always a deep response valley in-between the two response peaks. The presence of this valley will definitely deteriorate the performance of the harvester. Thus, an improved nonlinear 2-DOF harvester with magnets is developed, as presented in the following sections.

### 3. EXPERIMENTAL SETUP FOR NONLINEAR 2-DOF HARVESTER

Based on the design of the linear 2-DOF harvester, nonlinearity is introduced to the system by incorporating a pair of polar opposite magnets. The nonlinear 2-DOF harvester comprises a main beam (outer beam) and a secondary beam (inner beam) both with tip masses. Fig. 2 shows the developed harvester installed on the vertical shaker.

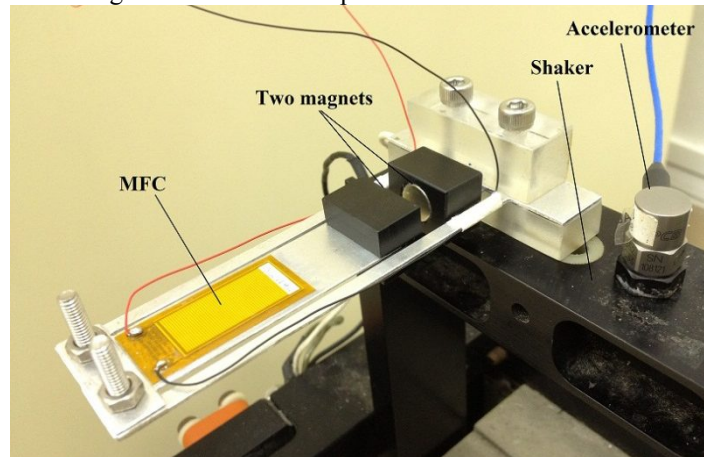


Figure 2. Nonlinear 2-DOF energy harvester installed on the vertical shaker

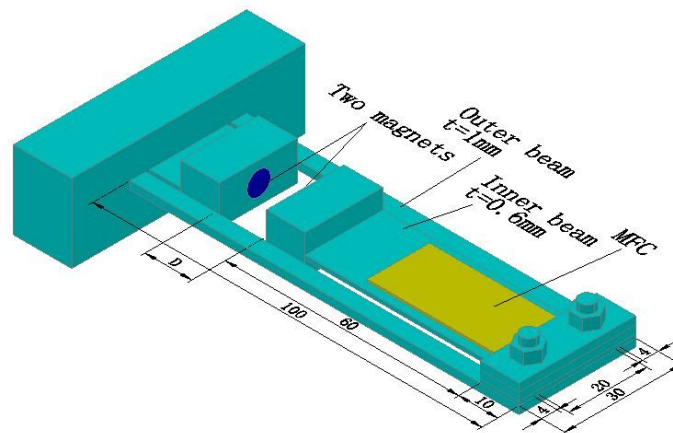


Figure 3. Illustration of nonlinear 2-DOF harvester (all dimensions in mm)

In this nonlinear design, the inner beam serves as an energy harvester with the attached one piece of  $d_{31}$  MFC sheet (M-2814-P2). Two NdFeB permanent magnets with diameter of 10 mm, thickness of 5 mm and surface flux of 3500 gauss embedded in plastic holders are installed on the harvester. In this experimental study, these two magnets are facing each other with the same pole at the distance of  $D$ , providing repulsive magnetic force. One holder with the magnet serves as the tip mass of inner beam ( $M_2=7.4$  g), while the other one is supported by a short beam clamped to the shaker. The length of the short support beam is adjustable, making it easy to change the distance between the two magnets. The outer and inner beams have the thickness of 1 mm and 0.6 mm, respectively, both of which are made of aluminum. For the convenience of fabrication, these two parts are fabricated separately and assembled with screws. The screws and several pieces of steel plate at the free end of the outer beam serve as the tip mass ( $M_1$ ), and this mass value is adjustable by adding or removing small steel plates. Each piece of small plate weighs about 1.9 g, while the minimum value of  $M_1$  including the screws and nuts is 3.6 g. The detailed dimensions are shown in Fig. 3.

In the experimental parametric study, three parameters are adjusted to study the response of the system. They are: the distance of the two magnets ( $D$ ), the weight of tip mass at outer beam ( $M_1$ ), and the base excitation level ( $A$ ). The groups of different values of these three parameters used in the experiment are listed in the Table 1.

Table 1. Values for parameters used in experiment study

Parameters	Values					
	Distance of two magnets, $D$ (mm)	N/A	14	12	11	10
Tip mass at the end of outer beam, $M_1$ (g)	13.1	11.2	9.3	7.4	5.5	3.6
Harmonic base excitation level, $A$ ( $m/s^2$ , RMS)	0.5		1		2	

While decreasing the distance between two magnets, the harvester gradually goes into bi-stable vibration from mono-stable vibration. In such condition, there are two equilibrium positions, each at one side of the central position, as shown in Fig. 4. The central position is the equilibrium position for the mono-stable vibration, which becomes the potential barrier for the bi-stable vibration. If the distance between two magnets is very small, the two equilibrium positions will be separated further away from each other, and the central potential barrier will increase. The vibration of the harvester could be either small-amplitude oscillation confined in one potential well or large-amplitude oscillation crossing two potential wells, or even be chaotic vibration, depending on the base excitation level and the initial condition. The critical point of the distance of two magnets ( $D$ ) for transition from mono-stable to bi-stable is between 10 and 9 mm, as observed in the experiment.

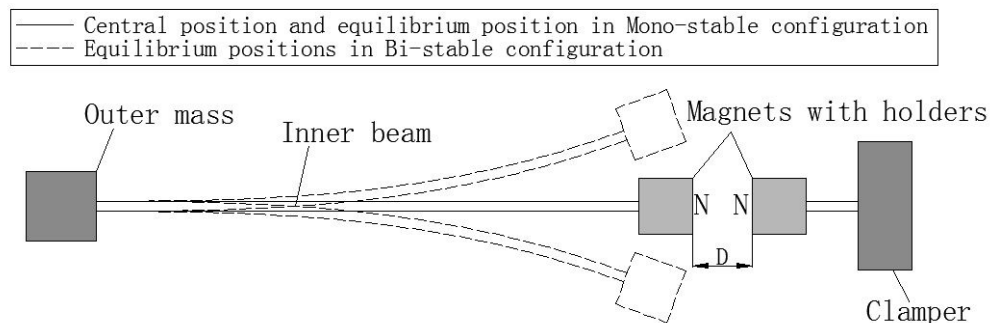


Figure 4. Illustration of equilibrium position for mono-stable and bi-stable vibrations (the outer beam is not shown in this figure)

During the experiment, frequency responses of open circuit voltage from the nonlinear 2-DOF energy harvester are recorded in terms of the root mean square (RMS) values. For some unique frequency responses, the transient responses are recorded as well. With various distances between the two magnets, linear response, quasi-linear response, mono-stable response and bi-stable response are observed, and an optimal configuration is found, which can achieve significantly wider bandwidth. The results are presented in the following sections.

## 4. FREQUENCY RESPONSES OF NONLINEAR 2-DOF HARVESTER

### 4.1 Frequency response of linear 2-DOF harvester

By removing the magnet clamped at the root, the linear responses are firstly recorded for this 2-DOF system. Three groups of experimental results with different mass values are shown in Fig. 5. The linear response is similar to the response presented in our previous paper [11]. However, the parameter for this linear 2-DOF harvester is not optimized to achieve two close peaks with adequate magnitudes. Hence, the two natural frequencies (one at 15Hz and the other at 28 Hz) are a bit far away from each other, and the magnitude of the first peak is quite small. Later, by adding the magnets into the system, the natural frequencies can be tuned much closer, as the linear part of the magnetic force will change the stiffness of the system, especially for the inner beam.

The peak shift due to the change of  $M_1$  is also illustrated in this figure. The frequency of first peak increases when  $M_1$  decreases while the other peak almost remains at the same position. Moreover, the voltage response under base acceleration of  $2 \text{ m/s}^2$  is less than twice of that under  $1 \text{ m/s}^2$  at the resonances. This is due to the geometric limitation that the amplitude of vibrating beam cannot be increased infinitely.

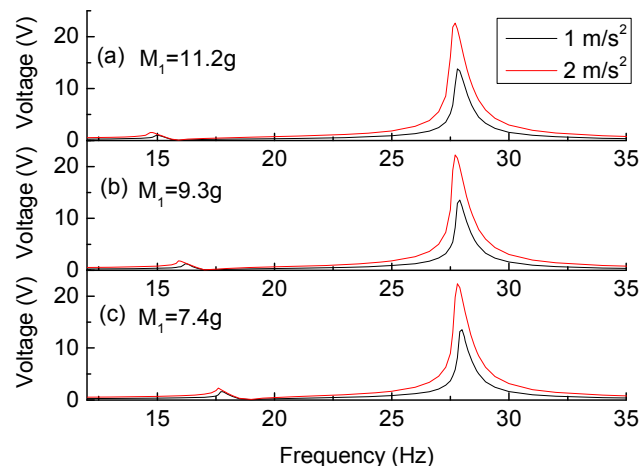


Figure 5. Frequency response of linear 2-DOF harvester, (a)  $M_1=11.2 \text{ g}$ , (b)  $M_1=9.3 \text{ g}$ , and (c)  $M_1=7.4 \text{ g}$ .

#### 4.2 Frequency response of mono-stable vibration ( $D \geq 10 \text{ mm}$ )

##### *Quasi-linear response for low level excitation*

In the mono-stable vibration, the nonlinear 2-DOF energy harvester works with hardening behavior because the repulsive magnetic force provides a positive nonlinear stiffness to this system. When the base excitation level is not so high, the harvester exhibits a quasi-linear behavior. Fig. 6 shows that the response of the harvester is still very similar to a linear 2-DOF system when the harvester is under a low excitation of  $0.5 \text{ m/s}^2$ , despite that the two magnets are quite close ( $D=10 \text{ mm}$ ). The voltage response indicates minor hardening nonlinear behavior as the curve bends slightly towards right (higher frequency) at the second peak. But the nonlinearity is not strong because the results for upward and downward frequency sweeps have negligible difference.

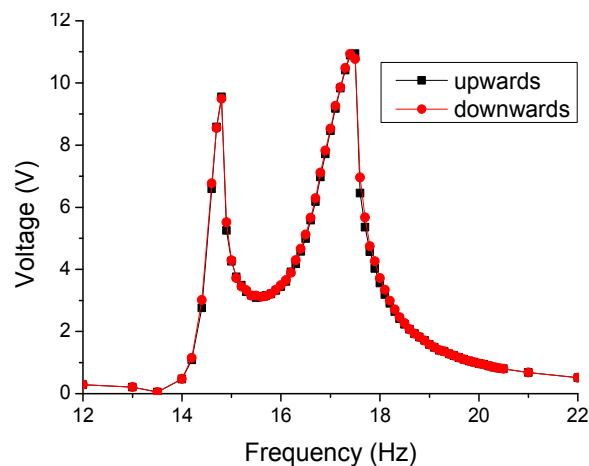


Figure 6. Quasi-linear frequency response for nonlinear 2-DOF harvester with  $M_1=11.2 \text{ g}$  and  $D=10 \text{ mm}$  under base excitation of  $0.5 \text{ m/s}^2$ .

Compared to the linear frequency response of 2-DOF harvester (Fig. 5), the second resonant frequency for this quasi-linear response (Fig. 6) has been shifted from 28 Hz to 18 Hz, while the other resonance is almost unchanged (around 15 Hz). Thus, two close resonance peaks are achieved with comparable and sufficiently large amplitude. This is similar to the results of linear 2-DOF harvester in [11], while a response valley still exists in-between the two peaks. The reason for

the frequency change from Fig. 5 to Fig. 6 is that the linear part of magnetic force reduces the linear stiffness of the inner beam, thus tunes the second resonance to the left. The first resonance does not change since the magnetic force has very minor effect on the linear stiffness related to the outer beam.

To further illustrate how the magnetic force changes the linear stiffness of the 2-DOF harvester, more quasi-linear responses of the harvester under low excitation of  $0.5 \text{ m/s}^2$  with the same  $M_1$  but different values of  $D$  are presented in Fig. 7. As there are negligible differences between the responses of upward and downward sweep, only the results of upward sweep are presented in this figure. As shown, with  $D$  decreasing from 14 to 10 mm, the second resonance is shifted from 28 Hz to about 18 Hz gradually, while the first resonance is always around 15 Hz. Furthermore, from Fig. 7, it is obvious that with larger distance between the magnets, the harvester behaves closer to a linear one.

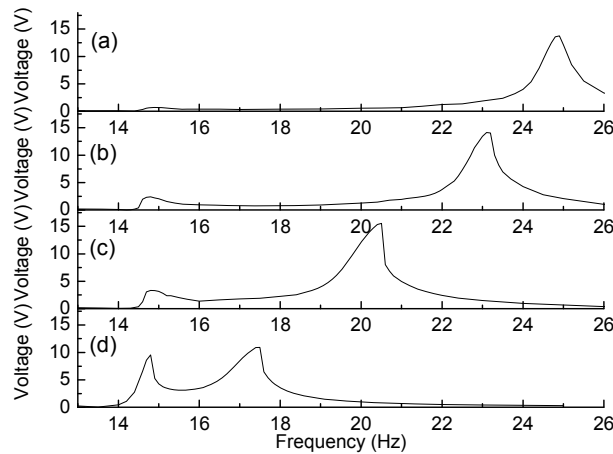


Figure 7. Quasi-linear frequency response for nonlinear 2-DOF harvester under base excitation of  $0.5 \text{ m/s}^2$  with  $M_1=11.2 \text{ g}$  and (a)  $D=14 \text{ mm}$ , (b)  $D=12 \text{ mm}$ , (c)  $D=11 \text{ mm}$  (d)  $D=10 \text{ mm}$ .

*Nonlinear frequency response for higher excitation level*

Fig. 8 shows the same trend that the second peak shifts towards left while the distance of two magnets decreases, with a higher excitation level of  $2 \text{ m/s}^2$ . It also clearly illustrates the effect of the nonlinear force from two magnets. Besides the resonance tuning effect, the smaller distance of two magnets will provide stronger nonlinear force, which will further fold the response curve to cover a wider frequency range.

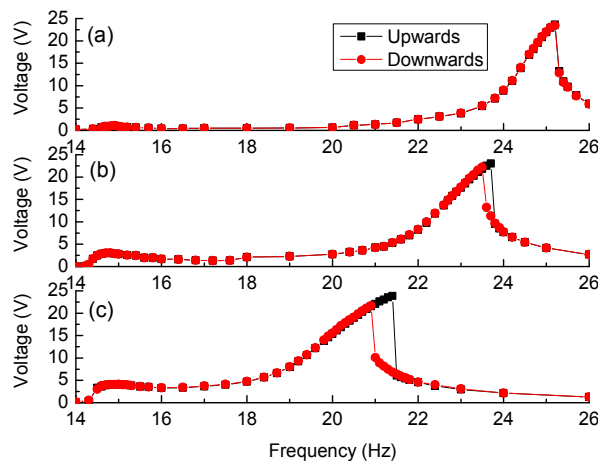


Figure 8. Comparison of frequency response for nonlinear 2-DOF harvester under base excitation of  $2 \text{ m/s}^2$  with  $M_1=11.2 \text{ g}$  and different distance of magnets (a)  $D=14 \text{ mm}$ , (b)  $D=12 \text{ mm}$  and (c)  $D=11 \text{ mm}$ .

By increasing the base excitation level, the nonlinear behavior of the harvester will be strengthened. Fig. 9 shows a comparison of the frequency responses of the harvester under different excitation levels while the other parameters are kept same. As discussed previously, for low excitation ( $0.5 \text{ m/s}^2$ ), the response is similar to the linear response and the

upward and downward sweeps generate the same response curve. With the increase of excitation, the response curve is bent over to the higher frequency direction, providing much wider bandwidth. As shown in the Fig. 9, with the high excitation of  $2 \text{ m/s}^2$ , the upper bound of the bandwidth reaches about 21.5 Hz for the upward sweep. When the excitation frequency is greater than 21.5 Hz, the large-amplitude oscillation cannot be maintained; the harvester suddenly drops into the low-amplitude oscillation state. For the downward sweep under the same excitation level, the harvester firstly vibrates in the low-amplitude orbit, and the large-amplitude orbit is re-captured at the frequency of 21 Hz. This phenomenon is consistent with those reported by other researchers. However, according to the experiment result, this nonlinear behavior will be observed only at the second peak (which is related to the inner beam), while the other peak will almost be similar to the linear response. In Fig. 9, if the small peak of first resonance is neglected, the result has no great difference from those of a mono-stable nonlinear SDOF system.

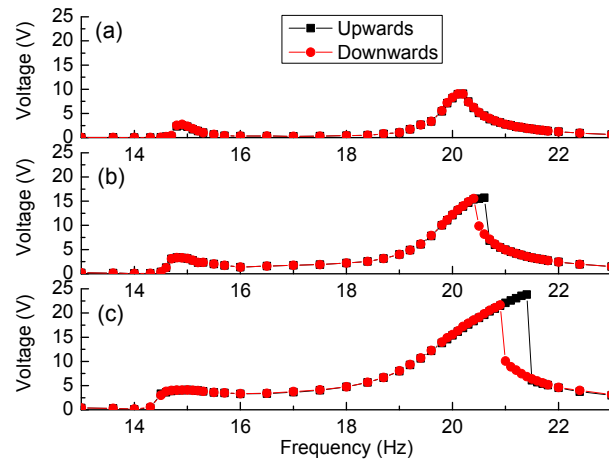


Figure 9. Comparison of frequency response for nonlinear 2-DOF harvester with  $M_1=11.2 \text{ g}$  and  $D=11 \text{ mm}$  under different base excitation level of (a)  $0.5 \text{ m/s}^2$ , (b)  $1 \text{ m/s}^2$  and (c)  $2 \text{ m/s}^2$

*Optimal configuration for broader bandwidth*

However, the biggest difference between the proposed nonlinear 2-DOF harvester and other nonlinear SDOF harvester is that two significant resonant response peaks can be achieved and tuned very close to each other. Thus, the optimal configuration of the proposed design should be able to produce a response curve where two peaks are sufficiently close and connected by the “bended” nonlinear response, generating a much wider useful bandwidth. According to our experimental parametric study, this optimal configuration is obtained and its frequency responses under different excitation levels are shown in Fig. 6 and Fig. 10.

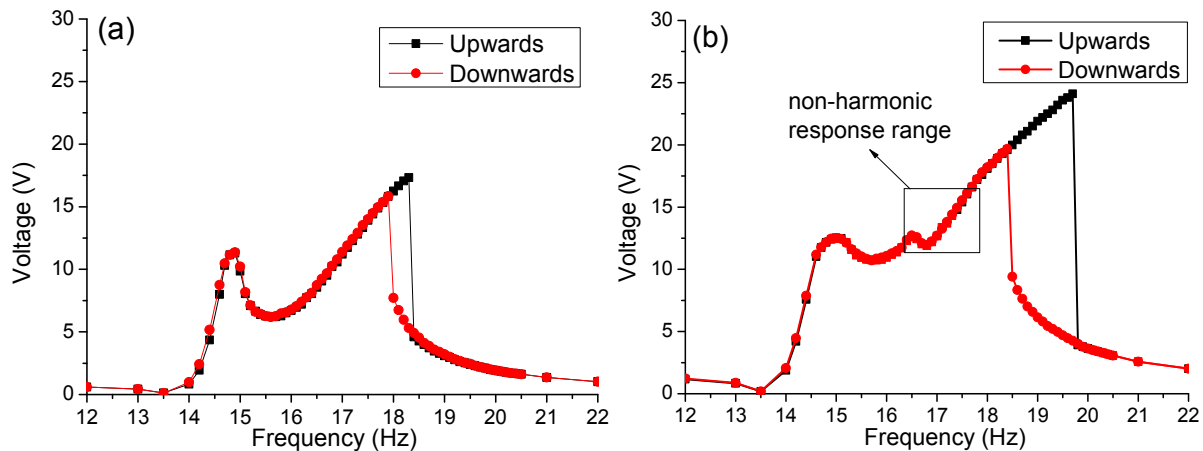


Figure 10. Frequency responses for nonlinear 2-DOF harvester with  $M_1=11.2 \text{ g}$  and  $D=10 \text{ mm}$  under excitation of (a)  $1 \text{ m/s}^2$  and (b)  $2 \text{ m/s}^2$



In this configuration, the value of  $M_1$  is 11.2g. With such outer mass value, two close resonant peaks with about 2 Hz difference are achieved for quasi-linear response (as shown in Fig. 6). The distance of two magnets are controlled as  $D=10$  mm in this configuration. This distance is the smallest value of mono-stable configuration in this experiment, and provides the largest nonlinear stiffness for mono-stable vibration. With the increase of the excitation level, the response curve bends more to higher frequency range to provide a wider bandwidth.

As shown in Fig. 10(b), if the output voltage level of 10 V is regarded as a useful working voltage, by considering the upward sweep, the bandwidth is about 5 Hz (15 to 20 Hz). However, the typical jump phenomenon for nonlinear vibration is also observed from about 18.5 to 20 Hz, where both the high and low energy oscillation orbits are stable and co-exist. When the external excitation falls in this frequency range, the harvester can be either attracted into the larger amplitude oscillation or the lower one, depending on the initial condition of the harvester. Thus the higher voltage response obtained from the upward sweep cannot be guaranteed in the practical application. By considering the downward sweep response curve, which is more robust for any initial condition, the bandwidth is still quite large of about 3.5 Hz (15 to 18.5 Hz).

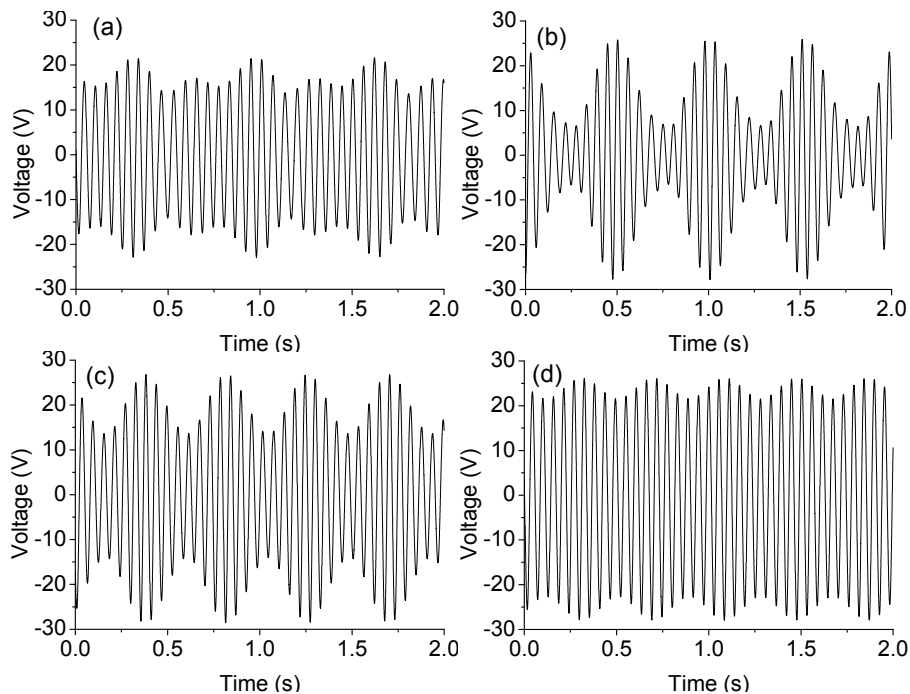


Figure 11. Time history voltage responses for nonlinear 2-DOF harvester in non-harmonic range (a) 16.4 Hz, (b) 16.9 Hz, (c) 17.4 Hz and (d) 17.8 Hz

In between these two peaks, there is a certain frequency range in which the output responses recorded are not harmonic although the input excitation is harmonic. However, the response in this frequency range is stable regardless of different initial conditions. This non-harmonic response range is from 16.0 to 18.0 Hz, as indicated in Fig. 10(b). Typical transient responses within this range are presented in Fig. 11.

The transient responses in Fig. 11 are no longer harmonic but still periodical, and each of them can be simply regarded as a combination of several harmonic oscillations. To find out the frequency of each harmonic component, a fast Fourier transform (FFT) is carried out for these responses. The results are shown in Fig. 12. There are mainly three peaks existing in the FFT results. First, there will be always a peak having the same frequency as the excitation source since the system is under the forced harmonic vibration. The frequency of this peak will definitely change with the sweep procedure. The first peak that is obvious in Fig. 12(a) and (b) is around 15 Hz, which matches the first resonant frequency of the harvester (as shown in Fig. 10). The third peak is related to the second resonance and the nonlinear vibration status. This is because the response curve at the second resonance bends to the right, resulting in that the second resonance is no longer fixed but depends on the vibration status.

This phenomenon only exists within this frequency range (indicated in Fig. 10(b)) for the optimal configuration with  $2 \text{ m/s}^2$  base excitation, which is not observed at any other frequency outside this range. For the same configuration under smaller excitation level, same phenomenon is also observed but within a narrower frequency range. This phenomenon also presents for some other configurations with two closer magnets. All these results suggest that strong magnetic force and high excitation level are two important factors contributing to this phenomenon.

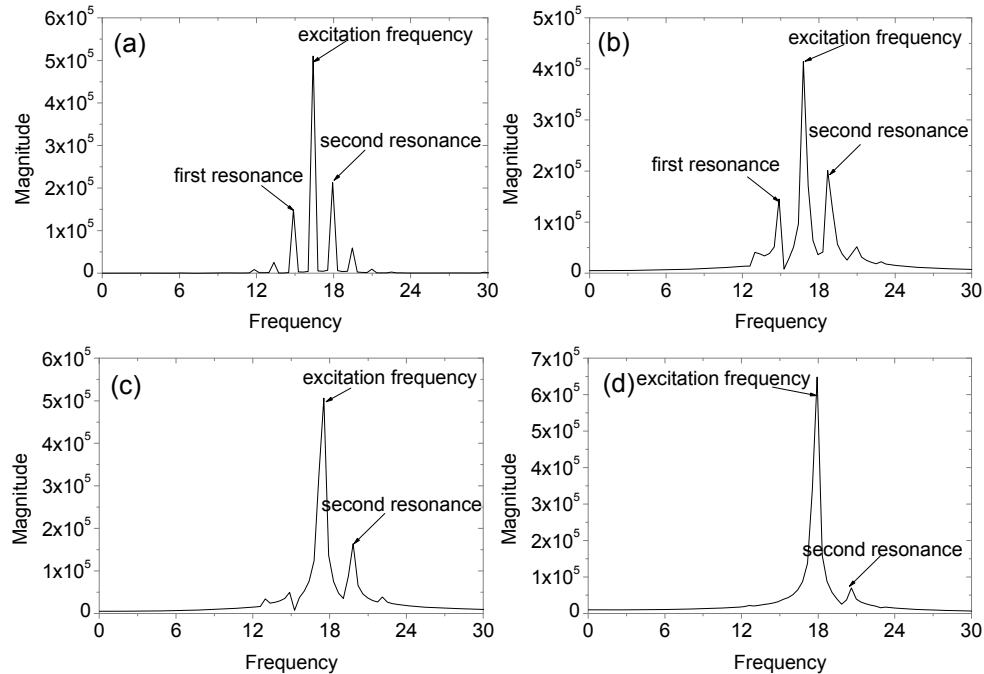


Figure 12. FFT for non-harmonic responses at (a) 16.4 Hz, (b) 16.9 Hz, (c) 17.4 Hz and (d) 17.8 Hz

*Other cases for  $D=10 \text{ mm}$*

For the other cases with the same magnetic distance and excitation level but different outer mass value, the voltage responses are shown in Fig. 13, which illustrates the dependency of the peak location on the mass value  $M_1$ . If the location of the peak is not chosen properly, the frequency response curve could have a narrower bandwidth or the two peaks could be separated far away with a low response valley in-between. Both are undesired in the design.

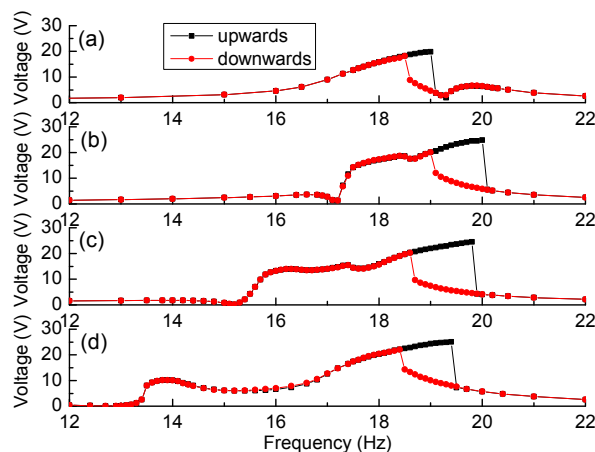


Figure 13. Frequency response for nonlinear 2-DOF harvester with  $D=10 \text{ mm}$  and (a)  $M_1=5.5 \text{ g}$ , (b)  $M_1=7.4 \text{ g}$ , (c)  $M_1=9.3 \text{ g}$  and (d)  $M_1=13.2 \text{ g}$ .

In summary, these results show that this mono-stable nonlinear 2-DOF harvester with the optimal configuration is definitely advantageous over its linear counterpart by eliminating the response valley in-between the two resonant peaks. However, the mechanism and analytical modeling is very much desired to explain this advantage, which is useful for further development of such harvester.

### 4.3 Frequency response for bi-stable vibration ( $D \leq 9\text{mm}$ )

With further decrease of the distance of two magnets, the harvester changes into the bi-stable vibration. The bi-stable vibration of the harvester is much more complicated than its mono-stable vibration. The motion of the harvester can present as large amplitude oscillation, small amplitude oscillation or even chaotic oscillation, depending on the excitation levels and initial conditions [15]. In the experiment, only one case of bi-stable vibration is studied with  $D=9\text{ mm}$ . The initial equilibrium position is observed slightly below the central position due to the gravity. The frequency response of the voltage on the MFC transducer is presented in Fig. 14. The obvious difference as compared to the mono-stable vibration is that the frequency response no longer behaves as the hardening response (bent to right). This is because the equilibrium position has changed, and the nonlinear stiffness at the new equilibrium position is no longer positive.

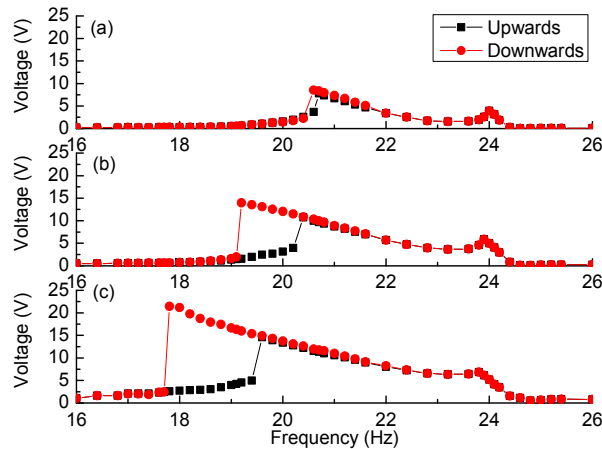


Figure 14. Frequency response of nonlinear 2-DOF harvester in bi-stable vibration with  $D = 9\text{ mm}$ ,  $M_1=3.6\text{ g}$  and (a)  $0.5\text{ m/s}^2$ , (b)  $1\text{ m/s}^2$  and (c)  $2\text{ m/s}^2$

## 5. CONCLUSIONS

To broaden the bandwidth for vibration energy harvesting, a nonlinear 2-DOF PEH is proposed based on the “cut-out” linear 2-DOF PEH and evaluated through experimental parametric study. Various configurations of this nonlinear 2-DOF harvester and its linear counterpart are tested and compared. Among them, one optimal configuration working in mono-stable vibration is found to produce the largest bandwidth, which is more advantageous over its linear counterpart as well as the conventional mono-stable nonlinear SDOF harvester. The deep response valley which always exists for linear 2-DOF harvesters is eliminated by this nonlinear 2-DOF design.

Analytical modeling of such design is highly desired to provide guidelines for analysis and design of such nonlinear harvesters. Also, the behavior of the harvester in the bi-stable vibration is illustrated with only a simple case in our experiment study. Thus, it is not conclusive whether the bi-stable or mono-stable vibration is more beneficial for the proposed nonlinear harvester. Further investigation will be carried out to ascertain this issue.

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