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Dual wavelength digital holography for improving the measurement accuracy

Jianglei Di\textsuperscript{a}, Weijuan Qu\textsuperscript{b}, Bingjing Wu\textsuperscript{a}, Xin Chen\textsuperscript{a}, Jianlin Zhao\textsuperscript{*a}, Anand Asundi\textsuperscript{c}

\textsuperscript{a}Shaanxi Key Laboratory of Optical Information Technology and The Key Laboratory of Space Applied Physics and Chemistry, School of Science, Northwestern Polytechnical University, Xi’an 710072, P. R. China;
\textsuperscript{b}Ngee Ann Polytechnic, 535 Clementi Road, Singapore 599489;
\textsuperscript{c}Centre for Optical and Laser Engineering, School of Mechanical and Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798

ABSTRACT

In dual wavelength digital holography, a synthetic wavelength is obtained by using two lasers with different wavelengths to expand the measurement range of samples’ step heights from nanometers to micrometers. However, its measurement accuracy reduces along with the expansion of measuring range and significant noise is introduced at the same time. For cases where the sample’s height is smaller than the wavelength of illumination light, the measurement accuracy is very important. In this paper, a new approach of dual wavelength digital holography is presented. The synthetic wavelength is smaller than the wavelength of the two different lasers. Higher measurement accuracy can thus be achieved. The analysis and experimental results show the validity of this method.

Keywords: Digital holography, dual wavelength, measurement accuracy, numerical reconstruction

1. INTRODUCTION

With the advantage of fast, nondestructive and full-field 3D measurement for reflecting or transmitted samples, digital holography has been widely used in many fields, such as vibration analysis, refractive index measurement, flow field visualization, microscopy, micro-optics and MEMS metrology, and so on\cite{1-9}.

In digital holography, the digital recording of the hologram and numerical reconstruction of the complex amplitude for the object wavefront provide us with a lot of flexibility in the processing of the hologram. For example, the quantitative phase information can be obtained directly from the reconstructed object wavefront, which is mainly determined by the optical path lengths for reflecting or transmitted samples. However, the phase ambiguity, caused by solving the phase angle of the complex amplitude of the object wavefront, limits the measurement range. Thus, dual wavelength digital holography is introduced to expand the measurement range, especially for samples with steps.

A synthetic wavelength can be obtained in dual wavelength digital holography by using two lasers with different wavelengths, and the synthetic wavelength is longer than either of the laser wavelengths\cite{10-14}. However, the measurement accuracy in dual wavelength digital holography reduces along with the expansion of measuring range and significant noise is also introduced. In some cases, the sample’s height is far less than the wavelength of illumination light. Therefore, the measurement accuracy is very important for this kind of samples.

In this paper, a new approach of dual wavelength digital holography is presented. A synthetic wavelength is also calculated by the multiplication of the complex amplitudes, and thus the synthetic wavelength is only about half of the wavelength from either of the two different lasers. So the wrapped phase fringes will be more closed and the phase information will be smooth with this new synthetic wavelength. Therefore, higher measurement accuracy can be achieved. The analysis and experimental results show the validity of this method.

*jlzhao@nwpu.edu.cn; phone 86 29 88431663; fax 86 29 88431663 801
2. PRINCIPLES

In digital holography, the intensity of the hologram recorded by CCD can be expressed as

\[ I(x, y) = |O(x, y) + R(x, y)|^2 \]

\[ = o_0^2(x, y) + r_0^2(x, y) + 2o_0(x, y)r_0(x, y)\cos[\phi_0(x, y) - \phi(x, y)] \]

where, \( O(x, y) = o_0\exp[j\phi_0(x, y)] \) and \( R(x, y) = r_0\exp[j\phi(x, y)] \) are the complex amplitudes of the object and reference waves, and \( \phi_0, \phi, o_0, r_0 \) are the phases and amplitudes of the object and reference waves, respectively. If the original reference wave \( R(x, y) \) and convolution method are used for the numerical reconstruction of the hologram, the reconstructed image wave field \( U(\xi, \eta) \) is given by

\[ U(\xi, \eta) = F^{-1}\{F[R(x, y)U(x, y)] \cdot F\{g(\xi, \eta, x, y)\}\} \]

where, \( g(\xi, \eta, x, y) \) is the impulse response function, \( F \) and \( F^{-1} \) represent the Fourier transform and the inverse Fourier transform, respectively. Then the intensity distribution \( I(\xi, \eta) \) and the phase distribution \( \phi(\xi, \eta) \) of the reconstructed image can be described as

\[ I(\xi, \eta) = |U(\xi, \eta)|^2 \]

and

\[ \phi(\xi, \eta) = \arctan \frac{\text{Im}[U(\xi, \eta)]}{\text{Re}[U(\xi, \eta)]} \]

For transmitted samples, the refractive index variation or thickness change alters the optical path length \( \Delta l(\xi, \eta) \). The phase change \( \Delta \phi(\xi, \eta) \) is given by

\[ \Delta \phi(\xi, \eta) = \frac{2\pi}{\lambda} \Delta l(\xi, \eta) = \frac{2\pi}{\lambda} n(\xi, \eta) L \]

where, \( L(\xi, \eta) \) is the geometrical path length, and \( n, n_0 \) are the refractive index of the air and the sample, respectively.

In dual wavelength digital holography, two lasers with different wavelengths \( \lambda_1 \) and \( \lambda_2 \) are used and the reconstructed image wave field \( U_1(\xi, \eta) \) and \( U_2(\xi, \eta) \) can be reconstructed respectively by using Eq.(2). Then the phase \( \Phi \) for the synthetic wavelength \( \Lambda \) can be got by calculating the arctangent of \( U_1 \) multiplying by \( U_2^* \), which can be written as

\[ \Phi = \arg(U_1U_2^*) = 2\pi \left( \frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2} \right) = 2\pi \frac{\Delta l}{A} \]

where \( \Delta l \) is the optical path lengths and \( A \) is the synthetic wavelength defined as

\[ A = \frac{\lambda_1\lambda_2}{|\lambda_2 - \lambda_1|} \]

The synthetic wavelength \( A \) is very large for a small difference \( |\lambda_2 - \lambda_1| \). By choosing two closer wavelengths to produce a longer synthetic wavelength \( A \), the wavelength can be extended in the range of \( 2\pi \) discontinuities in the phase map.

Different from this, in this paper we obtain the phase \( \Phi \) for a synthetic wavelength \( A \) by calculating the arctangent of \( U_1 \) multiplying by \( U_2 \), and accordingly the phase is

\[ \Phi = \arg(U_1U_2) = 2\pi \left( \frac{\lambda_2 + \lambda_1}{\lambda_1\lambda_2} \right) = 2\pi \frac{\Delta l}{A} \]

and the synthetic wavelength \( A \) is

\[ A = \frac{\lambda_1\lambda_2}{\lambda_2 + \lambda_1} \]
This synthetic wavelength depends on the wavelengths of two laser sources. When $\lambda_1$ and $\lambda_2$ are equal, the synthetic wavelength $\Lambda$ is equal to half the wavelength as is to be expected. By selecting appropriate laser sources, the synthetic wavelength $\Lambda$ will change accordingly. For example, for $\lambda_1=632.8\text{nm}$ and $\lambda_2=660\text{nm}$, the synthetic wavelength $\Lambda$ is 323.1nm. That means if the optical path length $\Delta l(\xi,\eta)$ equals to 323.1nm for transmitted samples with constant refractive index, the phase change $\Delta\phi(\xi,\eta)$ measured by digital holography is $2\pi$. But for $\lambda_1$ and $\lambda_2$, the phase change is only about $\pi$. In addition, the synthetic phase $\Phi$ has an averaging effect in the process of calculating the synthetic wavelength and thus smoothing the phase noise.

3. EXPERIMENTAL SETUP

Figure 1 shows the experimental setup of dual wavelength digital holography. The optical setup is based on a Mach-Zehnder interferometer. Laser beams from two different lasers (He-Ne laser with wavelength $\lambda_1=632.8\text{nm}$ and semiconductor laser with wavelength $\lambda_2=660\text{nm}$) are coupled into an optical fiber coupler FC and divided into two parts. One propagates along the fiber and divergence at the fiber exit to form a spherical light as the reference beam. The other is collimated by a lens L to illuminate the samples and then magnified by a 10X microscope objective MO as the object beam. Both beams pass through beam splitter BS and interfere with each other in a small incidence angle on CCD. The interferograms are recorded by a white-black CCD with $480\times640$ pixels and pixel size $4.65\mu\text{m}\times4.65\mu\text{m}$.

In this experiment, the sample is a small plastic sheet with microlenses, the refractive index of which is about 1.55. Two digital holograms for different wavelengths are recorded respectively by controlling the laser switch. Here different lasers are used with same optical device arrangement for recording holograms. The laser wavelength differences lead to different hologram fringes period.

![Experimental setup of dual wavelength digital holography for improving the measurement accuracy.](image)

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

Figure 2(a) shows one of the hologram recorded with $\lambda_1=632.8\text{nm}$. The outline of the sample can vaguely be seen from the figure. The spectrum of hologram with $\lambda_1=632.8\text{nm}$ and $\lambda_2=660\text{nm}$ are shown in Fig. 2(b) and Fig. 2(c), respectively. Because of the slight difference of the period of the hologram fringes, the distance from $\pm1$ order to zero order spectrum is different in the spectrum diagram. With the mathematical approach described in Section 2, we can numerically reconstruct the holograms. The reconstructed wrapped phase map of the hologram with $\lambda_1=632.8\text{nm}$ is shown in Fig. 2(d). From the figure, we can see the basic structure of the sample. The microlens is in the central part with clear wrapped phase fringes.

The central part of Fig. 2(d) is cut out and shown in Fig. 3(a). The phase map colors are red in the edge of microlens and yellow in the center. From edge to center along radial direction of the microlens the fringe order is 2.2. The same position of the wrapped phase map for hologram with $\lambda_2=660\text{nm}$ is shown in Fig. 3(b). The phase map colors are red in both edge and center, which show that the fringe order is about 2 and the phase fringes become sparse. From Eq. (5), we can see that the measured phase value will be reduced with the increase of the wavelength. That’s why the phase fringes are sparse in Fig. 3(b). In addition, the color of the background in Fig. 3 is not consistent due to the uneven thickness of the plastic sheet. At the same time, there are more phase noises in Fig. 3(a) because of the lasers’ large coherence length.
Figure 2. Digital hologram, spectrum and wrapped phase map. (a) Recorded hologram with \( \lambda_1 = 632.8 \text{nm} \); (b) spectrum of hologram with \( \lambda_1 = 632.8 \text{nm} \); (c) spectrum of hologram with \( \lambda_2 = 660 \text{nm} \); (d) reconstructed wrapped phase map of hologram with \( \lambda_1 = 632.8 \text{nm} \).

Figure 3. Part of wrapped phase map. (a) \( \lambda_1 = 632.8 \text{nm} \); (b) \( \lambda_2 = 660 \text{nm} \).

By utilizing unwrapping algorithm, we can unwrap the wrapped phase image in Fig. 3. The 2D and 3D unwrapped phase maps with \( \lambda_1 = 632.8 \text{nm} \) are shown in Fig. 4(a) and (b). We can see that the wrapped phase fringes have removed and the phase value range has expanded to 20. Figure 4(c) is the sectional view of phase map along the dash line in Fig. 4(a). The phase height of the microlens is about 15.2. According to Eqs. (5) and (8), the microlens height is about 2.78 \( \mu \text{m} \). The sectional view of phase map with \( \lambda_2 = 660 \text{nm} \) is shown in Fig. 4(d). The microlens phase height in the same position is about 14.7. The phase value reduces with the increase of the wavelength. We can also calculate the microlens height which is 2.80 \( \mu \text{m} \). The measurement results are almost same with different lasers.

Figure 5 shows the experimental results with dual wavelength digital holography method proposed in this paper. The laser wavelengths are 632.8nm and 660nm, the synthetic wavelength \( \Lambda \) is 323.1nm according to Eq. (9). We have obtained the reconstructed image wave field \( U_1(\xi, \eta) \) with \( \lambda_1 = 632.8 \text{nm} \) and \( U_2(\xi, \eta) \) with \( \lambda_2 = 660 \text{nm} \). By calculating the arctangent of \( U_1 \) multiplying by \( U_2 \), the wrapped phase map is shown in Fig. 5(a). Because the synthetic wavelength \( \Lambda \) is about half of the original wavelength, the phase fringes are more dense than that in Fig. 3. The 3D and 2D unwrapped phase maps with synthetic wavelength are shown in Fig. 5(b) and (c). In dual wavelength digital holography, the wrapped phase map contains all information of \( U_1(\xi, \eta) \) and \( U_2(\xi, \eta) \). Because the synthetic wavelength \( \Lambda \) is shorter and the phase fringes are denser, the phase noises become smaller in phase unwrapping operation. Compared with Fig. 4(a), it’s very clear that phase noises in Fig. 5(c) are very weak. Fig. 5(d) shows the sectional view of phase map along the...
dash line in Fig. 5(c). Here the phase height of the microlens is about 29.8. Then we get the microlens height which is about 2.79 \( \mu \)m by using Eqs. (5) and (8). The measurement results with synthetic wavelength are consistent with that of single wavelength. But in this method, higher measurement accuracy is achieved.

![Figure 4](image1.png)

**Figure 4.** Unwrapped phase map and its sectional view. (a) 2D phase map with \( \lambda_1=632.8 \)nm; (b) 3D phase map with \( \lambda_1=632.8 \)nm; (c) sectional view of phase map with \( \lambda_1=632.8 \)nm; (d) sectional view of phase map with \( \lambda_2=660 \)nm.

![Figure 5](image2.png)

**Figure 5.** Phase map and its sectional view with \( \Lambda=323.1 \)nm. (a) wrapped phase map; (b) 3D phase map; (c) 2D phase map; (d) sectional view of phase map.

5. CONCLUSIONS

A new dual wavelength digital holography method to improve the measurement accuracy is demonstrated. A synthetic
wavelength is calculated which is shorter than either of the original wavelengths. The measurement results with synthetic wavelength are consistent with that of single wavelength and higher measurement accuracy is achieved additionally. The analysis and experimental results show the validity of this method.

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