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Effect of the location and size of a single crack on first fundamental frequency of a cantilever beam using Fiber Optic Polarimetric sensors and characterisation of FBG sensors

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ABSTRACT
Fiber Optics Polarimetric Sensors (FOPS), utilizing first fundamental frequency mode and its harmonics, have already been used as damage detection tool. The FOPS technology is attractive in damage detection as it facilitates us with real time non-destructive health monitoring of different mechanical and civil structures. In this paper, the effects of the size and the location of a single crack on the frequency of first fundamental mode of a cantilever beam have been studied. A relation between the relative size of a crack and relative change in the first fundamental frequency has been established theoretically and then verified experimentally. Further, it has been shown that the cracks, close to the fixed end of the cantilever beam, have significant effect on the frequency of first fundamental mode and as the crack moves away from the fixed end, the effect on the frequency starts becoming diminished. Also the sensitivity of Fiber Bragg Grating (FBG) sensor against a single crack has been studied along both the directions; parallel to the axis of FBG sensor and perpendicular to the axis of FBG sensor. Experimental results show that the range of sensitivity in both the directions is almost the same but FBG is more efficient along its axis.

Keywords: FOPS, non-destructive, real time, FBG

1. INTRODUCTION
Fiber Optic Sensors (FOSs) have a lot of advantages over the other conventional, for instance, piezo-electric sensors. FOSs are light weight, independent of any sort of electromagnetic interference and have small diameter. These sensors can be easily embedded into composite and civil structures for SHM purposes. Metal structures do not allow embedment, so in this case FOSs are surface mounted. Conventional methods do not allow us the health monitoring of structures when they are in operation, as they are time consuming and require a lot of efforts\cite{1}. FOSs make such real-time online structural health monitoring very easy and effective. In most of the cases, structural integrity of the structures is very important and can’t be interfered by the presence of sensors. That is where non-destructive testing (NDT) methods come into picture. There are a number of NDT methods, such as acoustic emission (AE), ultrasonic scanning, shearography etc., but these classical NDT techniques are not capable of providing online structural health monitoring, because it is very difficult to make in situ implementation with them\cite{2}.

Different types of fiber optic sensing techniques are being used for different types of structural health monitoring methods. Extrinsic Fabry-Perot interferometer (EFPI) and Fiber Bragg Grating (FBG) sensors are used for local health monitoring. Multiplexing makes FBG suitable for global health monitoring. Fiber Optic Polarimetric Sensors (FOPSs) are the best suited for global health monitoring. Using FOPSs, both static and dynamic tests can be performed for the global health monitoring of different engineering structures.

A quantitative study, using FOPS, was performed for structural health monitoring and two factors; Static Damage Factor (SDF) and Dynamic Damage Factor (DDF), were defined for static and dynamic health monitoring respectively\cite{3}. But these factors give a very vague picture of state of damage. No systematic study has been performed so far. In this paper, a very systematic dynamic study of the effect of size and location of a single crack on the structural integrity of a cantilever beam, using FOPS, has been presented. A relation between the relative crack size and the normalized change in the frequency of the first fundamental mode of a cantilever has been established. Some well-designed sets of experiments were performed to verify this relationship. FBG sensors were examined for NDE of composite and aluminum structures\cite{2}. FBG were also used for the real-time cure monitoring of composite materials and were found very effective and efficient\cite{4}. In the present work, the sensitivity of FBG has been studied along both the directions; one
along the axis of FBG and another transverse to axis of FBG. In this paper, the gauge factors (GF) of two different strain gauges have also been calculated using FBG sensors.

2. PRINCIPLE

2.1 FOPS for dynamic test and crack theory

The block diagram of FOPS’s experimental set-up for dynamic test is shown in the figure 1. The experiments were done with the aluminum cantilever, as shown in the figure 1. The frequency of $n^{th}$ fundamental mode of a cantilever is given by the equation (1)\(^{(3)}\).

$$f_n = \left[\frac{a}{2}\right] \sin^2 \left(\frac{n\pi l}{2L}\right) \cdot \frac{EI}{\rho A}$$  \hspace{1cm} (1)

From equation (1), it can be clearly seen that the frequency of fundamental modes depends on the flexural stiffness (EI) of the cantilever. If the cantilever gets damaged, the stiffness will reduce and hence, the frequency of fundamental mode reduces.

Theoretically, a crack could be modelled by massless linear spring\(^{(5)}\). The cracked beam and its model are shown in the figures 2(a) and 2(b) respectively.

Experiments have been performed with an aluminum cantilever and the crack has been modelled by a spring of stiffness $K_x$. The change in the frequency of any fundamental mode is given by\(^{(6)}\):

$$\frac{\Delta f_n}{f_n} = \sin^2 \left(\frac{n\pi l}{2L}\right) \cdot \frac{EI}{K_x L}$$  \hspace{1cm} (2)

where EI is the stiffness, L is the length of the cantilever and l represents the distance of the crack from the fixed end of the cantilever.

The stiffness $K_x$ of the spring is given by\(^{(7)}\):

$$K_x = \frac{EI}{(5.346h) \cdot f(a/h)}$$  \hspace{1cm} (3)

where ‘h’ is the height, ‘a’ is the size of the crack and the function $f(a/h)$ is given by:

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Figure 1. Block diagram of FOPS for dynamic test.
\[ f(a/h) = 1.8624(a/h)^2 - 3.95(a/h)^3 + 16.375(a/h)^4 - 37.226(a/h)^5 + 78.81(a/h)^6 - 143.97(a/h)^7 + 66.56(a/h)^8 \]  
\( (4) \)

As the ratio \( a/h \) is small, higher orders of \( a/h \) could be neglected. Hence equation 4 reduces to:

\[ f(a/h) \approx 1.8624(a/h)^2 \]  
\( (5) \)

From equations (2) and (3) we have:

\[ \frac{\Delta f_n}{f_n} = \sin^2 \left( \frac{n \pi l}{2L} \right) \frac{5.346 \cdot h \cdot f(a/h)}{L} \]  
\( (6) \)

From equation (6) it is clear that the change in natural frequency depends on the location and size of the crack both. Equation (5) and equation (6) suggest that \( \Delta f_n/f_n \) is a quadratic function of \( a/h \).

**Figure 2:** (a) An aluminum cantilever with a single crack and (b) model of cracked cantilever beam.

### 2.2 Fiber Bragg Grating (FBG) sensors

In FBG sensors, grating (periodic refractive index profile) is inscribed into the core of the fiber. The Excimer laser (248 nm) and phase masks are used for this purpose. When light passes through a grating, a particular wavelength of light is reflected back by the grating. This wavelength depends on the distance between the consecutive grooves of the grating (grating period) and is called Bragg wavelength. The Bragg wavelength changes as the distance between the two consecutive grooves changes\(^8\). Bragg wavelength which is reflected back by FBG is given as:

\[ \lambda_B = 2n_{\text{eff}}\Lambda, \]  
\( (7) \)

where \( n_{\text{eff}} \) is the effective refractive index of the core of the fiber and \( \Lambda \) is the grating period of the sensor.

The block diagram of fiber Bragg grating sensing system is shown in the figure 3.
3. EXPERIMENTAL RESULTS AND DISCUSSION

3.1 Experimental verification of single crack theory by FOPS

Here an experiment has been performed with an aluminum cantilever of size $26.8 \times 0.32 \times 1.9$ cm$^3$. The vibrations were generated along Y-axis through impact (figure 2). A crack was created at different locations with different sizes. The frequency of first mode was noted every time. In figure 4, a snapshot from the oscilloscope showing the first fundamental frequency (35 Hz) of the aluminum cantilever has been presented.

Figure 4. A snapshot from oscilloscope showing the frequencies of first fundamental mode of the cantilever under experimentation.

Figure 5 shows the relative change in the frequency ($\Delta f_1/f_1$) of first fundamental mode with relative crack size (ratio of the crack size ‘a’ to the total height ‘h’ of the cantilever beam) at different crack locations. As $a/h$ is a small, equations (5) and (6) indicate that the quantity $\Delta f_1/f_1$ is a quadratic function of ($a/h$). From figures 5, it can be seen that however the function $\Delta f_1/f_1$ has both the quadratic and linear terms of $a/h$, the coefficient of linear term is very small in comparison to that of quadratic term. Hence, the linear term can be neglected. The presence of linear could be because of experimental errors like; the resolution of oscilloscope is not very high, relative change in the boundary conditions.
Sometimes oscilloscope is unable to tell the very exact values of frequencies also. Thus the function $\Delta f_n/f_n$ follows the relation established in equations (5) and (6).

**Figure 5.** Relative change in the first fundamental frequency of a cantilever with relative crack size at different crack locations.

**Figure 6.** Relative change in the first fundamental frequency with relative position of a crack with different crack sizes.
Similarly for different crack sizes, the relative change in the frequency of first fundamental mode (\( \Delta f_1/f_1 \)) has been plotted against the relative distance \((l/L)\) of the crack from the fixed end of the cantilever in figure 6. Clearly, the cracks of larger size have larger impact. The other thing which figure 6 brings into notice is that as the crack location is shifted away from the fixed end of the cantilever, the effect on the mode frequency diminishes.

### 3.2 Effect of a single crack on the sensitivity of an FBG along perpendicular and parallel to its axis

In this part it has been tried to find out that an FBG sensor is more effective in what direction; perpendicular or parallel to its axis? The specimen is shown in the figure 7. The weight (2 Kg) was put at different points along x-axis and y-axis both and Bragg wavelength has been measured for each points. Then a crack of 1.7 cm was made at the location \((0, 0)\) in the direction normal to FBG. The same weight was again put at the same points along both the axes and then the difference in Bragg wavelength was measured. Then the difference between the Bragg wavelengths of undamaged and damaged specimen

![Image of specimen with FBG](image)

**Figure 7.** Aluminum specimen with surface bonded FBG.

![Graph showing change in wavelength](image)

**Figure 8.** Change in wavelength along the direction normal to FBG.
against the loading position has been plotted in figures 8 and 9. In figure 8, the weight (2 Kg) was put along normal direction (y-axis) to the fiber axis and in figure 9, the weight was put along parallel (x-axis) to the fiber axis.

From figure 8 and figure 9, it is clear that although the sensitivity of FBG sensor and its range are approximately similar in both the directions, the drop in sensitivity is more regular along the axis parallel to FBG.

4. Conclusion

Experimental results show that the change in the frequency of fundamental mode follows a pattern and single crack theory points out that this pattern is predictable. Also, fundamental mode frequency is more sensitive for a crack closer to the fixed end of the cantilever. This theory could be used for the detection of location and size of a crack in the cantilever beams. Sensitivity of FBG sensor is independent of direction, but still the change in sensitivity is more regular and predictable in the direction parallel to FBG.

REFERENCES


