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Novel Adaptive Soft Input Soft Output demodulator for Serially Concatenated CPM Signals

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Abstract—In this paper, we propose a novel adaptive soft input soft output (SISO) demodulator that addresses the problem of joint data detection and channel estimation for serially concatenated continuous phase modulation (SCCPM) operating in a time-varying frequency-flat fading channel. Due to its ability of incorporating prior symbol probabilities and its soft output decisions, the proposed SISO demodulator is well suited for serially concatenated CPM systems in attaining better performance through iteration. The bit error rate (BER) results of the proposed iterative receiver and the BER comparison with SISO demodulators based on the per-survivor processing (PSP) and single estimation processing (SEP) methods are presented.

Index Terms—CPM, Joint Data Detection and Channel Estimation, Soft Input Soft Output(SISO), Concatenated Systems

I. INTRODUCTION

Bandwidth limitations have motivated considerable investigations into continuous phase modulation (CPM) techniques for communication systems. Moreover, owing to the constant envelope property, nonlinear amplifiers can be used with CPM for cost saving without undesired effects on the shape of transmitted signals. In this paper, it is assumed that the CPM receiver is moving at a constant velocity and an infinite number of reflected waves arrive at the receiving antenna in omnidirectional with a uniformly distributed angle. Therefore, the channel can be modeled as a Rayleigh flat fading channel [1]. Note that in fading channels, even with the perfect knowledge of channel information at the receiver, the performance may not satisfy communication requirements. As a solution, channel coding is introduced to protect the transmitted information sequence from the adverse effect of channel fading by introducing redundancy into the transmitted sequence. After the introduction of turbo codes, serially concatenated convolutional codes (SCCC) have shown to be suitable for iterative receivers and have comparable performance with turbo codes. Due to the bandwidth and power efficiency of CPM signals, the serially concatenation of convolutional encoder and CPM modulation will not only have comparable performance to SCCC, but also achieve better bandwidth efficiency.

A joint demodulation and decoding scheme, which involves the entire memory of both the demodulator and the decoder, thus generating a super trellis, can provide the optimum performance for SCCPM. However, the considerable complexity of the super trellis makes it difficult to use in real applications. Fortunately, an iterative receiver which exchanges information between the demodulator and the decoder alternately can achieve near optimum performance with a moderate complexity. In the literature, SCCPM with an iterative receiver has been analyzed [2]. PSP in [3] and SEP in [4] are two typical ways to perform joint data detection and channel estimation but their drawback is that they can only produce hard decisions which are not suitable for iterative receiver. In concatenated systems, a SISO module is essential for iterative processing. For this reason, a family of adaptive SISO (A-SISO) algorithms based on the PSP and SEP methods have been proposed in [5], [6]. We denote them as PSP based A-SISO and SEP based A-SISO respectively. They fall into two categories: the forward only and forward-backward algorithms. It has been proven in [6] that the PSP based A-SISO algorithm is better than the SEP based A-SISO when channel variation is considerable. Meanwhile, in PSP based A-SISO algorithm, the superiority of A-SISO forward-only (A-SISO-FO) over the forward-backward (A-SISO-FB) counterpart has been proven in [7] for CPM signals.

As the previous PSP based A-SISO and SEP based A-SISO can not guarantee the channel tracking capability and the accuracy of survivor branch selection, in the proposed method, channels over an observation interval are predicted. This prediction over an interval guarantees high tracking capability and at the same time maintains the accuracy of survivor path selection. Then a decision is made at the first symbol of the observation interval. The detected symbol is fed back for channel updating and state updating for the next symbol detection. BER comparisons with existing algorithms are presented to demonstrate the efficacy of the proposed algorithms.

II. PRELIMINARY

A. System description for SCCPM

The block diagram of the transmitter and the receiver of a SCCPM system is depicted in Fig 1 [8]. The information bit...
sequence $I_n$ is first encoded by a convolutional encoder. The output coded bit sequence $C_n$ is input to a random interleave. The CPM modulator takes the interleaved bit sequence $a_n$ and pilots symbols (for channel initialization) as its input and outputs the modulated signal $x_n$. At the receiver, the proposed SISO module acts not only as a CPM demodulator but also a channel estimator. Its ability of incorporating a priori symbol probability and the output of soft information makes it suitable for iterative processing. In each iteration, the SISO demodulator takes the soft information $L_n(a_n)$ provided by the decoder from the previous iteration as its priori probability to perform joint symbol by symbol MAP demodulation and channel tracking. The output of SISO demodulator is the log likelihood ratio (LLR) of coded bit $a_n$, denoted as $L(a_n)$. The LLR related to a priori probabilities and a posteriori probabilities is given by $L(a_n) = L_e(a_n) + L_o(a_n)$ where $L_e(a_n)$ is the extrinsic information. Note that in iterative processing, the a priori information to a stage cannot include the information added by that stage in the previous iteration [9]. To achieve this, the output LLRs of demodulator and decoder must be subtracted from its input priori LLR and then delivered to the next stage [8]. Therefore, we have $L_e(a_n) = L(a_n) - L_o(a_n)$. These extrinsic LLRs $L_e(a_n)$ are then deinterleaved to obtain the LLRs of $C_n$, which is denoted as $L_n(C_n)$. The deinterleaver is essential to make the errors independent and thereby the subsequent decoder is able to correct the errors. At the end a SISO decoder employing the BCJR algorithm outputs both the final decision of information symbol $\hat{I}_n$ and the LLRs of the coded sequence $L(C_n)$. Before passing to the next stage, $L_e(C_n) = L(C_n) - L_o(C_n)$ is evaluated and $L_e(C_n)$ is interleaved to provide the updated priori probability for the SISO demodulator for the next iteration.

### B. CPM signal

The complex envelope of a CPM signal is $x(t) = \sqrt{2E/T}e^{j\phi(t,A)}$ [10] where $E$ is the average signal energy per symbol, $T$ is the symbol duration time and $A$ is the sequence of transmitted symbol $a$. The information-carrying phase of $x(t)$ between $nT$ and $(n+1)T$ is given by

$$\phi(t,A) = \pi h \sum_{i=0}^{n} a_i + 2\pi h \sum_{i=n-L+1}^{n} a_i q(t - iT)$$ (1)

where $h$ is the modulation index, $a_i \in \{\pm 1, \pm 3, \ldots \pm (M-1)\}$ is the symbol to be transmitted, $L$ is the CPM correlation length, $M$ is the alphabet size and $q(t)$ is the phase response which has values of 0 and 1/2 when $t$ is equal to 0 and $LT$ respectively. According to Eq. (1), for any symbol interval from $nT$ to $(n+1)T$, the information-carrying phase is uniquely defined by the phase state $a_n$, the correlative state vector $\{a_{n-1}, a_{n-2}, \ldots a_{n-L+1}\}$ and the present input symbol $\alpha_n$. It is well known that the CPM modulator can be described by a finite state machine [11]. As a consequence, it can be presented in a trellis diagram with associated states and state transitions. The states at the $n$th symbol interval can be represented as $s_n = \{\theta_n, a_{n-1}, a_{n-2}, \ldots a_{n-L+1}\}$

In this paper, we consider communication channel conditions which can be generally modeled as Rayleigh flat fading. The frequency flat fading channel will cause a multiplicative distortion to the transmitted signal. Therefore, after symbol rate sampling, the received baseband signal transmitted over a Rayleigh flat fading channel can be represented as $r_n = h_n x_n + v_n$ where $h_n$ is the channel coefficient at time $nT$, $v_n$ is the additive white Gaussian noise (AWGN) with covariance $\sigma^2$ and $x_n$ is the sampled CPM signal. The channel model is same as the one in [12].

### III. The Proposed SISO Demodulator

We define three vectors in a block $A_{1n} = \{a_1, a_2, \ldots, a_{N_b}\}$, $R_{1n} = \{r_1, r_2, \ldots, r_{N_b}\}$, $H_{1n} = \{h_1, h_2, \ldots, h_{N_b}\}$ where $N_b$ is the block length, $A_{1n}$ and $R_{1n}$ represent the input data sequence at the transmitter and the received sequence at the receiver, $H_{1n}$ is a vector of time-varying flat fading channel coefficients. Due to the correlated fading and the symbol dependence introduced by CPM modulation, the symbol by symbol MAP detection of $a_k$ in a block $A_{1n}$ under known $H_{1n}$ is

$$\hat{a}_k = \arg\max_{\tilde{a}_k \in M} \sum_{\tilde{A}_{1n}^{N_b}} P(R_{1n}^{N_b} | \tilde{A}_{1n}^{N_b}, H_{1n}^{N_b}) P(\tilde{A}_{1n}^{N_b})$$ (2)

where $\hat{a}_k$ and $\tilde{a}_k$ are the estimate and hypothesis of $a_k$. When hard output is used, symbol $a_k \in \{\pm 1, \pm 3, \ldots \pm (M-1)\}$ with the highest $P(\tilde{a}_k)$ will be the detected symbol. On the other hand, if soft output is used, the log likelihood ratio of $a_k$ denoted as $L(a_k)$ is delivered to the next step for further processing. Direct evaluation of Eq. (2) has a prohibitively high complexity. The BCJR algorithm proposed in [13], is computationally efficient for solving this problem in a trellis structure. Similar to the approximation method in [12], the decision criterion of Eq. (2) can be approximated by

$$\hat{a}_k = \arg\max_{\tilde{a}_k \in M} \sum_{\tilde{A}_{1n}^{N_b}} P(R_{1n}^{k+N_b-1} | \tilde{A}_{1n}^{k+N_b-1}, \tilde{A}_k^{k-1}, \tilde{H}_k^{k+N_b-1}) P(\tilde{A}_{1n}^{k+N_b-1})$$ (3)

where $\tilde{H}_k^{k+N_b-1}$ is the estimated channel and we use a Kalman filter (KF) for channel prediction over the observation length $N_{o}$. As the input of the CPM modulator is the output of the interleaver, the input symbols can be considered as independent. Thus, we have

$$P(\tilde{A}_{1n}^{k+N_b-1}) = \prod_{n=k}^{k+N_b-1} P(a_n)$$ (4)

Using the definition of conditional probability function and substituting Eq. (4) into Eq. (3), we get

$$\hat{a}_k = \arg\max_{\tilde{a}_k \in M} \sum_{\tilde{A}_{1n}^{k+N_b-1}} \prod_{n=k}^{k+N_b-1} P(r_n | R_n^{k-1}, \tilde{A}_k^{k-1}, \tilde{H}_k^{n}) P(\tilde{a}_n)$$ (5)

For simplicity, here we only consider binary transmission which has $M = 2$. When soft outputs are required for subsequent processing, the LLRs would be the output evaluated.
by
\[
L(a_k) = \frac{\sum_{\hat{a}_k^{-1} = a_{k-1} + 1}^{k+N_o-1} p(r_n | R_{n-o}^{n-1}, \hat{A}_k^{n-1}, H_{n-o}^{n-1}) P(a_n)}{\sum_{\hat{a}_k^{-1} = a_{k-1} + 1}^{k+N_o-1} p(r_n | R_{n-o}^{n-1}, \hat{A}_k^{n-1}, H_{n-o}^{n-1}) P(a_n)}
\]

Eqs. (5) and (6) are suboptimal MAP criteria derived from the optimal MAP criterion in Eq. (2). As the quantity in Eq. (2) can be evaluated by the fast computational method of the BCJR algorithm, here we present a truncated BCJR algorithm which can be used to calculate Eq. (6) efficiently. Following [13], it can be shown that

\[
L(a_k) = \frac{\sum_{ST^{k+1}} \gamma(s_k \rightarrow s_{k+1}) \alpha(s_k) \beta(s_{k+1}) P(a_k = 1)}{\sum_{ST^{k+1}} \gamma(s_k \rightarrow s_{k+1}) \alpha(s_k) \beta(s_{k+1}) P(a_k = -1)} \tag{7}
\]

where \(ST^{k+1}\) and \(ST^{k-1}\) are the branches connecting state at time \(k\) to \((k + 1)\) which are caused by the input symbol +1 and -1, \(\alpha\) and \(\beta\) are the forward and backward coefficients, respectively.

For the \(k\)th symbol detection, APP is calculated under a truncated interval which starts at the \(k\)th symbol time and ends at \((k + N_o - 1)\)th symbol time with channel information \(H_k^{k+N_o-1}\) predicted from the estimated \(H_1^{k-1}\). In this paper, we use \(s_k\) to denote state at \(k\)th time. As the data decision is made at the first symbol of the observation interval, only the initialization is necessary when considering the evaluation of the forward coefficients for every truncated interval. This initialization can be easily obtained by the previously decided symbols and we denote \(s_k(\hat{A}_k^{k-1})\) as the corresponding state of the decided data sequence \(\hat{A}_k^{k-1} = \{\hat{a}_1, \hat{a}_2, ..., \hat{a}_{k-1}\}\). Based on the approximation of Eq. (5), we have \(\alpha(s_k = s_k(\hat{A}_k^{k-1})) = 1\) and for other states at the \(k\)th symbol time, \(\alpha\) will be 0. The initialization of \(\beta(s_k+N_o)\) = 1 for all possible states at the \((k+N_o)\)th symbol time. The forward coefficient \(\alpha\) and backward coefficient \(\beta\) can be calculated recursively based on \(\gamma\). As data detection uses the predicted channel, it has an irreducible error covariance. A more accurate APP can be obtained if we consider and incorporate the channel estimation error. It has been shown in [12] that Kalman procedure can provide exact prediction error covariance \(P_{\Delta h}\) which is an essential requirement for the symbol by symbol MAP detection. We can thus modify the truncated BCJR algorithm incorporating the channel estimation error as follows

\[
\gamma(s_k \rightarrow s_{k+1}) = \exp\left(\frac{1}{2\sigma_h^2} \left| r_k - \hat{h}_k x(s_k \rightarrow s_{k+1}) \right|^2 \right) \tag{8}
\]

where \(x(s_k \rightarrow s_{k+1})\) denotes the CPM signal corresponding to state transition from \(s_k\) to \(s_{k+1}\). With the estimated \(\gamma\), forward coefficient \(\alpha\) and backward coefficient \(\beta\), both the hard output \(\hat{a}_k\) and soft output \(L(a_k)\) can be easily obtained by Eq. (7). The transmitted CPM signal during the \(k\)th symbol time \(s(k)\) can be uniquely determined by the value of \(\hat{a}_k\) and \(s_k\). Therefore, the KF updating equations in [12] can be adopted for channel updating. Soft output \(L(a_k)\) is presented to the decoder for further processing and the hard decision \(\hat{a}_k\) is fed back for two purposes: one is for establishing initial state for the next symbol estimation in the truncated BCJR procedure and the other is for regenerating the input CPM signal during the \(k\)th symbol time for evaluating channel coefficients in the channel updating procedure. Subsequently, the soft information of \(L(a_{k+1})\) can be obtained using the predicted channel and the predicted channel error covariance.

### IV. Simulation Results

In the following simulations, the convolutional encoder has a generator polynomial \([5, 7]\)s, constraint length 3 and rate 1/2. A random interleaver is applied, and for simplicity MSK is used as the CPM modulator. Fig. 2 shows the different requirements of interleaver size for different fading rates. The BER curves are obtained after five iterations under \(E_b/N_0 = 8\) dB. When the channel is slow time-varying such as \(f_dT = 0.001\), the BER curve lowers along with the increase of interleaver size where \(f_dT\) is the normalized doppler frequency. However, for a faster time-varying channel with \(f_dT = 0.01\), when interleaver size \(N_i > 4000\), performance does not improve. It can be concluded that the proposed iterative receiver in a slow time-varying channel needs a larger interleaver size than the one in a fast fading channel. The reason is the following. The time when the channel goes through a deep fade is long when the channel is slow fading, causing a long error burst at the output of the proposed SISO demodulator. Therefore, a large interleaver size is required to make these errors occur in a non-contiguous manner.

In Figs. 3 and 4, we illustrate the bit error rate performance with different iteration numbers of the proposed SISO module in time-varying fading channels with \(f_dT = 0.001\) and \(f_dT = 0.01\), respectively. In the figures \(ite\) refers to the iteration number. The frame length is 32 symbols containing one pilot symbol for initialization. The performance with known channel conditions is also included in these figures for comparison. The largest performance gap for both figures occurs between the zeroth iteration and the first iteration. Through iterations, performance can be improved. However, the performance improvement after the fifth iteration is negli-
In this paper we propose a novel adaptive SISO demodulator which addressed the problem of joint data-channel estimation for SCCPM signals in time-varying flat fading channels. In this method, the prediction over an interval guarantees zero decision delay and at the same time maintains the accuracy of the survivor path selection. Due to its ability of incorporating priori symbol probabilities and the use of soft outputs, the proposed SISO demodulator is well suited for an iterative receiver such as the receiver used with turbo codes. The effects of the interleave size has been analyzed. BER comparisons with existing algorithms were presented to demonstrate the efficacy of the proposed algorithms.

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