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<td>Author(s)</td>
<td>Xu, Y.; Huang, G. H.; Xu, T. Y.</td>
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Inexact Management Modeling for Urban Water Supply Systems

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Received 27 December 2010; revised 13 September 2011; accepted 20 October 2011; published online 27 September 2012

ABSTRACT. The water shortage problems have become main obstacles for sustainable socio-economic development of many cities. There was an urgent need to develop effective decision-support tools for supporting water-supply schemes under multiple uncertainties. In this study, an interval-parameter stochastic chance-constrained programming (IPSCCP) model was developed for urban water supply system. It integrated stochastic chance-constrained programming (SCCP) and interval linear programming (ILP) into a general optimization framework. IPSCCP could deal with uncertainties expressed as both discrete intervals and probability distributions; meanwhile, it was also useful for helping analyze the reliability of satisfying system constraints. A multi-layer urban water supply system, including water resources, collection and treatment facilities, reservoirs, and consuming zones, was used to demonstrate the feasibility and applicability of proposed method. The results indicated that IPSCCP was capable of helping understand the effects of uncertainties and was useful for urban water managers to gain an in-depth insight into the tradeoffs between system cost and reliability of constraints satisfaction. The study would be a new attempt in advancing an integrated uncertainty-analysis tool for urban water supply system. It was also suggested that other uncertain approaches are integrated into an IPSCCP framework for reflecting more complex conditions.

Keywords: urban water resources management, stochastic chance-constrained programming, interval linear programming, uncertainty, optimization

1. Introduction

The shortage of water resources has become main obstacles for sustainable socio-economic development in many cities over decades. Currently, about 700 million people in the world live below the benchmark, which is a threshold for maintaining the operation of socio-economic and environmental system; this figure is expected to reach 3 billion by 2025 as water stress intensifies (Human Development Report, 2006). The water shortage problems were mainly caused by the increase of water demand due to rapid population growth, development in industrial and agricultural production, and shrinkage of water supplies. It is thus necessary to develop effective decision-support tools for supporting urban water supply management system (Yang et al., 2005; Ping et al., 2010; Fattahi and Fayyaz, 2010). However, the system is complicated with uncertainties that may exist in many system components and these complexities are further compounded by interactions among various system parameters. This would bring significant difficulties in formulating the management models and generating effective solutions. Therefore, based on a comprehensive water supply system framework, the incorporation of effective uncertain optimization methods is desired to help evaluate the effects of various urban water management policies.

During the past decades, many inexact optimization methods were developed to describe and tackle uncertainties associated with various management system (Slowinski, 1986; Jenkins and Lund, 2000; Liu et al., 2007a,b, 2008; Xi et al., 2008; Liu et al., 2009; Sun and Huang, 2010; Xu and Qin, 2010; Cao and Huang, 2011; Huang and Cao, 2011; Qin, 2011). The majority of these methods were related to stochastic mathematical programming (SMP), fuzzy mathematical programming (FMP) and interval linear programming (ILP). For example, Wilchfort and Lund (1997) developed a shortage management model, where a two-stage stochastic programming was used to tackle the uncertainties involved in cost and hydrologic aspects. The model was expanded in several case studies and has advantage in incorporating the effects of the seasonal shortages and uncertainties related to long-term and short-term management options. Bender and Simonovic (2000) proposed a fuzzy compromise approach for supporting water sources planning under uncertainty, where fuzzy ranking measures was used to reflect the decision maker’s attitudes to the risk and determine the importance of different decision alternatives. These studies demonstrated that FMP and SMP are suitable in describing and handling uncertain information in water management systems.

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ISSN: 1726-2135 print/1684-8799 online
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FMP is mainly used to reflect the ambiguous coefficients and relations in optimization models as well as the vague information of decision makers’ implicit knowledge through expert consultation or public survey. SMP was used to tackle the stochastic uncertainties expressed as random variables with probabilistic distribution functions (PDFs) based on complete long-term historical records. However, the process of collecting and analyzing the above information of both two methods is time-consuming and requires additional manpower in many practical applications. Moreover, it is also difficult to solve a large-scale SMP or FMP model, even though such information is available.

ILP, which was proposed by Huang et al. (1992), was integrated with other inexact methods and was extensively applied in water resources management fields. For example, Huang and Loucks (2000) developed an inexact two-stage stochastic programming method which could tackle uncertainties expressed as the discrete intervals and random variables. The reasonable solutions have been obtained. Li et al. (2009) proposed an inexact multistage joint-probabilistic programming method for tackling uncertainties presented as interval values and joint probabilities. The results demonstrated that reasonable solutions for continuous and binary variables had been generated. From previous studies, it was indicated that integrated uncertain methods were more suitable in tackling real-world problems where ILP was a good complement for FMP and SMP, especially when available uncertain information was limited. However, the structure and components of water resources management system tended to be simplified in order to better describe and reflect the characteristics of uncertain method. This would affect its feasibility and applicability in real-world applications.

Based on the facts mentioned above, this study aims to develop an interval-parameter stochastic chance-constrained programming (IPSCCP) model and apply it to an urban water supply system under multiple uncertainties. The proposed model can effectively deal with uncertainties expressed as not only probabilistic distributions but also discrete intervals. It allows some constraints with random variables are satisfied at a prescribed range of probability levels. A variety of cost-effective interval solutions can be obtained. An urban water supply management case will be used to demonstrate the applicability of the proposed method. The objective entails: (i) formulation of an IPSCCP model based on both SCCP and ILP models; (ii) application of the developed model to an urban water supply management case; (iii) analysis of the results and discussion of the model applicability.

2. Methodology

2.1. Stochastic Chance-constrained Programming

Stochastic Chance-constrained programming (SCCP), as a main SMP approach, is advantageous that it does not require all of the constraints be absolutely satisfied. Instead, they only need to be satisfied at a prescribed range of probability levels, such that a variety of cost-effective solutions are generated (Loucks et al., 1981). A general SCCP model can be written as follows (Charnes et al., 1972):

Minimize $f = \sum_{j=1}^{J} c_j x_j$  \hspace{1cm} (1a)

Subject to:

$P \left[ \sum_{j=1}^{J} a_{ij} x_j \leq b_i(s) \right] \geq q_i, \forall i$  \hspace{1cm} (1b)

$x_j \geq 0, \forall j$  \hspace{1cm} (1c)

$c_i, a_{ij} \neq 0, \forall i, j$  \hspace{1cm} (1d)

where $j$ is the index of decision variables, where $j = 1, 2, ..., J$, and $J$ is the total number of decision variables; $i$ is the index of chance-constraints, where $i = 1, 2, ..., I$, and $I$ is the total number of chance-constraints; $z$ is the index of acceptable probability levels of constraints satisfaction, where $z = 1, 2, ..., Z$, and $Z$ is the total number of given probability level; $x_j$ are deterministic decision variables $b_i(s)$ is the random numbers with probability distribution functions $p(s)$; $c_j$ and $a_{ij}$ are deterministic coefficients, respectively; $P [ \cdot ]$ denotes probability of events in $\left[ 0, 1 \right]$; $q_i$ is acceptable probability levels. Theoretically, it is possible that the probability level of satisfying random constraints could be any value ranging from 0 to 1. In fact, a low probability level means that the random constraints can hardly be satisfied and this could lead to an increase of system-failure risk. Under such a case, the feasibility and reliability of the obtained solutions would decrease. In real-world applications, the benchmark values of $q_i$ are normally set by the decision maker at 0.8 or 0.9. In model (1), Equation (1a) is an objective function with deterministic coefficients and decision variables. Equation (1b) is a chance-constraint with random parameters in the right-hand side and crisp parameters in the left-hand side, respectively. The procedures (1c) and (1d) are technical constraints, respectively. According to (Charnes et al., 1972), the chance-constraint (1b) can be transformed to their respective crisp equivalents:

$\sum_{j=1}^{J} a_{ij} x_j \leq b_i^{-q_i}, \forall i$  \hspace{1cm} (2a)

$b_i^{-q_i} = F_i^{-1}(q_i), \forall i, q_i$  \hspace{1cm} (2b)

where $F_i^{-1}(q_i)$ is given the cumulative distribution functions (CDFs) of $b_i$, i.e. $F_i(b_i)$. According to Cheng et al. (2009), the constraints (1b) can be transformed to their respective crisp equivalents (2) only for some specific distributions and certain levels of $p_i$, such as the cases when (i) $a_{ij}$ are deterministic and $b_i$ are random (for all $p_i$ values); (ii) $a_{ij}$ and $b_i$ are discrete random coefficients; (iii) $a_{ij}$ and $b_i$ have Gaussian distributions. In this paper, $a_{ij}$ are deterministic and $b_i$ are random, such that constraint (1b) can be transformed into (2). Finally, the deterministic objective function values and decision variables (i.e. $f_{opt}$ and $x_{ij, opt}$ at different probability levels can be obtained.
2.2. Interval Linear Programming

Interval linear programming (ILP), which is based on interval number theory, was firstly proposed by Huang et al. (1992). In a typical ILP model, all or part of the model parameters expressed as interval numbers can be directly incorporated within its optimization process and resulting solution. Referring to Huang et al. (1992), an ILP model can be written as follows:

\[
\text{Minimize } f^\pm = \sum_{j=1}^J c^\pm_j x^\pm_j \tag{3a}
\]

Subject to:

\[
\sum_{j=1}^J a^\pm_i x^\pm_j \leq b^\pm_i, \forall i \tag{3b}
\]

\[
x^\pm_j \geq 0, \forall j \tag{3c}
\]

\[
c^\pm_i, a^\pm_i \neq 0, \forall i, j \tag{3d}
\]

where \( x^\pm_j \) is a vector of decision variables expressed as interval numbers; \( a^\pm_i \), \( b^\pm_i \) and \( c^\pm_i \) are interval-format vectors of coefficients in objective function and constraints. The term “interval number” is expressed as \([a^-_i, a^+_i]\) where \( a^-_i \) and \( a^+_i \) are lower and upper bounds of \( a^\pm_i \), respectively. As proposed by Huang et al. (1992), the interactive two-step method can be used to solve model (3). Finally, the objective value and decision variables as discrete intervals are obtained, being \( f^\pm_{\text{opt}} = [f^-_{\text{opt}}, f^+_{\text{opt}}] \) and \( x^\pm_{\text{opt}} = [x^-_{\text{opt}}, x^+_{\text{opt}}] \), respectively.

2.3. Interval-parameter Stochastic Chance-constrained Programming

In many real-world problems, it is very difficult to find that all uncertain variables are presented as uniform uncertain format. Therefore, the system uncertainties should be tackled by various uncertain analysis methods based on uncertain properties and quality of available data information. ILP is capable of handling the boundary uncertainties with lower data requirement than those of SCCP, however, it may become infeasible when the right-hand side parameters in constraints are highly uncertain. To realize optimal objective function values, the constraints should be satisfied within an acceptable range. SCCP can effectively handle the above problems. Therefore, an interval-parameter stochastic chance-constrained programming (IPSCCP) model is proposed and is formulated as follows:

\[
\text{Minimize } f^\pm = \sum_{j=1}^J c^\pm_j x^\pm_j \tag{4a}
\]

Subject to:

\[
\sum_{j=1}^J \left( a^\pm_i x^\pm_j \right) \leq b^\pm_i, \forall i \tag{4b}
\]

\[
x^\pm_j \geq 0, \forall j \tag{4c}
\]

\[
c^\pm_i, a^\pm_i \neq 0, \forall i, j \tag{4d}
\]

where all coefficients and variables in the objective function (4a) is presented as interval numbers. Equation (4b) is the constraint with interval and random variables. Based on Eqs (2), the constraint (4b) can be transformed into the interval-format constraints as follows:

\[
\sum_{j=1}^J a^\pm_i x^\pm_j \leq b^\pm_i, \forall i \tag{5}
\]

where \( b^\pm_i = F^{-1}(b_i) \) and \( F_i^{-1}(b_i) \) is given the cumulative distribution functions (CDFs) of \( b_i \), i.e. \([F_i(b_i)]\). As a result, model (4) can be transformed into a general ILP models. As proposed by Huang et al. (1992), the solution for ILP model can be obtained through a two-step method. A sub-model corresponding to \( f^- \) (when the objective function is to be minimized) is first formulated and solved, and then the relevant sub-model corresponding to \( f^+ \) can be formulated based on the solution from the first sub-model. Finally, the objective values and decision variables expressed as discrete intervals at various acceptable levels of constraints satisfaction will be obtained. Figure 1 shows the general framework of an IPSCCP model. The detailed procedures of formulating and solving the model are summarized as follows:

Step 1: Identify the uncertain variables and acquire the related probabilistic distribution and discrete-interval information.

Step 2: Formulate an IPSCCP model.

Step 3: Convert the stochastic chance-constraints to their respective crisp equivalents.

Step 4: Reformulate and solve sub-model one, which corresponds to \( f^- \) if the objective function is to be minimized;
Step 5: Formulate and solve sub-model two, which corresponds to $f^+$, based on the obtained solutions from $f^-$.
Step 6: Generate the final solutions of $f^\pm_{opt} = [f^+_{opt}, f^-_{opt}]$ and $x^\pm_{opt} = [x^+_{opt}, x^-_{opt}]$.

Figure 2. Integrated urban water supply management system.

3. Case study

3.1. Overview of the Study Case

In this study, an urban water supply system case will be used for demonstrating the applicability of proposed method. This case was adapted from the real case discussed by Fattahi and Fayyaz, (2010). The water shortage problem of research region is intensified by unequal distribution of rainfalls in different seasons and continuous decreasing of the rainfalls in recent years. It is necessary to develop effective tools for assisting urban water service providers and government agencies to generate rational water resources management scheme based on a comprehensive management framework of urban water supply system.

Figure 2 shows the general diagram of the water supply system, which is a water supply network with four layers, including water sources, treatment facilities, storage facilities and water users. Many nodes representing sub-components with similar characteristics are included in each layer. For example, the municipal, agricultural and industrial sectors belong to the layer of water users. In a typical water supply network, some optional water-distribution paths are needed to be prescribed by local authorities, and they are reflected by the lines linking the nodes in different layers. The operation process of the water supply system is that the available water from the two resources (i.e. surface water and groundwater) should firstly be collected and transferred to treatments for purifying. Then the purified water are transported to the reservoirs and finally pumped to consuming zones. The water managers are responsible for solving the following problems: how much water must be extracted and supplied from each water resources during the planning periods? How much water can be treated by different treatment plants? How much water can be transferred to reservoirs subjected to the limitation of their storage capacities and requirement from the consuming zones? Which paths can be used in entire water supply system? To tackle such problems, optimization models are needed.

Many system parameters in water supply system, such as water demand amounts from consuming zones, available water amounts from water resources, purified and stored capacities of treatment plants and reservoirs may appear uncertain. Generally, the water demand amounts of consuming zones, the storage capacities of reservoirs, the treatment capacities of treatment plants and maximum extracted water amounts from two water resources own long-term historical record for generating the PDFs, thus they may be assumed as random variables. Table 1 shows the water demand amounts at one year. Other parameters, such as the inventory amounts of the water resources, treatments and reservoirs during the first period, and the cost of purchase, transfer and treatment suffer from a lack of complete data survey and record. These parameters are more suitable to be described by discrete intervals. Table 2 shows the inventory amounts and maximum capacities of water resources, treatments and reservoirs, respectively.

3.2. Formulation of an IPSCCP Model

Based on the general IPSCCP model, a specific water supply model for the research region can be formulated as follows (Fattahi and Fayyaz, 2010):

Objective function:

$$\min f^+ = \sum_{j=1}^{J} \sum_{k=1}^{K} JT_{j,k} PR_{j,k} + \sum_{j=1}^{J} \sum_{k=1}^{K} JT^+_{j,k} CJT_{j,k}^+ + \sum_{r=1}^{R} \sum_{z=1}^{Z} XTR_{r,z} CTR_{r,z} + \sum_{r=1}^{R} \sum_{z=1}^{Z} XZ_{r,z} CRZ_{r,z}$$

Subject to:

Consuming constraints:

$$P_{s} \left( \left\{ \sum_{k=1}^{K} \left( 1-LXZ_{r,z}^k \right) * XZ_{r,z}^k \geq D_{s} \right\} \right) \geq q_{s}, \quad \forall k, q_{s}, \quad (6b)$$

$$XRZ_{r,z} \leq ZRZ_{r,z} * U \quad \forall r, z, k \quad (6c)$$

Reservoirs constraints:

$$IR_{r,z}^+ = IR_{r,z}^{-} + \sum_{k=1}^{K} \left( 1-LXZ_{r,z}^k \right) * XTR_{r,z}^k - \sum_{k=1}^{K} XRZ_{r,z}^k, \quad \forall r; k = 2, 3, \ldots, K \quad (6d)$$
Table 2. Information of the Water Resources, Treatment Plants and Reservoirs

<table>
<thead>
<tr>
<th>Information of water resources</th>
<th>Beginning inventory ($\times 10^3$ m³)</th>
<th>Maximum capacities ($\times 10^3$ m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dam</td>
<td>[15000, 19000]</td>
<td>(4600, 480)</td>
</tr>
<tr>
<td>Well</td>
<td>[2050, 2950]</td>
<td>(3800, 365)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Information of treatment plants</th>
<th>Beginning inventory ($\times 10^3$ m³)</th>
<th>Maximum capacities ($\times 10^3$ m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment plant 1</td>
<td>[5, 7.5]</td>
<td>(1900, 220)</td>
</tr>
<tr>
<td>Treatment plant 2</td>
<td>[10, 13]</td>
<td>(3400, 245)</td>
</tr>
<tr>
<td>Treatment plant 3</td>
<td>0</td>
<td>$+\infty$</td>
</tr>
<tr>
<td>Treatment plant 4</td>
<td>0</td>
<td>$+\infty$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Information of reservoirs</th>
<th>Beginning inventory ($\times 10^3$ m³)</th>
<th>Maximum capacities ($\times 10^3$ m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir 1</td>
<td>[16, 26]</td>
<td>(4500, 420)</td>
</tr>
<tr>
<td>Reservoir 2</td>
<td>[6.5, 13.5]</td>
<td>(720, 60)</td>
</tr>
<tr>
<td>Reservoir 3</td>
<td>[1, 3.5]</td>
<td>(230, 15)</td>
</tr>
<tr>
<td>Reservoir 4</td>
<td>[6.5, 13.5]</td>
<td>(440, 35)</td>
</tr>
<tr>
<td>Reservoir 5</td>
<td>[2, 4.5]</td>
<td>(230, 15)</td>
</tr>
<tr>
<td>Reservoir 6</td>
<td>[22, 38]</td>
<td>(700, 50)</td>
</tr>
<tr>
<td>Reservoir 7</td>
<td>[4, 6.5]</td>
<td>(440, 35)</td>
</tr>
</tbody>
</table>

*Where $[a_l, a_u]$ represents the interval numbers where $a_l$ and $a_u$ are the lower and upper bounds, respectively; $(m_l, d_l)$ represents the random variables where $m_l$ and $d_l$ are the mean values and standard deviation values, respectively.

\[ \begin{align*}
IR_{ro}^k &= IRO^k + \sum_{r=1}^{T} (1 - LX_{tr}^r) * XTR_{ro}^k, \forall \ r \in R \tag{6e} \\
XTR_{ro}^k &\leq ZTR_{ro}^k * U \quad \forall \ t, \ r, \ k \tag{6f} \\
\end{align*} \]
Leakage rate constraints:

\[ \sum_{r_1 = 1}^{R} \sum_{t_1 = 1}^{T} \sum_{j_1 = 1}^{J} X_{JTK} \cdot \sum_{r = 1}^{R} \sum_{t = 1}^{T} \sum_{j = 1}^{J} X_{JTR} \cdot \sum_{z = 1}^{Z} X_{ZRK} \leq T_{L} \]  

(6a)

Technical constraints:

\[ X_{JTK} \geq 0, \quad X_{JTR} \geq 0, \quad X_{ZRK} \geq 0, \quad \forall j, t, r, z, k \]  

(6b)

where \( f \) is net system cost ($); \( k (k = 1, 2, \ldots, K) \) is index of time periods where \( K \) is number of time periods; \( j, t, r \) and \( z (j = 1, 2, \ldots, J; t = 1, 2, \ldots, T; r = 1, 2, \ldots, R; z = 1, 2, \ldots, Z) \) are indexes of specific water resources, treatment plants, reservoirs and consuming zones, respectively; \( J, T, R, Z \) and \( N \) are numbers of water resources, treatment plants, reservoirs and consuming zones; \( B_{RJ} \) is the recovered water for each water resource \( j \) in each season \( k \) (m³); \( C_{TR} \), \( C_{RK} \), and \( C_{ZK} \) are the transferred cost of water from water resources \( j \) to treatment plants \( t \), from treatment plants \( t \) to reservoirs \( r \) and from reservoirs \( r \) to consuming zones \( z \), respectively ($); \( D_{RJK} \) is the amount of water required for consuming zone \( z \) in season \( k \) (m³); \( I_{ROj} \) and \( I_{ROk} \) are the inventory of each reservoir \( r \) at the first of the planning horizon and the end of each season \( k \), respectively; \( I_{TO} \) and \( I_{TT} \) are the inventory of each treatment \( t \) at the first of the planning horizon and the end of each season \( k \), respectively (m³); \( I_{ROj} \) and \( I_{ROk} \) are the inventory of each water resource \( j \) at the first of the planning horizon and the end of each season \( k \), respectively (m³); \( L_{XJtr} \), \( L_{XT} \) and \( L_{XZr} \) are the leakage rate of water in network from water resources \( j \) to treatments \( t \), treatments \( t \) to reservoirs \( r \) and reservoirs \( r \) to consuming zone \( z \), respectively (%); \( M_{J} \) is the maximum amount of water can be exited from water resources \( j \) at each season \( k \) (m³); \( P_{R} \) is the purchasing cost of water from water resources \( j \) at each season \( k \) ($); \( q \) is acceptable level of constraints-satisfaction. \( T\) is the allowed maximum leakage amounts (m³); \( F_{R} \) and \( F_{T} \) are the capacity of reservoirs \( r \) and treatment \( t \) at each season \( k \) (m³); \( X_{JTR}, X_{ZRK} \) and \( X_{ZRK} \) are decision variables representing the amount of water transferred from water resources \( j \) to treatments \( t \), treatments \( t \) to reservoirs \( r \) and from reservoirs \( r \) to consuming zones \( z \) at each season \( k \), respectively (m³); \( Z_{RZr} \) and \( Z_{RZr} \) are binary variables (i.e. expressed as 1 or 0, representing yes or no answers) used to define paths from reservoirs \( r \) to consuming zone \( z \), from water resources \( j \) to treatments \( t \) and from treatments \( t \) to reservoirs \( r \), respectively. Based on model (4), the chance-constraints in model (5) (i.e. constraints 5b, 5g, 5k and 5n) can be transformed to their respective crisp equivalent as follows:
\[
\sum_{i=1}^{g} (1 - LjX^s)^k \cdot XRZ_{nk}^k \geq D^s \quad \forall k, q, r \quad (7a)
\]

\[
IR_{rk}^s \leq V^r_{rk} \quad \forall r, k, q, r \quad (7b)
\]

\[
IT_{rk}^s \leq VT_{rk} \quad \forall t, k, q, r \quad (7c)
\]

\[
\sum_{i=1}^{n} X_{jk}^s \leq MJ_{jk}^s \quad \forall j, k, q, r \quad (7d)
\]

Finally, the transformed ILP models can be formulated and solved, such that the objective values and decision variables expressed as discrete intervals at various probability levels will be obtained.

### 4. Result Analysis

Tables 3 to 5 present the solutions at some acceptable probability levels of constraints satisfaction (i.e., 0.9, 0.95 and 0.99) obtained through IPSCCP model. Figures 3 and 4 show the obtained solutions of decision variables and objective function values at various probability levels. The related solutions indicate that the water supply patterns are affected by multiple factors.

Firstly, the objective function values and part of the decision variables from IPSCCP would be presented as discrete intervals. As shown in Figure 4, at a probability level of 0.9, the objective function represents an optimal decision scheme with the lowest cost; correspondingly, the obtained decision variables of water amounts transferred among different layers would reach their lower bounds. Conversely, the solution corresponding to the higher bound of system cost is of conservative consideration where the alternatives with higher supplied amounts would be generated to satisfy the strict requirements of the water users. Based on obtained interval solutions, a variety of alternatives can be generated through adjusting within their solution intervals. Considering extensive uncertainties exist within the urban water supply system, the decision variables presented as discrete intervals are more flexible and suitable to generate the effective decision alternatives.

The obtained solutions from Tables 3 to 5 also demonstrate that under a fixed probability level, the prescribed transferring routines by local managers are main factors for determining water supply schemes. The investigation results of the water supply network show that the water from the treatment plant 1 must be transferred to the reservoirs 2, 3 and 5, respectively. The water from the treatment plant 2 must be transferred to the reservoirs 4, 6 and 7, respectively. Moreover, the leakage rates also have notable influences on the planning results.
Table 5. Part of the Water Amounts Allocated from the Reservoirs to Consuming Zones at Different Probabilistic Levels ($\times 10^3$ m$^3$)

<table>
<thead>
<tr>
<th>k</th>
<th>p = 0.9</th>
<th>p = 0.95</th>
<th>p = 0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>z = 2</td>
<td>z = 4</td>
<td>z = 6</td>
<td>z = 2</td>
</tr>
<tr>
<td>1</td>
<td>242.36</td>
<td>317.76</td>
<td>247.37</td>
</tr>
<tr>
<td>2</td>
<td>332.27</td>
<td>617.33</td>
<td>433.23</td>
</tr>
<tr>
<td>3</td>
<td>228.04</td>
<td>486.60</td>
<td>304.25</td>
</tr>
<tr>
<td>4</td>
<td>312.64</td>
<td>580.18</td>
<td>410.38</td>
</tr>
<tr>
<td>5</td>
<td>231.81</td>
<td>494.91</td>
<td>308.79</td>
</tr>
<tr>
<td>6</td>
<td>317.80</td>
<td>590.09</td>
<td>416.51</td>
</tr>
<tr>
<td>7</td>
<td>228.04</td>
<td>486.60</td>
<td>304.25</td>
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* Specific water amounts allocated from reservoirs to consuming zones are listed; t represents treatment plants; r represents reservoirs; k represents months.

Figure 3. The total water amounts transferred from reservoirs to consuming zones. The obtained results from Table 4 at a probability level of 0.9, the water amounts transferred from the treatment plant 2 to the reservoirs 6 and 7 at month 1 are [988.29, 1231.94] and [171.37, 188.87] ($\times 10^3$ m$^3$), respectively. This is because the leakage rate of transferring path from the treatment plant 2 to the reservoir 6 (0.07, 0.15) is higher than that of water transferring path from the treatment plant 2 to the reservoir 7 (0.01), although the water demand amount of consuming zone 6 (186.29 ($\times 10^3$ m$^3$)) is slightly higher than that of consuming zone 7 (167.30 ($\times 10^3$ m$^3$)).

Figure 4. Comparison of solutions between IPSCCP and SCCP models.

In addition, the selective results among multiple available routines are mainly depended on transferred cost, leakage rate and water demand amounts of consuming zones. According to the investigation results of the water supply network, the reservoir 4 can receive water from treatment plants 2, 3, and 4. The obtained solutions from Table 4 indicate that the water from the treatment plant 2 would be transferred to the reservoir 4 at month 1, being [519.86, 656.81] ($\times 10^3$ m$^3$); however, the transferred water amounts from other two treatment plants to reservoir 4 are zero. This is mainly because that the transferred cost from the treatment plant 2 to the reservoir 4 is the smallest, being $[220, 325]$. The transferred costs from treatment plants 3 and 4 to the reservoir 2 are $[530, 830]$ and $[386, 590]$, respectively. Meanwhile, the leakage rate of path from the treatment plant 2 to the reservoir 4 is the smallest, being 0.01, which ensures the water quality and aquatic ecosystem service level.
ment plant 2 to the reservoir 4 is also the smallest, being [0.03, 0.06]. The leakage rates of other two paths are [0.08, 0.13] and [0.03, 0.06], respectively.

It is also indicated that, the interactive relationships among the system components would lead to the mutual influences among decision variables. For example, as shown in Table 3, at a probability level of 0.95, the water amounts transferred from the water resource 1 to the treatment plant 1 at month 1 are lower than those to the treatment plant 2, being 1,157.46 and [1,689.32, 2,272.58] (×10^3 m^3), respectively. This is mainly due to the facts that the water from the treatment plant 1 must be transferred to reservoirs 2, 3 and 5. Meanwhile, the storage water in reservoirs 2, 3 and 5 are used to satisfy the water demand from consuming zones 2, 3 and 5, respectively. The total demand amount of consuming zones 2, 3 and 5 is 492.20 (×10^3 m^3). As for the treatment plant 2, the water should be allocated to reservoirs 4, 6 and 7 for satisfying demand amounts of consuming zones 4, 6 and 7, being 690.75 (×10^3 m^3).

Figures 3 and 4 are used to demonstrate that the variations in the probability levels would result in changes of the water supply patterns. From Figure 3, at month 1, the total water amounts transferred from the reservoirs to the consuming zones at various probability levels (i.e. 0.99, 0.95 and 0.99) are [3,050.21, 3,987.00], [3,098.34, 4,048.39] and [3,188.62, 4,163.57] (×10^3 m^3), respectively. This is mainly because that as the increases of probability levels (from 0.9 to 0.99), the constraints in the water demand from the consuming zones would become stricter. Figure 4 reflects the variation of system cost at various probability levels. Generally, the system cost would increase as the increasing of probability levels. For example, at different probability levels, the system costs are [38.68, 74.26], [39.19, 75.27] and [40.15, 77.17] (×10^6), respectively. A tradeoff between total system cost and probability levels of constraints satisfaction can help decision makers gain an in-depth insight into the characteristics of urban water supply system and generate rational water supply alternatives.

Generally, the above results demonstrate that IPSCCP has advantages in: (i) addressing uncertainties in urban water supply systems expressed as discrete intervals and probability distributions; (ii) generating the cost-effective interval solutions due to combinative application of probability levels of constraints satisfaction and interactive two-step interval algorithm; (iii) providing supports for decision makers to analyze the trade-offs between system cost and reliability of constraints satisfaction. Moreover, IPSCCP is capable of generating a spectrum of decision alternatives for decision makers, where the trade-off between system economy and reliability could be analyzed. If the decision makers only need one concrete planning pattern for water supply management, they should clearly identify their preferences on system economy and reliability, and specify the corresponding alternative for meeting their requirement.

5. Discussions

To better reflect the advantages of proposed IPSCCP model, a general SCCP model would be generated for comparison purpose where the deterministic parameters are derived by averaging the upper and lower bounds of intervals from IPSCCP model. The total cost at different probability levels are $70.88, 71.88 and 73.76 (×10^6). Obviously, the solutions of SCCP model are special cases in the solutions obtained from IPSCCP model. In such case, the decision alternative would be restricted to a single solution, leading to the negative influence on its application in real-world. As the proposed methodology offers solutions under various scenarios, it is possible that similar decision alternatives are generated under different combinations of probability levels and deterministic decision values. For example, the lower system costs under the higher probability levels are possibly leading to the same results with those from the higher system costs under the lower probability levels. But this does not mean that the model consideration is the same. In real applications, the decision makers should determine the probability levels and adjust the standard of choosing proper decision variables based on their own preferences, in light of system economy and reliability.

However, IPSCCP also shows a number of limitations. For example, the obtained solutions through IPSCCP can reflect the trade-off between system cost and reliability of constraints satisfaction. It is challenging to choose reasonable solutions and form a decision-support base. Moreover, the urban water supply management system needs a comprehensive consideration of all related aspects, e.g. social, environmental, institutional, political, and financial (Zarghami et al., 2008). The single objective function in this study may not be sufficient to reflect the balance between various objectives in decision makings. In such a case, multi-objective programming (MOP) techniques could be applied for dealing with such a difficulty. Therefore, how to incorporate multi-objective programming (MOP) techniques into the proposed method framework is important and deserve an in-depth study.

6. Conclusions

In this study, an interval-parameter stochastic chance-constrained programming (IPSCCP) model has been developed for integrated urban water supply management system under uncertainty. The IPSCCP model can effectively deal with uncertainties expressed as discrete intervals and random variables. Moreover, it incorporates the interval uncertainties and prescribed probability levels of constraints satisfaction into its optimization process. Finally, the decision alternatives can be generated through adjusting within the solution intervals. An integrated urban water supply management system has been used to demonstrate the applicability of proposed method.

The proposed model could help decision makers establish rational water supply patterns under complex uncertainties, and gain in-depth insights into the trade-offs between system cost and reliability. This study was the first attempt for the urban water supply management system through development of IPSCCP. The results suggested that other uncertain approaches, such as FMP and MOP, could be integrated into an IPSCCP framework for reflecting more complex conditions.
Acknowledgments. This research was supported by the National Natural Science Foundation of China (51009004, 51900095) and the Program for Innovative Research Team in University (IRT1127). We deeply appreciate the anonymous reviewers for their insightful comments and suggestions.

References


