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Investigation of PMD in direct-detection optical OFDM with zero padding

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Abstract: We investigate the polarization-mode dispersion (PMD) effect of zero padding OFDM (ZP-OFDM) in direct-detection optical orthogonal frequency division multiplexing (DDO-OFDM) systems. We first study the conventional equalization method for ZP-OFDM. Then an equalization method based on sorted QR decomposition is proposed to further improve the performance. It is found that the performance improvement of ZP-OFDM is due to the frequency domain oversampling (FDO) induced inter-carrier interference (ICI). Numerical simulation results show that compared with cyclic prefix OFDM (CP-OFDM), ZP-OFDM has a significantly higher tolerance to PMD in DDO-OFDM systems when the channel spectral nulls occur at certain differential group delay (DGD) values.

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References and links


1. Introduction

Optical orthogonal frequency division multiplexing (OFDM) has been extensively studied for high-speed optical fiber transmission due to its advantages such as high spectral efficiency, and effectiveness in channel estimation and equalization for mitigating channel impairments caused by chromatic dispersion (CD) and polarization mode dispersion (PMD). Coherent optical OFDM (CO-OFDM) has been demonstrated to achieve high speed long-haul optical fiber transmission [1–3]. However, the high performance of CO-OFDM is generally
associated with expensive polarization-diversity receiver, local oscillator and complex carrier phase estimation algorithm to mitigate the effects of laser phase noise [1, 3].

Direct-detection optical OFDM (DDO-OFDM) requires much simpler transmitter and receiver architecture and is immune to laser phase noise [4–6]. One critical challenge for DDO-OFDM systems is that the frequency-selective fading caused by PMD will reduce the optical signal-to-noise ratio (OSNR) of affected subcarriers [7, 8]. In order to combat the PMD impairments, the polarization-diversity receiver was applied to DDO-OFDM systems together with an adaptive polarization controller [9]. However, the additional expensive optical components required for polarization-diversity and automatic polarization controlling in the systems will make the DDO-OFDM lose its advantages over CO-OFDM.

In this paper, we investigate zero padding OFDM (ZP-OFDM) in the DDO-OFDM systems to deal with the PMD induced channel spectral nulls. The channel equalization method for ZP-OFDM based on frequency domain oversampling (FDO) is studied. An effective equalization method based on sorted QR decomposition is then proposed to further improve the performance. It is found that the inter-carrier interference (ICI) induced by FDO is the main reason that ZP-OFDM can overcome PMD induced channel spectral nulls. The bit error rate (BER) performances of ZP-OFDM and cyclic prefix OFDM (CP-OFDM) signals transmitting through a PMD limited channel are compared. It is shown that ZP-OFDM has a significantly higher PMD tolerance than CP-OFDM when channel spectral nulls occur at certain DGD values.

In the rest of the paper, notations (·)T, (·)H and (·)† represent transpose, Hermitian and pseudoinverse operators; F_M represents a unitary M × M DFT matrix; Diag(x) stands for a diagonal matrix whose diagonal is vector x; Bold letters denote vectors and matrices.

2. Channel equalization for ZP-OFDM

The channel equalization method for ZP-OFDM has been presented in [10]. The transmitted symbol frame of ZP-OFDM is shown in Fig. 1, where the i-th OFDM symbol in frequency domain is denoted by a N × 1 vector S_N(i) with N subcarriers. After being pre-coded by the IFFT matrix F_NN, the time domain transmitted signal ẑ_N(i) is formed by inserting a ZP with length D between each OFDM symbol to avoid inter-symbol interference (ISI). Therefore, the symbol length in time domain is P = N + D. We define a P × N matrix F_zp = [F_N, 0_{N×(P−N)}]H, which correspond to the combined multicarrier modulation with ZP insertion. The ZP-OFDM symbol at the transmitter side can then be simply expressed as P×1 vector ẑ_zp(i) = F_zpS_N(i).

![Fig. 1. OFDM symbol frames for ZP-OFDM.](image)

The first-order PMD channel is modeled as an Lth-order (L ≤ D) FIR filter with channel impulse response being a P × 1 vector h = [h_0, h_1, ..., h_{L−1}, 0, ..., 0]T. The received time domain ZP-OFDM signal is P × 1 vector which can be expressed as:

\[
\tilde{x}_z(i) = HF_{zp}S_N(i) + H_{ISI}F_{zp}S_N(i−1) + \tilde{n}_p(i)
\]  

(1)

where H is a P × P circulant matrix with first column h; H_{ISI} is a P × P upper triangular Toeplitz matrix with first row [0, ..., 0, h_1, ..., h_L]; and \tilde{n}_p(i) is a P × 1 noise vector. For ZP-OFDM, the ISI can be eliminated since H_{ISI}F_{zp} = 0. Then (1) can be rewritten as
\[ \tilde{x}_p(i) = H P S_N(i) + \tilde{n}_p(i) \quad (2) \]

Unlike the equalization method in [10], we split \( \tilde{x}_p(i) \) into its upper part \( \tilde{x}_u(i) = H_{sz} \tilde{x}_s(i) \) \((U \times 1\) vector \((U \geq N)\)) and its lower part \( \tilde{x}_l(i) = H_{sz} \tilde{x}_l(i) \)((P-U) \times 1\) vector \((P \geq U)\)), where \( H_{sz} \) denotes the corresponding \( U \times U \) \(((P-U) \times U)\) partition of \( H \), and \( \tilde{x}_s(i) \) is the \( U \times 1 \) partition of \( \tilde{x}_p(i) \). Padding \( 2U-P \) zeros in \( \tilde{x}_l(i) \) and adding the result to \( \tilde{x}_u(i) \), we have

\[
\tilde{x}_e(i) = \tilde{x}_u(i) + \begin{bmatrix} \tilde{x}_l(i) \\ 0_{(U-P) \times 1} \end{bmatrix} = \left(H_s + \begin{bmatrix} H_l \\ 0_{(U-P) \times U} \end{bmatrix}\right) \tilde{x}_s(i)
= H_U \tilde{x}_s(i) = H_U \begin{bmatrix} F_N \cdot 0_{(N+U-N)} \end{bmatrix}^T S_N(i)
\quad (3)
\]

where \( H_s \) is a \( U \times U \) circulant matrix with first column \([h_0, h_1, \ldots, h_{U-1}, 0, \ldots, 0]^T \). The noise term \( \tilde{n}_p(i) \) in (2) is ignored in following derivation for simplicity. Since the circulant matrix can be diagonalized by DFT matrix \( F_U \), which leads to

\[
X'_u(i) = F_U \tilde{x}_e(i) = \text{Diag} \left( \tilde{h}_U \right) F_U \begin{bmatrix} F_N \cdot 0_{(N+U-N)} \end{bmatrix}^T S_N(i)
\quad (4)
\]

where \( \tilde{h}_U = F_U^H h \) is a \( U \times 1 \) vector, \( F_U^H \) is the first \( L \) columns of \( F_U \). By defining \( M_U = \text{Diag} \left( \tilde{h}_U \right) F_U \begin{bmatrix} F_N \cdot 0_{(N+U-N)} \end{bmatrix}^T \), the zero forcing (ZF) recovery is given by

\[
S_N^{ZF}(i) = \left[ \text{Diag} \left( \tilde{h}_N \right) F_U \begin{bmatrix} F_N \cdot 0_{(N+U-N)} \end{bmatrix}^T \right]^{-1} X'_u(i) = \left[ M_U \right]^{-1} X'_u(i)
= S_N^{ZF}(i) = \left[ \text{Diag} \left( \tilde{h}_N \right) \right]^{-1} X'_u(i)
\quad (5)
\]

For comparison, the ZF equalizer for CP-OFDM is given by [1]

\[
S_N^{CP}(i) = \left[ \text{Diag} \left( \tilde{h}_N \right) \right]^{-1} X_N(i)
\quad (6)
\]

where \( \tilde{h}_N = F_N^H h \) is a \( N \times 1 \) vector, \( F_N^H \) is the first \( L \) columns of \( F_N \); \( X_N(i) \) is the received OFDM signal in frequency domain after CP removal. From (6), we can find that the subcarriers of CP-OFDM cannot be correctly recovered if any entry of \( \tilde{h}_N \) is zero (or close to zero). However, with ZP-OFDM, \( M_U \) in (5) is always invertible, which guarantees the ZF recovery of \( S_N \), regardless of the channel spectral nulls. From (5), we can see that the equalizer in [10] is actually a special case of (5) with \( U = P = N + D \), and another equalization method for ZP-OFDM based on overlap-add (ZP-OFDM-OLA) [10] is also a special case with \( U = N \). It is observed in (5) that the channel for ZP-OFDM is over-sampled in frequency domain, since \( U > N \). Therefore, it can be expected that it is the FDO that improves the system performance. ZP-OFDM-OLA shows no obvious performance improvement because \( U = N \) in this case. To explore the FDO for performance improvement, we consider the case of \( U > P \). Padding \( (U-P) \) zeros at the end of \( \tilde{x}_p(i) \), we have \( \tilde{x}_U(i) = \left[ \tilde{x}_U(i), 0_{(U-P)} \right]^T \). Following the derivations from (3) to (5), we can see that (5) can be used to recover the original signal. The expression of \( \tilde{x}_U(i) \) can be altered with different values of \( U \). Compared with CP-OFDM in [1], the calculation of the pseudoinverse of a \( U \times N \) matrix in (5) for ZP-OFDM can be pre-computed after channel estimation process.
The following equalization process is related to the value of $U$ which can be reduced in (5). Therefore, there is a tradeoff between the performance and computational complexity.

It is easy to see that $M_U$ in (5) is a full column rank matrix as long as $U > N$. Since the QR decomposition [11] can be applied to the full column matrix, (4) can be re-written as

$$X_U(i) = M_U S_N(i) = Q R S_N(i)$$

where the $U \times N$ matrix $Q$ has orthogonal columns with unit form and the $N \times N$ matrix $R$ is upper triangular. Multiplying $X_U(i)$ with $Q^H$, then (7) can be re-written as

$$\tilde{X}_U(i) = Q^H X_U(i) = R S_N(i)$$

Due to the upper triangular structure of $R$, the $k$-th element of $\tilde{X}_U(i)$ is given by

$$\tilde{X}_U^k(i) = r_{k,k} S_N^k(i) + \sum_{n=k+1}^{N} r_{k,n} S_N^n(i)$$

As shown in (9), $\tilde{X}_U^k(i)$ is free of interference data in subcarriers $1,\ldots,k-1$. Therefore, $\tilde{X}_U^k(i)$ is free of interference and can be used to estimate $S_N^k(i)$. From (9), the data in the $k$-th subcarrier $S_N^k(i)$ can be recovered via the data in subcarriers $k+1,\ldots,N$, which is given by

$$S_N^k = \frac{\tilde{X}_U^k(i) - \sum_{n=k+1}^{N} r_{k,n} \langle S_N^n(i) \rangle}{r_{k,k}}$$

where $\langle S_N^n(i) \rangle$ is the decision of data in the subcarrier $S_N^n(i)$. The detection sequence of $S_N(i)$ can also be optimized by re-ordering the columns in matrix $M_U$ to arrange the diagonal elements in matrix $R$ in ascending order. We refer to the equalization method based on sorted QR decomposition in (9) and (10) as “ZP-OFDM-sQR”.

### 3. Simulation results and discussions

The simulation setup to evaluate the performance of CP- and ZP-OFDM systems is illustrated in Fig. 2, which is similar to that in [8] where optical single sideband (OSSB) signal is considered (inset of Fig. 2). The system performance is evaluated using Monte Carlo simulations and the PMD emulator in Optisystem 10.0 [12], considering only the first order PMD. The OFDM modulation and demodulation are both processed in Matlab, with the compensated baseband QPSK signal used to compute the BER. The optical power at the input of PMD emulator is fixed at 0dBm. The noise is added at the output of the PMD emulator.
The noise value is varied to obtain a different ratio of energy per bit to noise power spectral density ($E_b/N_0$). As shown in Fig. 2, the simulation parameters are as follows: 1) input state of polarization angle $\theta = \pi/4$, corresponding to the worst case [8]; 2) the number of subcarriers $N$ is 64 and all the subcarriers are filled with data; 3) the CP or ZP size is set to be 8; 4) the sampling rate of DAC is 20GSamples/s; 5) the up-conversion RF frequency is 20 GHz; 6) 1600ps/nm of chromatic dispersion is added; 7) 20 training symbols are used for every 300 data symbols for channel estimation. Therefore, the total transmission bit rate is 20GSa/s × 64/(64 + 8) × 2b/Sa × 300/(300 + 20) = 33.3 Gb/s.

The BER performance of CP-OFDM, ZP-OFDM and ZP-OFDM-sQR with $U = P = 72$ versus $E_b/N_0$ for different DGD values is shown in Fig. 3(a). With the interference cancellation in each step, ZP-OFDM-sQR shows the best performance among the three cases. At DGD values of 15 ps and the 7% forward error coding (FEC) threshold ($3.8 \times 10^{-3}$), ZP-OFDM-sQR shows ~1.6 dB $E_b/N_0$ improvement than CP-OFDM. It can be seen that the performance improvement of ZP-OFDM (ZP-OFDM-sQR) is more significant at DGD of 20 and 100 ps than 15 ps. This is because the spectral null is more obvious at larger DGD values as shown in Fig. 3 (b)-(e). The spectral dips occur at $f_n = (2k + 1)/2\Delta\tau$ ($k$ is a non-negative integer), which confirms the PMD analytical results [7, 8], shown in red line. As shown in Fig. 3(a), the ZP-OFDM (ZP-OFDM-sQR) can always achieve the 7% FEC threshold at DGD values of 20 and 100 ps when $E_b/N_0 \geq 25$ dB. However, CP-OFDM shows a clear BER floor and cannot achieve error free transmission with FEC at the corresponding DGD values.

![Fig. 3. (a) BER performance of CP-OFDM, ZP-OFDM and ZP-OFDM-sQR for several DGD values; (b)-(e) OFDM QPSK RF spectra at DGD values of (b) 0, (c) 15ps, (d) 20 ps, (e) 100ps.](image)

Figure 4 investigates the BER performance of ZP-OFDM with $U = 68, 72$ and 128 versus $E_b/N_0$ for DGD values of 20 and 100 ps, respectively. The BER performances of ZP-OFDM with all three $U$ values are better than that of CP-OFDM. The performance is slightly improved with an increase in $U$, but the difference is negligible at the $E_b/N_0$ of 25 dB. Therefore, the equalizer with $U = 68$ can recover the signal effectively with lower computation complexity.

The BER performances of CP-OFDM, ZP-OFDM-sQR and ZP-OFDM with the three $U$ values at the $E_b/N_0$ of 30 dB versus DGD are shown in Fig. 5. As shown in Fig. 5, the BER performance of ZP-OFDM (ZP-OFDM-sQR) can achieve 7% FEC threshold when DGD varies from 20ps to 120ps. This means that ZP-OFDM (ZP-OFDM-sQR) can always achieve error free transmission with FEC. However CP-OFDM cannot achieve error free transmission even with FEC in the range of 20ps to 120ps. The BER fluctuation in Fig. 5 can be explained as the frequency spacing between the subcarriers and channel notches is varied at different
DGD values. When some subcarriers are located exactly at the channel spectral nulls, the OSNR of those subcarriers will be close to zero, resulting in a significant increase in BER.

Fig. 4. BER performance at several DGD values versus $E_b/N_0$ for ZP-OFDM with $U = 68$, 72 and 128.

Fig. 5. BER performance of CP-OFDM, ZP-OFDM-sQR and ZP-OFDM with $U = 68$, 72 and 128 versus DGD when $E_b/N_0$ is 30 dB.

The performance improvement of ZP-OFDM is mainly due to the fact that ICI is introduced by FDO, as shown in (9). It means that other subcarriers can be used to recover the data in specific subcarriers which are undermined by channel spectral nulls. For ZP-OFDM, most of matrix computations are only related to the channel state information that can be computed before signal transmission. Considering the simple receiver structure of ZP-OFDM without additional optical components, the ZP-OFDM with our proposed equalizer offers a cost-effective solution to reduce the effect of PMD in DDO-OFDM systems.

4. Conclusion

We have studied the equalization method for ZP-OFDM in DDO-OFDM systems where the impairment caused by the first-order PMD is considered. A new efficient equalizer based on FDO for ZP-OFDM is proposed to combat the PMD induced spectral nulls in optical fiber transmission. The proposed equalizer can achieve a good tradeoff between the performance and implementation complexity. Sorted QR decomposition based equalizer is also proposed to further improve the performance. The numerical simulation results have shown that ZP-OFDM with the new equalizers significantly outperforms the CP-OFDM when the subcarriers are located at or near the channel spectral nulls caused by PMD. Since the ZP-OFDM requires no additional expensive optical components in DDO-OFDM systems, it is very promising for future low cost, multicarrier high-speed optical fiber transmission systems.

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