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<td><a href="http://hdl.handle.net/10220/18341">http://hdl.handle.net/10220/18341</a></td>
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A laser is a physical system which, when subjected to an energy flux (pump), self-organizes at a threshold value of the pump to produce narrow-band coherent electromagnetic radiation. In the absence of inhomogeneous broadening and quantum fluctuations, this radiation has zero linewidth. Above the first lasing threshold, lasers are non-thermal, quantum fluctuations, this radiation has zero netic radiation. In the absence of inhomogeneous broad- 

The effect arises from the interaction of optical absorption and wave interference and corresponds to moving a zero of the elastic $S$ matrix onto the real wave vector axis. It is thus the time-reversed process of lasing at threshold. The effect is demonstrated in a simple Si slab geometry illuminated in the 500–900 nm range. Coherent perfect absorbers act as linear, absorptive interferometers, which may be useful as detectors, transducers, and switches.

Coherent Perfect Absorbers: Time-Reversed Lasers

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(Received 25 March 2010; revised manuscript received 14 June 2010; published 26 July 2010)

We show that an arbitrary body or aggregate can be made perfectly absorbing at discrete frequencies if a precise amount of dissipation is added under specific conditions of coherent monochromatic illumination. This effect arises from the interaction of optical absorption and wave interference and corresponds to moving a zero of the elastic $S$ matrix onto the real wave vector axis. It is thus the time-reversed process of lasing at threshold. The effect is demonstrated in a simple Si slab geometry illuminated in the 500–900 nm range. Coherent perfect absorbers act as linear, absorptive interferometers, which may be useful as detectors, transducers, and switches.

DOI: 10.1103/PhysRevLett.105.053901

PACS numbers: 42.25.Bs, 42.25.Hz, 42.55.Ah
specific modes. The CPA process arises from the interplay of interference and absorption: In the presence of specific amounts of dissipation, there exist interference patterns that trap the incident radiation for an infinite time. If the resonator of the CPA has high $Q$, then even small rates of single-pass absorption can lead to perfect absorption. Hence, media that normally do not absorb radiation well at certain frequencies can be made to do so, as we demonstrate below for Si (silicon). Finally, we show that absorption can also be reduced by illuminating the CPA with eigenvectors of the $S$ matrix with constructive interference for escape.

Before moving to specific examples, we discuss a general framework for finding CPA zeros based on the “$R$-matrix” theory of Wigner and Eisenbud [9]. In this approach, the $S$ matrix is given by

$$S(k) = -e^{2i\sin k r_0} [I - i n_0 k R(k)]^{-1} [I + i n_0 k R(k)],$$  (3)

where $r_0 > r_1$ is an arbitrarily chosen boundary far from the origin, and $R(k)$ (the $R$ matrix) takes the form

$$R_{mn}(k) = \sum_{a,a'=1}^{\infty} \phi^m_a F^{-1}_{aa'} \phi^{m'}_{a'},$$  (4)

$$F_{aa'} = (k^2_a - k^2) \delta_{aa'} - i k^2 \gamma_{aa'},$$  (5)

where each $\phi^m_a \in \mathbb{R}$ is a Wigner-Eisenbud basis function evaluated at $r = r_0$ and decomposed into the $m$th channel [10], and $k_a \in \mathbb{R}$ is the corresponding eigenvalue. The dissipation matrix $\gamma_{aa'} = \int d\vec{r} Im(n^2)/(\vec{r}) \phi_a(\vec{r}) \phi_{a'}(\vec{r})$ is real and positive-definite.

When $Q \gg 1$, each $S$-matrix zero or pole is determined by approximating the $R$ matrix by a single term $a$; in this case the zeros and poles of the $S$ matrix occur at

$$(1 + i \gamma_{aa}) k^2 = i \gamma_a n_0 k - k^2 = 0,$$  (6)

where $\gamma_a = \sum_m (\phi^m_a)^2 > 0$. Without dissipation ($\gamma_{aa} = 0$), this implies that all zeros have positive imaginary parts, as already noted. Furthermore, zeros cross the real axis exactly when $\gamma_{aa} = \varphi_a/k_a$, at frequency $k = k_a$. In Fig. 1, we show the exact pole motion, for the case of the two-channel CPA (see inset and discussion below), finding excellent agreement with the single-pole $R$-matrix prediction, even though the cavity only has $Q \sim 30$.

The simplest possible CPA is a single port reflector, in which a single channel fiber or waveguide is terminated by a cavity tuned to the correct value of $n^\prime$. This device is similar, although not identical, to the “critically coupled fiber-resonator” systems widely studied in integrated optics [11,12]. In a CPA, the loss induced in the resonator is completely due to absorption in the loss medium and not due to outcoupling (e.g., bending) loss. This makes the one-port CPA is potentially useful as an on-chip photovoltaic or calorimetric detector or transducer. However, the optical control properties of the CPA are revealed only when there is more than a single input channel of the incident field. We therefore study a simple two-channel case to illustrate the concept fully. Consider a single-mode fiber or waveguide with index $n_0$ containing a resonator consisting simply of a segment of thickness $a$ and uniform refractive index $n$ (see inset in Fig. 1); in this case, there are two input channels for each propagating $k$, corresponding to incident radiation from the left and right. An almost equivalent system would be a slab of thickness $a$ illuminated on both sides by a narrow beam at normal incidence, in which case $n_0 = 1$. In both cases, it is straightforward to calculate the $2 \times 2$ $S$ matrix for arbitrary complex index $n$ and find its two eigenvalues $s_1, s_2$. The total scattering intensity for each eigenmode is $|s_{1,2}|^2$, and a zero eigenvalue of $S$ occurs when

$$e^{inka} = \pm \frac{n - n_0}{n + n_0}.$$  (7)

When $k a \gg 1$, we can find an infinite number of discrete solutions of this equation, $n_\nu = n_\nu' + i n_\nu''$, as

$$n_\nu' = \frac{\pi \nu}{n_0 ka}, \quad \nu = 1, 2, 3, \ldots,$$  (8)

$$n_\nu'' = \frac{1}{n_0 ka} \ln \left( \frac{n_\nu' + n_0}{n_\nu' - n_0} \right).$$  (9)

In Fig. 2, we take $n_0 = 1$ and plot the solutions for $ka = 664.7$. Note that we have here restated the CPA problem so as to find the $\{n_\nu(k)\}$ which produces a zero of the $S$ matrix for a fixed $k$. This can be achieved by letting both the real and imaginary parts of $n_\nu$ vary; earlier, we varied the imaginary part $n''$ at fixed $n'$, leading to zeros at different $k$ points. This is a useful reformulation because it suggests one practical means to realize a CPA. If the frequency-dependent $n(k)$ of the loss medium can be tuned appropriately by scanning $k$, one may achieve the CPA resonance condition, i.e., $n(k) = n_\nu(k)$ for some integers $\nu$.  

![FIG. 1 (color online). Motion of exact $S$-matrix zeros (blue crosses) and poles (blue triangles) in the complex-$k$ plane, as dissipation increases from zero, for a two-channel resonator of length $a$ and uniform index $n$. In the external region, $n_0 = 1$. For $n = 3.0.05i$, the fourth zero from the left touches the real axis, yielding a CPA zero. Also shown are the zeros (red circles) and poles (red squares) predicted from Eq. (6). Inset: Schematic of system; each input is a single-mode fiber.](image-url)
A crucial point is that the system exhibits perfect absorption only if it is coherently illuminated with the zero eigenmode of the \( S \) matrix for the resonant \( k \) value. Because of the mirror symmetry of this uniform index CPA, the two eigenmodes are parity eigenstates, meaning that the left and right beams have the same wave function. The location of these minima is \( \alpha \)-dependent and hence tunable within a given material; we can derive a tight lower bound for the \( S \)-matrix eigenvalue intensities:

\[
|s(a)|^2 \simeq \left[ \frac{2(n^2 - n_0) \sinh(n^n/k a) - 4n' n_0}{(n' + n_0)^2 e^{n^n/k a} + (n' - n_0)^2 e^{-n^n/k a}} \right]^2,
\]

which goes to zero at the \( n \) values given in (7) and locates the interesting operating regions. For the uniform Si CPA, the optimal wavelength occurs at around 750 nm for \( a = 10 \) \( \mu \)m and around 1000 nm for \( a = 150 \) \( \mu \)m; the exact value depends on \( n(k) \), which in turn depends on the doping. If \( a \approx 20 \) \( \mu \)m, the spacing between CPA zeros, given in (8), becomes small, and the material index always passes close to one or more of them without fine-tuning \( a \), making the behavior shown in Fig. 3 robust. Other indirect band gap semiconductors such as GaP show similarly good results.

The perfect absorption of a CPA arises from a combination of interference and dissipation: The reflected part of the first incident beam interferes destructively with the transmitted part of the second incident beam, and vice versa, and therefore the radiation is trapped in an interference pattern within the slab and lost entirely to dissipation. One sees from Fig. 3 that the other, orthogonal eigenmode of the \( S \) matrix is significantly less than if the system is coherently illuminated from both sides; the reflected part of each beam interferes constructively with the transmitted part of the other beam, causing the radiation to escape the slab more quickly. Thus, a CPA allows resonant control of absorption, either an increase to nearly 100% or a reduction to <1% for some resonators (see Fig. 4).

To illustrate the role of interference, we write down the transfer matrix, a \( 2 \times 2 \) matrix with unit determinant:

\[
T(n, k) = \frac{1}{r} \begin{bmatrix} r^2 - r^2 & r \\ r & 1 \end{bmatrix},
\]

where \( r(n, k) \) and \( t(n, k) \) are the reflection and transmission amplitudes, respectively, for a single wave of unit amplitude incident from either direction. Perfect absorption occurs when \( T_{11} = 0 \), i.e., when \( r^2 = r^2 \). For the output beams to interfere destructively in the manner described above, not only must they have equal intensities \( |r|^2 = |t|^2 \), they must also have the correct relative phase. This should be impossible to satisfy when \( n \) is real due to energy conservation, and the \( T \)-matrix analysis confirms this as follows. When \( n \) is real, \( T^* = T^{-1} \), and consequently (11)
implies that, even if we achieve the condition $|r|^2 = |t|^2$, there is always a $\pm \pi/2$ phase difference between $r$ and $t$ and not the required 0, $\pi$. This analysis is easily generalized to systems lacking mirror symmetry.

A very important property of the CPA which follows from this analysis is that we can change the output intensity simply by changing the relative phase of the input beams. For the parity symmetric two-channel CPA just discussed, simply by changing the relative phase of the input beams.

For the absorbing eigenmode and for the other eigenmode. The absorption in the second mode is exceptionally low because the field inside the slab is concentrated in the SiO$_2$ regions, which are nonabsorbing.

CPAs are potentially useful as transducers, modulators, or optical switches, for example, in on-chip integrated optical circuits based on Si waveguide or resonator technology [14]. We have verified that with realistic nonoptimized parameters a Si single-mode waveguide with 0.9 $\mu$m distributed Bragg reflector mirrors and a 4 $\mu$m “loss region” of pure intrinsic Si exhibits a CPA absorption resonance at 947 nm with contrast of roughly 90%. Operation likely can be extended into the communications wavelengths around 1.5 $\mu$m by designing devices with index tuning via free carrier injection as has already been achieved in other Si-based resonant photonic circuits [14]. However, solar photovoltaic or stealth applications appear unlikely, as CPAs are narrow-band devices and the oscillatory frequency dependence of their absorption tends to average out the response to a broadband signal.

This work was partially supported by NSF Grants No. DMR-0808937 and No. DMR-0908437, and by seed funding from the Yale NSF-MRSEC (DMR-0520495). We thank Hakan Türeci, Marin Soljačić, John Joannopoulos, Eric Ippen, Qinghai Song, Heeso Noh, and Michal Lipson for helpful discussions.

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[10] We choose the basis set corresponding to the dissipation-less system, i.e., $n = n'$, so that the imaginary part of the $R$ matrix is expressed explicitly in terms of $\text{Im}(n^2)$.


