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Functionally graded shells subjected to underwater shock
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FUNCTIONALLY GRADED SHELLS SUBJECTED TO UNDERWATER SHOCK

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Abstract. This paper deals with the problem of functionally graded (FG) cylindrical shells subjected to underwater shock. A computational approach to predict the dynamic response of the FG cylindrical shells to underwater shock is presented. The effective material properties of functionally graded materials (FGMs) for the cylindrical shells are assumed to vary continuously through the shell thickness and are graded in the shell thickness direction according to a volume fraction power law distribution. Based on Doubly Asymptotic Approximation (DAA) method, the fluid-structure interaction equation for a submerged structure is derived, in which the constitutive relation for functional graded material is implemented. The coupled fluid-structure equations, relating structure response to fluid impulsive loading, are solved using coupled finite-element and boundary-element codes. The computational procedure for the prediction of transient response of the FG graded cylindrical shells subjected to underwater shock is described, with a discussion of the results.

Keywords: Underwater shock, functionally graded material, stress analysis.

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INTRODUCTION

Functionally gradient materials are composite materials, microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. This is achieved by gradually varying the volume fraction of the constituent materials. The advantage of using these materials is that they are able to withstand high temperature gradient environments while maintaining their structural integrity [1].

The previous research has been performed on analysis of the transient response of functionally graded shells to vibration and impact [2, 3]. And also some works were reported on the problems of composite submersible hull and ship section subjected to underwater shock loading using coupled finite element and DAA-boundary element methods [4, 5, 6, 7]. However, relatively little has been reported on the problem of functionally graded cylindrical shells exposed to underwater shock. This paper provides some understanding of the transient response of FG shells to underwater shock, which would benefit some relevant studies for composite marine structure designs as well as biomedical research [8, 9, 10, 11, 12].

EFFECTIVE MATERIAL PROPERTIES

The functionally gradient materials (FGMs) is made by combining two or more materials, mainly made from a mixture of ceramic and metals. The properties of the combined materials can be expressed as

\[ P_F(T, \xi) = \sum_i P_{mi}(T) V_{mi}(\xi), \quad (1) \]

where \( P_F \) is the effective material property of functionally gradient material, \( P_{mi} \) is the temperature dependent property of the \( i \)-th material and \( V_{mi} \) is the volume fraction of the \( i \)-th material. \( \xi \) is the coordinate in the thickness direction and \( T \) is temperature.
The properties for each material $P_{mi}(T)$ in Eq. 1 can therefore be expressed in the form of

$$P_{mi}(T) = p_0 \left( \frac{p_{-1}}{T} + 1 + p_1 T + p_2 T^2 + p_3 T^3 \right).$$

(2)

where $p_0$ is the constant appearing in the cubic fit of the material property with temperature, and $p_{-1}$, $p_1$, $p_2$, and $p_3$ are the coefficients of $T$, $T^2$, and $T^3$ obtained after factoring out $p_0$ from the cubic curve fit of the property. The material properties are expressed in this way so that the higher order effects of the temperature on material properties would be readily discernible.

The volume fraction $V_{mi}(\xi)$ is a spatial function and the sum of the volume fractions of all the constituent materials makes one whole, i.e., $\sum V_{mi}(\xi) = 1$. The simple power law exponent of volume fraction distribution is used to provide a measure of the amount of the $i$-th material in functionally gradient materials. Considering a FG cylindrical shell with $L_k$ layers, the volume fraction of the $i$-th material in the $k$-th lamina can be expressed as

$$V_{mi}(z, z_{k-1}, z_k) = \left[ f_{mi}^2(z, z_{k-1}, z_k) \right]^q,$$

(3)

where $(-1 \leq f_{mi} \leq 1)$, $z_{k-1} \leq z \leq z_k$, and $k \in \{1, 2, \ldots, L_k\}$. $V_{mi}(z, z_{k-1}, z_k)$, a function of variables $z$, $z_{k-1}$, $z_k$, varies continuously from 0 to 1, or from 1 to 0, and can be different from layer to layer. The power law exponent $2q$ is used to control the constituent volume fraction. The square of $f_{mi}$ facilitates a description of symmetrical laminated shells made of FGMs if necessary. For a mixture of two materials at the $k$-th lamina, the effective material properties of functional gradient materials can be expressed as:

$$P_{F}(k, z) = [P_{m1}^k(T) - P_{m2}^k] V_{m1}^k(z) + P_{m2}^k(T),$$

(4)

where $k \in \{1, 2, 3, \ldots, L_k\}$.

For the $k$-th lamina containing two constituent materials, the Young’s modulus $E_F^k$, Poisson ratio $\nu_F^k$, and density $\rho_F^k$, can be obtained from Eq. 4 in conjunction with Eq. 3 as follows

$$E_F^k(z) = (E_1^k - E_2^k) [f_{mi}^2(z, z_{k-1}, z_k)]^q + E_2^k,$$

$$\nu_F^k(z) = (\nu_1^k - \nu_2^k) [f_{mi}^2(z, z_{k-1}, z_k)]^q + \nu_2^k,$$

$$\rho_F^k(z) = (\rho_1^k - \rho_2^k) [f_{mi}^2(z, z_{k-1}, z_k)]^q + \rho_2^k,$$

(5)

The equations in 5 above are applied for all layers, where $f_{mi}(z, z_{k-1}, z_k)$ varies continuously from 0 to 1. When $z = z_k$, the Eqs. in 5 yield $E_F^k = E_2^k$, $\nu_F^k = \nu_2^k$ and $\rho_F^k = \rho_2^k$; and when $z = z_1$, the Eqs. in 5 yield $E_F^k = E_1^k$, $\nu_F^k = \nu_1^k$ and $\rho_F^k = \rho_1^k$; i.e., the material properties vary continuously from material 2 at the inner surface $(z = z_{k-1})$ of the $k$-th lamina to material 1 at the outer surface $(z = z_k)$ of the $k$-th lamina.

COUPLED FLUID-STRUCTURE INTERACTION EQUATIONS

The finite element equation of motion for the structure can be expressed in the form of

$$[M_s][\dot{x}] + [K_s][x] = [f],$$

(6)

where $[M_s]$ and $[K_s]$ are matrices of element mass and stiffness; $[x]$ is the vector of unknown nodal displacements and rotations and $[f]$ is the nodal acceleration vector and $[f]$ is the force vector given by $[f] = -[G][A_f]([\{p_I\} + \{p_S\}])$, where $[p_I]$ is the incident nodal pressure vector and $[p_S]$ is the scattered-wave pressure vector. $[A_f]$ is the diagonal area matrix associated with elements in the fluid mesh, and $[G]$ is the transformation matrix that relates the structural and fluid nodal surface forces.

The Doubly Asymptotic Approximation (DAA) method is employed to simplify the analysis of the transient interaction between a flexible structure and a surrounding infinite medium. The DAA may be written in classical matrix notation by $[M_f][\dot{p}_S] + \rho c [A_f]([p_S] = \rho c [M_f][u_S]$, where $[M_f]$ is the symmetric fluid mass matrix for the fluid-surface mesh. $\{u_S\}$ and $c$ are the density and sound velocity of the fluid. $\{u_S\}$ is the vector of scattered-wave fluid particle velocities normal to the structure’s surface. This is the term that is coupled to the structural response by $\{u_S\} = [G]^T \{\dot{x} - \{u_T\}$, which yields the coupled fluid-structure interaction Eqs. [4, 5, 6, 7]:

$$[M_s][\dot{x}] + [K_s][x] = -[G][A_f]([\{p_I\} + \{p_S\})$$

(7)

$$[M_f][\dot{p}_S] + \rho c [A_f]([p_S] = \rho c [M_f][G]^T \{\dot{x} - \{u_T\}).$$

For the $k$-th lamina of FGMs, the stiffness matrix is represented by

$$\{K_F^k(z)\} = \int_v [B(x)]^T [T_{k\sigma}]^{-1} [C(x)] [T_{k\sigma}] [B(x)] dv,$$

(8)
where the matrix \([B^{(k)}]\) is defined by the relationship between the strain vectors and the nodal displacements and rotations \(\{\varepsilon^{(k)}\} = [B^{(k)}]\{x^{(k)}\}\). \(T^{(k)}\) and \(T^{(k)}_e\) are the stress and strain transformation matrix \([3]\). \([C^{(k)}]\) is a known stiffness matrix referred to the material axes. The stiffness coefficients \(C_{ij}^{(k)}\) for the \(k\)-th layer may be expressed in terms of the engineering constants by:

\[
C_{11}^{(k)}(z) = C_{22}^{(k)}(z) = C_{33}^{(k)}(z) = \frac{1 - (v^{(k)}(z))^2}{\Delta^{(k)}(z)(E^{(k)}(z))^2},
\]

\[
C_{12}^{(k)}(z) = C_{13}^{(k)}(z) = C_{23}^{(k)}(z) = \frac{v(1 + v^{(k)}(z))}{\Delta^{(k)}(z)(E^{(k)}(z))^2},
\]

\[
C_{44}^{(k)}(z) = C_{55}^{(k)}(z) = C_{66}^{(k)}(z) = C^{(k)}(z),
\]

\[
\Delta^{(k)}(z) = \frac{1 - 3(v^{(k)}(z))^2 - 2(v^{(k)}(z))^3}{(E^{(k)}(z))^3}.
\]

By substituting Eq. 8 and the Eqs. in 9 into Eq. 7, the effective properties of functionally gradient materials can be implemented in the fluid-structure interaction equation.

**COMPUTATIONAL PROCEDURE**

To solve the coupled fluid-structure interaction equations, coupled finite-element and DAA-boundary-element codes were employed. Structural calculations were carried out using the finite element code while the fluid-structure interaction was handled by the boundary element code. The finite element code was first used to generate the mass and stiffness matrices of the FGMs cylinder and define the wet surface that is in contact with the fluid. Then the DAA-boundary element code was used to extract the geometry data of the wet surface. From the extracted data, a fluid added mass matrix and directional vectors of the shock loading on the FGMs cylinder can be computed. The computed fluid added mass matrix is incorporated to the structural mass matrix to form an augmented mass matrix.

The shock pressure generated by the explosion is determined by the empirical equation

\[
P(t) = P_{\text{max}}e^{-\left(\frac{t-t_1}{\theta}\right)}, \quad t \geq t_1
\]

where \(t - t_1\) is the time elapsed after the arrival of the shock, \(P_{\text{max}}\) is the peak magnitude of the pressure in the shock front and \(\theta\) is the exponential decay constant. The peak pressure and decay constant in Eq. 10 are given by

\[
P_{\text{max}} = K_1\left(\frac{W}{R}\right)^\frac{1}{A_1},
\]

\[
\theta = K_2\left(\frac{W}{R}\right)^\frac{1}{A_2},
\]

where \(K_1, K_2, A_1\) and \(A_2\) are constants which depend on explosive charge type; \(W\) is the weight of the explosive charge and \(R\) the distance from the explosive charge to the target. The coupled finite element and boundary element codes perform the staggered time integration to solve the fluid-structure coupled equations.

**RESULTS AND DISCUSSION**

The functionally graded (FG) cylindrical shell with a single layer is the similar to the FG shell used in reference [3]. It is composed of Stainless Steel and Silicon Nitride and has Stainless Steel on its outer surface and Silicon Nitride on its inner surface. The material properties for Stainless Steel and Silicon Nitride can also be found in reference [3].

Figures 1 and 2 show the variations of the volume fractions \(V_{m1}\) and \(V_{m2}\) with radial position \(z\) in the thickness direction. For the FG cylindrical shell, \(V_{m1}\) represents the volume fraction of Stainless Steel and \(V_{m2}\) the volume fraction of Silicon Nitride. Its volume fraction of Stainless Steel is zero at the inner

\[\text{FIGURE 1. Variation of volume fraction } V_{m1} \text{ with radial position } z \text{ in the thickness direction.}\]
FIGURE 2. Variation of volume fraction $V_{m2}$ with radial position $z$ in the thickness direction.

surface ($z/h = -0.5$) and increases continuously to 1 at the outer surface ($z/h = 0.5$). On the contrary, the volume fraction of Silicon Nitride decreases from 1 at the inner surface ($z/h = -0.5$) to zero at the outer surface ($z/h = 0.5$).

Figure 3 shows the variation of the effective stress of the single layer cylinder with time. A comparison of the strain responses between a Stainless Steel and a Silicon Nitride shell shows that the peak strain of the Stainless Steel shell is about 1.5 times as high as that of the Silicon Nitride shell. It is observed from Fig. 3 that the stress response decreases as the power law exponent $n(n = 2q)$ increases, with the maximum value of the stress reduced by 17% from $n = 0.3$ to $n = 5$. This can be explained by the fact that small values of $n$ correspond to a large volume fraction of Stainless Steel whereas large values of $n$ correspond to a large volume fraction of Silicon Nitride. The stress responses approach those of the Stainless Steel cylindrical shell as $n$ become smaller and close to those of Silicon Nitride when $n$ becomes larger. Hence, the stress responses fall between those of the Stainless Steel cylindrical shell and the Silicon Nitride shell.

CONCLUSIONS

An approximate approach was developed to study FG cylindrical shells subjected to underwater shock. The effects of the constituent volume fraction and the FGM configuration on the transient response of the FG cylindrical shell induced by underwater shock were illustrated. The present analysis has a potentiality for further application in composite marine structure designs as well as some biomedical research.

REFERENCES