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Seismic Behavior of Reinforced Concrete Columns with Light Transverse Reinforcement under Different Lateral Loading Directions
by Thanh Phuong Pham and Bing Li

Considerable progress has been made on the strength and ductility research of reinforced concrete (RC) columns over the last two decades. However, the seismic performance of RC columns under lateral loading in different directions remains limited. This paper presents an experimental and numerical investigation carried out on RC columns with light transverse reinforcement with an emphasis on how varying the directions of seismic loading influences the seismic failure mechanisms of the columns. Seven half-scale RC columns were tested with the columns subjected to axial load and cyclic forces under reversed double-curvature bending. The parameters varied in the test program included the axial load ratio and loading directions. The overall performance of each specimen is examined in terms of cracking patterns, hysteretic response, initial stiffness, shear strength, drift ratio at axial failure, and energy dissipation capacity. A three-dimensional (3-D) finite element (FE) model is developed to supplement the experimental results. The direction of seismic loads is found to have a significant effect on the drift capacities and failure modes of both rectangular and square columns. The extent of change in the shear resistance capacity resulting from the different angles of seismic load could be predicted. A simple upper limit for shear strength contributed by concrete is proposed.

Keywords: axial failure; axial load; column; light transverse reinforcement; loading direction; reinforced concrete.

INTRODUCTION
The detailing of reinforced concrete (RC) moment-resisting frames in regions where the probability of a seismic event is not significant exhibits a lack of consideration for ductile response, therefore rendering these frames vulnerable to the effects of seismic loadings. Column transverse reinforcement is typically poorly configured and widely spaced, resulting in inadequate confinement of the longitudinal reinforcement and column core for demands related to axial load, flexure, and shear. Furthermore, columns may be weaker than adjacent framing components, such that inelastic demands may concentrate in the columns under severe earthquake loading. The consequences of these inadequacies range from severe damage to complete collapse and are evident in the earthquake reconnaissance literature.1–4 There have been extensive experimental studies conducted on RC columns subjected to seismic loading, most of them focused on the principal loading direction. Thus far, few research works have been done on multi-directional loading. However, real structures behave multi-directionally when subjected to earthquake excitation; therefore, it is essential to evaluate the effect of multi-directional loading on the seismic performance of RC columns to develop more reliable design procedures. Furthermore, most tests have been terminated shortly after the loss of lateral load resistance, with few tests carried out to the point of axial failure, which has resulted in a limited understanding of the failure and collapse mechanisms of nonseismically detailed structures. To better understand the seismic behavior of nonseismically detailed columns under multi-directional loadings, a study was undertaken at Nanyang Technological University, Singapore. Seven half-scale RC columns with light transverse reinforcements were tested to the point of axial failure under a combination of a constant axial load and quasi-static cyclic loadings simulating earthquake actions. The experimental results included crack patterns, hysteretic responses, shear strength, shear failure, axial failure, and cumulative energy dissipation. The results obtained were then compared with a nonlinear finite element (FE) analysis. The results are useful in understanding the failure and collapse mechanisms of nonseismically detailed RC columns under different loading directions. Understanding their deficiencies would make it possible to improve the survivability of such columns in the event of earthquakes.

RESEARCH SIGNIFICANCE
Most research studies on RC columns have been concerned with the seismic behavior of specimens after loading along the principal directions of the column sections and were stopped shortly after the loss of lateral load resistance. It is of great interest to explore the seismic behavior of RC columns with light transverse reinforcement when loaded to the point of axial failure under different loading directions to attain a better understanding of collapse mechanisms and the effect of seismic loading directions.

EXPERIMENTAL PROCEDURE
Specimen details and test setup
Figure 1 and Table 1 illustrate the schematic diagram with a summary of dimensions and detailing of the test specimens. Seven test specimens were divided into three series: two series of square columns with aspect ratios of 2.4 and 1.7 (S2.4 and S1.7) and one series of rectangular columns with an aspect ratio of 1.7 (the aspect ratio is defined as the ratio of the shear span and the depth of the column cross section a/h). The main variable was the lateral loading direction. Longitudinal reinforcement consisted of eight T20 deformed bars characterized by a yield strength fy of 545 MPa (79.0 ksi), resulting in a steel ratio of 2.05%. The transverse reinforce-
ment of all test specimens comprised R6 mild steel bars with 135 degrees bent-spaced at 125 mm (4.92 in.) that were characterized by a yield strength $f_y$ of 511 MPa (74.1 ksi). The theoretical flexural strengths $V_u$ were estimated using the material properties obtained through tests and in accordance with the recommendations provided by ACI 318-08. The nominal shear strengths $V_n$ of the test specimens were calculated based on the ASCE/SEI 41-06 suggestion combined with the approach proposed by Umehara and Jirsa. The values of $V_u$ and $V_n$ of the test specimens are tabulated in Table 1.

Figure 2 shows a schematic diagram of the loading apparatus. A reversible horizontal load was applied to the top of the column using a 1000 kN (224.8 kip) capacity actuator that was mounted onto a reaction wall. The actuator was pinned at both ends to allow rotation during the test, while the base of the column was fixed to a strong floor with four post-tensioned bolts. The axial load was applied to the column by using two 1000 kN (224.8 kip) capacity actuators through a transfer beam.

**Test procedure and instrumentation**

The column axial load was slowly applied to the specimens until the designated level was achieved. During each test, the column axial load was maintained by manually adjusting the vertical actuators after each load step. The lateral load was applied cyclically through the horizontal actuator in a quasi-static fashion, as shown in Fig. 2. The loading procedure consisting of displacement-controlled steps is illustrated in Fig. 3.

The test specimens were extensively instrumented with measuring devices internally and externally. Lateral displacement was measured by two horizontal linear variable differential transformers (LVDTs) mounted parallel to the horizontal actuator. Shear and flexural deformation was recorded based on the reading of a number of LVDT sets located throughout the height of the specimens. Strain gauges were mounted to capture strains on both longitudinal and transverse reinforcements at critical locations, as shown in Fig. 1.

**EXPERIMENTAL RESULTS**

The results from seven specimens tested in diagonal directions are presented and compared with experimental results obtained previously from testing three horizontally loaded columns in only the principal directions. The previous and

| Table 1—Summary of test specimens |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Specimen | $b$, mm | $h$, mm | $L$, mm | $s$, mm | $f_y$, MPa | $f_y'$, MPa | $\phi$, degrees | $P/ f_y' A_g$ | $V_u$, kN | $V_n$, kN | $V_{max}$, kN | $E$, kNm |
| S2.4-30 | 350 | 350 | 1700 | 125 | 56.52 | 31.4 | 30 | 0.2 | 247.1 | 244.0 | 226.4 | 38.2 |
| S2.4-45 | 350 | 350 | 1700 | 125 | 56.52 | 29.3 | 45 | 0.2 | 236.5 | 236.5 | 226.0 | 42.6 |
| S1.7-30 | 350 | 350 | 1200 | 125 | 56.52 | 30.1 | 30 | 0.2 | 348.3 | 308.6 | 260.0 | 17.2 |
| S1.7-45 | 350 | 350 | 1200 | 125 | 56.52 | 29.3 | 45 | 0.2 | 348.3 | 304.7 | 284.0 | 18.6 |
| R1.7-30 | 250 | 490 | 1700 | 125 | 56.52 | 31.4 | 30 | 0.35 | 347.8 | 297.3 | 253.6 | 28.2 |
| R1.7-45 | 250 | 490 | 1700 | 125 | 56.52 | 30.1 | 45 | 0.35 | 270.6 | 240.3 | 221.1 | 30.6 |
| R1.7-60 | 250 | 490 | 1700 | 125 | 56.52 | 29.3 | 60 | 0.35 | 194.9 | 206.7 | 190.0 | 33.4 |

Notes: $E$ is cumulative energy dissipation; 1 kN = 0.2248 kip; 1 mm = 0.04 in.; 1 MPa = 0.145 ksi; 1 kNm = 8.85 kip.in.
current tests were conducted using the same setup—the only difference is the loading direction of the shear force.

**Crack patterns and failure modes**

The crack patterns of the test specimens at shear failure and axial failure are shown in Fig. 4, in which shear failure was recorded when the columns lost over 20% of lateral resistance; axial failure occurred once the specimens were no longer able to resist a certain applied axial force. The important features in the crack development are highlighted.

All of the tested specimens developed fine flexural cracks at both ends of the columns when loaded up to a drift ratio of 0.4%. In subsequent cycles, diagonal cracks were observed as the flexural cracks extended from the two edges in the loading direction of the columns to the middle area before the columns reached maximum shear force. In Series S2.4 (S2.4-30 and S2.4-45), the first flexural cracks were observed at a drift ratio of 0.25%, regardless of loading angles. Those columns also experienced the first diagonal crack at the same drift ratio of 1.25%. A similar observation was recorded at another square section group, S1.7 (S1.7-30 and S1.7-45), whose first flexural and diagonal cracks appeared at a drift ratio of 0.2% and 0.4%, respectively. However, in the rectangular series, R1.7 (R1.7-30, R1.7-45, and R1.7-60), there were substantial changes in the drift ratio corresponding to those two points upon the change in loading angles. Vertical cracks developed throughout the length of the longitudinal bars when loaded in nonprincipal directions, while the columns tested in the principal direction only developed inclination cracks at the end of the test.

There were two main axial failure modes. In the first mode, as observed in Series S2.4 (Specimens S1.7-0 and R1.7-0), the steep shear crack that had developed in the column from the previous stages became wider. The column’s height was then divided into two parts by this shear crack. There was sliding between the two parts in the crack surfaces, leading to the buckling of longitudinal reinforcing bars and fracturing of transverse reinforcing bars along this shear crack. The axial failure occurring in those specimens was believed to be due to the shear friction demand exceeding the shear friction capacity. In the second mode, as found in Specimens S1.7-30, S1.7-45, R1.7-30, R1.7-45, and R1.7-60, vertical cracks appeared from the early cycles and continued to develop without distinct diagonal cracks. This is due to sliding in the surface of longitudinal reinforcements and surrounding concrete when the bond stress exceeds the bond splitting strength. The cross section of the specimens was then gradually reduced, leading to shear failure and axial failure. It was observed in Series S2.4 that the axial failure mode did not change when the loading direction of lateral force was rotated; the only change observed was the angle of the diagonal cracks, which was 60 degrees (with respect to the column axis) in Specimens S2.4-0 and S2.4-30 to 45 degrees in Specimen S2.4-45. However, the axial failure mode was changed from Mode 1 to Mode 2 at Series S1.7 and R1.7 when loading out of the principal directions.

**Hysteretic responses**

The hysteretic responses of the test specimens with the theoretical flexural strength and nominal shear strength are shown in Fig. 5; comparisons of envelope responses are shown in Fig. 6. Overall, the responses were more ductile when the angle of the loading direction was changed from 0 to 45 degrees. Specimens with a brittle response (S1.7-30, S1.7-45, and R1.7-30) did not reach nominal shear strength. The hysteretic loops of all test specimens showed the degradation of stiffness and load-carrying capacity during repeated cycles due to the cracking of the concrete and yielding of the reinforcing bars together due to the extra bending resulting from the applied axial load. The pinching effect was observed in the hysteretic loops of all of the test specimens.

**Strains in reinforcement**

The strain gauge readings were found to agree well with the bending moment patterns in all specimens before the columns failed in shear. As shown in Fig. 7, the largest recorded tensile strain of 2670 μ was observed at Location L3e in Specimen S2.4-45 as a typical illustration, which was 250 mm (9.84 in.) from the column-top base interface at the corner bar in the loading direction plane. It can be seen from Fig. 7(a) that the corner bar reached yield strain just after the specimen reached maximum shear force at a drift ratio of 1.42%.

The measured strains in the transverse reinforcing bars of Specimen S2.4-45 are illustrated in Fig. 7(b). Up to a drift ratio of 2, the measured strain did not increase significantly with an increase of the drift ratio. No yielding of transverse reinforcement was observed prior to shear failure occurring. In subsequent cycles, however, the strain at Position V6 had a sudden spike and reached the fracture strain. Compared to the crack patterns, the strain gauge reading matched the stress distribution on the specimens well.
Initial stiffness

As a crucial parameter in the displacement-based design, initial stiffness was determined based on a point obtained from the measured force-displacement envelope with a shear force that is equal to the theoretical yield force. This is defined as either the first yield that occurs within the longitudinal reinforcement or when the maximum compressive strain of the concrete attains a value of 0.002 at any critical section of the column. However, this definition would not apply for columns whose shear strength does not substantially exceed its theoretical yield force (S1.7-30). For such columns, the initial stiffness is defined based on a point on its measured force-displacement envelope with a shear force that equates to 80% of the obtained maximum shear force.

The initial stiffness of all test specimens is illustrated with respect to its loading direction in Fig. 8. It is shown that the initial stiffness of Series S1.7 was almost unchanged at approximately 25 kN/mm (142.7 kip/in.), where the highest value was reached at the loading angle of 30 degrees. A similar trend was observed for the other square section (S2.4).
with a slightly smaller stiffness strength at 10.2 and 11.8 kN/mm (58.2 and 67.4 kip/in.) when loading from the principal and 45-degree directions, respectively. Overall, initial stiffness was observed to increase by 100% when the aspect ratio (in the principal direction) was decreased from 2.4 to 1.7 (under applied axial forces of $0.2f'_cA_g$). In the rectangular series (R1.7), the highest initial stiffness also resulted from the lateral force angle of 30 degrees to the strong direction of the column. This value decreased by approximately 20% when the loading angle increased to 45 and 60 degrees.

**Shear strengths**

The shear strengths of the test specimens are shown in Table 1. The shear strength of Series S2.4 was achieved when the specimens were loaded up to a drift ratio of approximately 1.4%. Similar observations were also recorded in the other specimen group (S1.7), where shear forces reached the maximum at drift ratios of 1.05% and 0.97% in Specimens S1.7-45 and S1.7-30, respectively. It is noted that the two square series were subjected to the same level of axial force of $0.2f'_cA_g$. It can be concluded that the shorter columns reach the maximum shear force at lower drift ratios. For the third series (R1.7), the shear strengths of three rectangular columns also attained close values of drift ratios of 1.21, 1.25, and 1.26% when the loading angles of shear forces were 30, 45, and 60 degrees to the strong direction of the columns. This data, together with the previous discussion, clearly indicates that the seismic shear strength of a column will reach a particular drift ratio regardless of loading directions.

Figure 9 shows the relationship between the shear strength of columns and the direction of horizontal forces from both the test and predicted results. In Fig. 10, the lines connecting two shear strength values in principal directions (taken from

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**Fig. 5—Hysteretic responses of specimens from test and FE model. (Note: 1 kN = 0.2248 kip; 1 mm = 0.04 in.)**
previous test results) are plotted as the equation of a circle, with the center as the center point of the columns and the radius as the shear strength in the principal directions. Only a 5% difference for Series S1.7 was observed, while almost no change was observed in Series S2.4. This provides evidence that for square sections, columns in nonprincipal directions will reach the same value of maximum shear force regardless of loading directions.

For the rectangular section case (R1.7), it is obvious that with an increase in the loading angle from the strong direction to the weak direction, the shear strength of columns will be decreased. The recorded data were 253.6, 221.1, and 190.0 kN (57, 49.7, and 42.7 kip) for loading angles of 30, 45, and 60 degrees, respectively. However, by fitting the curves, it is found that those values can be linked by the ellipse equation as follows

\[
\frac{V_x^2}{V_{x0}^2} + \frac{V_y^2}{V_{y0}^2} = 1 \tag{1}
\]

where \(V_x, V_y\) are the shear strengths in the strong and weak directions; \(V_{x0}, V_{y0}\) are the components of strength in the diagonal direction when projecting to the strong and weak directions; and the shear strength in the diagonal direction is calculated as \(\sqrt{V_{x0}^2 + V_{y0}^2}\). In other words, the shear strength of rectangular columns in nonprincipal directions can be predicted based on the shear strength in principal directions.
This is similar to the finding on short columns by Umehara and Jirsa. It is noted that the shear strength predicted per the combination of ASCE/SEI 41-06 using the approach proposed by Umehara and Jirsa is found to be overestimated in all specimen series.

**Drift ratios at axial failure**

Axial failure occurs when columns are unable to resist a certain level of applied axial force ($0.2f_c' A_g$ for the square series and $0.35f_c' A_g$ for the rectangular series); the drift ratios at axial failure of the test specimens are illustrated in Fig. 10. Generally, an increase in the angle of the loading direction corresponds to a growth in the drift ratio at axial failure. There was an increment of approximately 2.15% when the angles changed from 0 to 30 degrees at all square columns, whereas that of Series S2.4 increased from 2.82 to 4.96% and that of Series S1.7 changed to 4.0 from 1.82%. These two groups reached the maximum drift ratio at axial failure while being at 45 degrees. The shorter series (S1.7) continued along the same gradient when loading from 0 to 45 to 60 degrees, while the longer series (S2.4) increased at a smaller rate to reach 5.41% at 45 degrees. An analogous trend was also observed in the specimens of Series RC-1.7, whose drift ratios experienced an increase of approximately 26 and 34% when rotating the loading angle from the strong direction to 30 and 45 degrees. This series was found to produce the largest drift ratio when loaded at 60 degrees.

It should be noted that most of the existing models predicting the ultimate drift ratio are based on an assumption of a shear plane failure in which the equilibrium of component forces is formed on a shear-friction plane; axial failure is defined when the shear-friction demand exceeds the shear-friction resistance along this plane. Those methods seem unsuitable for application on columns failing in the second mode without forming the shear plane.

**Cumulative energy dissipation**

The maximum cumulative energy dissipation of all specimens obtained from the tests is shown in Table 1. The results from the tests in the principal directions of these columns were 34.9, 13.5, and 26.5 kNm (308.87, 119.48, and 234.53 kip.in.) for Specimens S2.4-0, S1.7-0, and R1.7-0, respectively. It can be seen that an increase in the maximum cumulative energy dissipation was recorded for both the S2.4 and S1.7 series specimens as the loading direction rotated from 0 to 45 degrees. It is also shown that the maximum cumulative energy dissipations of Series S1.7 were almost 60% lower than those of Series S2.4 for the same loading directions. This can be explained by the more brittle characteristics of RC columns with a smaller aspect ratio. An analogous trend was observed for the rectangular series in which the maximum cumulative energy dissipations gradually increased with an increment of approximately 2 kNm (17.7 kip.in.) when the loading directions changed from 0 to 30 degrees and 45 to 60 degrees. It can therefore be concluded that the maximum cumulative energy dissipations of the columns had a close relation to the drift ratio at axial failure.

**FE ANALYSIS**

The complex behavior of RC columns under seismic loadings requires further analysis to supplement the experimental
The compressive stress after the cylinder strength of concrete; the definition of ultimate strain \( \varepsilon_u \) is developed from regression analysis of test data in the range of peak strain \( \varepsilon_{cc} \) to ultimate strain \( \varepsilon_u \). The parameter \( K \) is set to 0.2 for the confining part and zero for the concrete cover.

### Model of concrete

To consider the effect of low confining reinforcement ratios, the Sakai and Kawashima Concrete Model\(^{10} \) was used in this study. As depicted in Fig. 11, the stress-strain curve of this model consisted of three parts: an ascending branch, a falling branch, and a sustaining branch. The ascending branch of confined concrete reflected four boundary conditions: initial condition \( f_c = 0 \) at \( \varepsilon_c = 0 \), initial stiffness condition \( df_c/\varepsilon_c = E_c \) at \( \varepsilon_c = 0 \), peak condition \( f_c = f_{cc} \) at \( \varepsilon_c = \varepsilon_{cc} \), and \( E_c \) taken as 5000\((f_{co})^{0.5} \) MPa (57,000\((f_{co})^{0.5} \) psi). The tensile strength of concrete was neglected. For unconfined concrete, compressive strength and Young’s modulus of the material tests were used. For confined concrete, compressive strength and strain at pick, \( f_{cc} \) and \( \varepsilon_{cc} \), were calculated as follows

\[
\frac{f_{cc}}{f_{co}} = 1 + 3.8\alpha \frac{P_{f_{sh}}}{f_{co}} \quad ; \quad \varepsilon_{cc} = 0.002 + 0.033\beta \frac{P_{f_{sh}}}{f_{co}}
\]

where \( f_{co} \) is the cylinder strength of concrete; \( \alpha \) and \( \beta \) are modification factors depending on the confined sectional shape (for circular, \( \alpha = 1.0 \) and \( \beta = 1.0 \); for square, \( \alpha = 0.2 \) and \( \beta = 0.4 \)); and \( P_{f_{sh}} \) are the ratio and yield strength of transverse reinforcement.

The falling branch of the stress-strain curve \( (f_c, \varepsilon_c) \) is approximated as a straight line and is formulated as

\[
f_c = f_{co} - E_{des} (\varepsilon_c - \varepsilon_u)
\]

where \( E_{des} \) is the deterioration rate, which is developed from regression analysis of test data in the range of peak strain \( \varepsilon_{cc} \) to ultimate strain \( \varepsilon_u \). The deterioration rate \( E_{des} \) was found experimentally as

\[
E_{des} = 0.061 + 0.002 f_{cc}
\]

The parameter \( K \) defines the minimum compressive stress that concrete burdens in the zone that exceeds the ultimate. The compressive stress after \( \varepsilon_c \) becomes \( Kf_{cc}' \) by using the maximum compressive stress \( f_{cc} \). The parameter \( K \) is set to 0.2 for the confining part and zero for the concrete cover.

### Modeling of reinforcement

The stress-strain relation of steel reinforcement was modeled by a trilinear curve, as shown in Fig. 12. That curve is described by the Giuffre-Menegotto-Pinto Model\(^{11} \), which incorporates the Bauschinger effect for cyclic loading. For simplicity, the bond-slip relationship in the reinforcements was not included in the model. Steel yield strengths were based on tensile tests of the reinforcements. A hardening ratio of 2% was assumed.

### Load-displacement response of specimens

Comparisons of results obtained from FE analysis and the experiment are shown in Fig. 5. Overall, the behavioral trends observed during FE analysis matched the test results well. All specimens expressed pinching effects with good correlation as recorded from the experiment. The recorded points of interest were maximum shear force, shear failure, and axial failure, which usually define the key behavior changes of RC columns subjected to cyclic loading.

The analytical results showed a good correlation with the experimental data in terms of shear strengths for all test specimens; the average ratio of the experimental-to-analytical shear strength was found to be 1.014 and its coefficient of variation was 0.061. However, the drift ratios at maximum shear force of the square column Series S2.4 and S1.7 were not captured well by FE analysis. The program underestimated almost 50% of the drift ratios for these columns. Better results were found for Series R1.7, especially when the specimen was loaded horizontally at angles of 45 or 60 degrees. It should be noted that for the rectangular Series R1.7, when the loading direction changed from 0 degrees (strong direction) to 90 degrees (weak direction), the specimens tended to change their behavior from shear-dominant to flexural-dominant. Therefore, it is evident that FE analysis does not provide a good prediction in terms of drift ratios at maximum shear force for shear-critical columns.

### Parametric studies

Among the critical parameters that influence the shear strength of RC columns, the effects of loading directions and axial force have not been well-investigated. In this section, these effects are studied and the results are compared with test results in terms of shear strength.

**Influence of directions of horizontal force**—The effects of the loading directions were tested in two groups—rectangular and square columns—by varying the angles of
the horizontal force from 0 to 90 degrees while all other parameters were kept constant. The results depicted in Fig. 13 confirmed that the shear strength of RC columns with square sections did not depend on the directions of the horizontal force. It was also found that the shear strength of rectangular columns matched well with elliptical curves when the loading directions were changed. This observation was found even at different levels of axial force.

Influence of axial loads—Previous experimental studies have shown that axial force is beneficial to the shear strength of RC columns. It is expressed in existing codes\(^5,6\) with linearly increasing shear strength when the applied axial force is increased, as shown in Fig. 14. However, by using the Bayesian parameter estimation technique, Sasani\(^12\) pointed out that shear strength remained at a certain value when the axial force was raised beyond the level \(P_c/Ag = 0.42\). In this study, parametric studies were conducted to find out how axial loads influence the shear strength when applied at high levels.

Figure 15 shows the numerical results in comparison to the test results of Series R1.7 conducted at different levels of axial force (from 0.05 to 0.5). It can be seen that the shear strength increased gradually by approximately 11.1%, 9.6%, and 8.56% (by mean) when the axial force was increased from 0 to 0.1, 0.1 to 0.2, and 0.2 to 0.35\(f_c'Ag\), respectively. However, there were only increments of approximately 1.3 and 1.5% when axial force increased to 0.4 and 0.5\(f_c'Ag\).

Experiments of this series\(^3\) also observed similar results, where shear strength was almost unchanged when the applied axial force was beyond the level of 0.35\(f_c'Ag\).

From the previous discussion, considering that ASCE/SEI 4-06 provides the closest estimations for shear strength when axial force is applied up to 0.35\(f_c'Ag\), the modified ASCE/SEI 41-06 equation for shear strength is therefore written as (all units are in MPa)

\[
V_c = \frac{0.5\sqrt{f_c'}}{M/Vd} \left[1 + \frac{P}{0.5\sqrt{f_c'Ag}} \right] 0.8A_g
\]

The upper limit of \(V_c\) is

\[
[V_c]_{\text{max}} = \frac{0.5\sqrt{f_c'}}{M/Vd} \sqrt{1 + 0.7\sqrt{f_c'}0.8A_g}
\]

and the upper limit of shear strength is

\[
[V_c]_{\text{max}} + V_c = \frac{0.5\sqrt{f_c'}}{M/Vd} \sqrt{1 + 0.7\sqrt{f_c'}0.8A_g} + \frac{A_fA_d}{s}
\]

Fig. 13—Influence of loading direction and axial force on shear strength. (Note: 1 kN = 0.2248 kip.)

Fig. 14—Calculation of shear strength per existing codes. (Note: 1 kN = 0.2248 kip.)

Fig. 15—Influence of axial force on shear strength. (Note: 1 kN = 0.2248 kip.)

The modified ASCE/SEI 41-06 equation for shear strength is therefore written as

\[
V_c = \begin{cases} 
\frac{A_fA_d}{s} + \lambda_k \left[ \frac{0.5\sqrt{f_c'}}{a_d} \left(1 + \frac{P}{0.5\sqrt{f_c'Ag}} \right) 0.8A_g \right] & \text{for } P < 0.35f_c'Ag \\
\frac{A_fA_d}{s} + \lambda_k \left[ \frac{0.5\sqrt{f_c'}}{M/Vd} \sqrt{1 + 0.7\sqrt{f_c'}0.8A_g} \right] & \text{for } P \geq 0.35f_c'Ag 
\end{cases}
\]
\[ k_1 = \begin{cases} 1 & \text{for transverse steel spacing less than or equal to } d/2; \\ 0.5 & \text{for spacing exceeding } d/2 \text{ but not more than } d; \\ 0 & \text{otherwise}; \end{cases} \\
\]
\[ k_2 \text{ is taken as } 1 \text{ for displacement ductility less than } 2, \text{ as } 0.7 \text{ for displacement ductility more than } 4, \text{ and varies linearly for intermediate displacement ductility;} \\
\]
\[ l = 1 \text{ for normal weight concrete.} \]

Applying the proposed equation to previous tests on columns with high axial loads \((P \geq 0.35f'_cA_g)\), the results are shown in Fig. 16 and Table 2. It was found that the average ratio of predicted-to-test results for shear strength by the proposed equation was 1.02, while ASCE/SEI 41-06 gave 1.19. The coefficients of variation were 0.33 and 0.43 for the proposed method and the ASCE/SEI 41-06 method, respectively. The comparisons clearly show that the proposed equation produces a better estimation of shear strength for RC columns when high axial force is applied.

CONCLUSIONS

An experimental program was carried out on seven RC columns with light transverse reinforcement under simulated gravity and different directions of seismic loading. Numerical studies were conducted to supplement the test results and further investigate the effects of seismic loading directions and axial force. The conclusions drawn are as follows:

1. The loading direction had significant effects on axial failure modes and the angle of critical diagonal crack. It was of key importance to the drift ratio at axial failure and maximum energy dissipation capacity of test specimens. However, it did not have any significant effect on the initial stiffness and drift ratio at maximum shear force.

2. The shear strength of columns in nonprincipal directions can be estimated once their shear strength in the principal direction is found; the results matched the circle-interaction line for square columns and the ellipse-interaction line for rectangular columns.

3. There were two modes of axial failure observed in the test specimens. The development of critical shear cracks caused failure in the first mode, while the second failure mode resulted from vertical bond-splitting cracks. Existing models to estimate the drift ratio at axial failure were found to be unable to be applied for those columns that had failed in the second mode.

4. An upper limit of shear strength was proposed by modifying the contribution of the concrete part in ASCE/SEI 41-06 \((f'_cA_g)\) when the applied axial force was greater than 0.35\(f'_cA_g\).

ACKNOWLEDGMENTS

The financial assistance provided by the School of Civil and Structural Engineering at Nanyang Technological University, Singapore, is gratefully acknowledged.

NOTATION

- \(A_x\) = cross-sectional area
- \(A_v\) = total transverse reinforcement area within spacing

Table 2—Comparisons of shear strength between proposed equation and ASCE/SEI 41-06 equation

<table>
<thead>
<tr>
<th>Authors</th>
<th>Specimen</th>
<th>(b, \text{mm})</th>
<th>(h, \text{mm})</th>
<th>(f'_c, \text{MPa})</th>
<th>(f_{yt}, \text{MPa})</th>
<th>(A_v, \text{mm}^2)</th>
<th>(s, \text{mm})</th>
<th>1.9</th>
<th>(P/f'_cA_g)</th>
<th>(V_{L/\text{ASCE}})</th>
<th>(V_{L/\text{pro}})</th>
<th>(V_{\text{test}})</th>
<th>(V_{\text{test}}/V_{\text{ASCE}})</th>
<th>(V_{\text{test}}/V_{\text{pro}})</th>
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<tr>
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<td>RC-1.7-0.35</td>
<td>250</td>
<td>490</td>
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<td>32.6</td>
<td>29.6</td>
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Mean: 1.02, 1.19
Coefficient of variation: 0.33, 0.43

Notes: 1 kN = 0.2248 kip; 1 mm = 0.04 in.; 1 MPa = 0.145 ksi; 1 mm² = 0.00155 in.²

Fig. 16—Comparisons of shear strength between proposed equation and ASCE/SEI 41-06 equation. (Note: 1 kN = 0.2248 kip.)
\[ a = \text{shear span (distance from maximum moment section to point of inflection)} \]
\[ b = \text{width of column section} \]
\[ d = \text{distance from extreme compression fiber to centroid of tension reinforcement} \]
\[ E = \text{cumulative energy dissipation} \]
\[ E_c = \text{modulus of elasticity of concrete} \]
\[ f'c = \text{compressive strength of concrete} \]
\[ f_y = \text{compressive strength of longitudinal reinforcement} \]
\[ f_{yt} = \text{compressive strength of transverse reinforcement} \]
\[ h = \text{height of column section} \]
\[ M/V = \text{ratio of moment to shear at critical section} \]
\[ P = \text{applied axial load} \]
\[ s = \text{spacing of transverse reinforcement} \]
\[ V_c = \text{shear force carried by concrete} \]
\[ V_{\text{max}} = \text{maximum of applied horizontal force} \]
\[ V_r = \text{nominal shear strength of columns} \]
\[ V_t = \text{theoretical flexural strength of columns} \]

REFERENCES


5. ACI Committee 318, “Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary,” American Concrete Institute, Farmington Hills, MI, 2008, 473 pp.


