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Multicast Traffic Grooming in Tap-and-Continue WDM Mesh Networks

Rongping Lin, Wen-De Zhong, Sanjay Kumar Bose, and Moshe Zukerman

Abstract—Multicast applications are expected to be major drivers of Internet traffic growth. As most multicast connections require much lower bandwidth than the capacity offered by a wavelength, multicast traffic grooming is needed to efficiently use network resources. Recent research on multicast grooming has focused on light-trees because of their natural advantage for multicast traffic. However, using light-trees may lead to some serious negative side effects because of light splitting. In this paper, we investigate the multicast traffic grooming problem in tap-and-continue (TaC) networks, where a node can tap a small amount of incoming optical power for the local station while forwarding the remainder to an output. We first propose a simple and efficient node architecture with the TaC mechanism. We use this in an integer linear programming (ILP) formulation with the objective of minimizing the network cost in terms of the number of higher layer electronic ports and the number of wavelengths used. Since the ILP is not scalable, two heuristic algorithms, multicast trail grooming (MTG) and multiple destination trail-based grooming (MDTG), are proposed. Using the ILP, we show that having more costly nodes with multicast capability does not improve the performance significantly. The solutions obtained by MTG and MDTG are close to the ILP optimal solution. MTG and MDTG are shown to work efficiently for typical network topologies such as NSFNET, with MTG showing better performance than MDTG.

Index Terms—Multicast traffic grooming; Tap-and-continue (TaC); Trail; Wavelength-division multiplexing (WDM).

I. INTRODUCTION

Multicast applications such as IPTV, telemedicine, video conferencing, distance learning and streamed video broadcasts are expected to be major drivers of Internet traffic growth, and may become key network applications in the near future. It is also likely that, in the foreseeable future, the growth, and may become key network applications in the near future will be transported over wavelength channels. Traffic generated by these applications will be transported over WDM networks, where a node can tap a small amount of incoming optical power for the local station while forwarding the remainder to an output. We first propose a simple and efficient node architecture with the TaC mechanism. We use this in an integer linear programming (ILP) formulation with the objective of minimizing the network cost in terms of the number of higher layer electronic ports and the number of wavelengths used. Since the ILP is not scalable, two heuristic algorithms, multicast trail grooming (MTG) and multiple destination trail-based grooming (MDTG), are proposed. Using the ILP, we show that having more costly nodes with multicast capability does not improve the performance significantly. The solutions obtained by MTG and MDTG are close to the ILP optimal solution. MTG and MDTG are shown to work efficiently for typical network topologies such as NSFNET, with MTG showing better performance than MDTG.

A. Related Work

Minimizing the required number of wavelengths to meet the traffic demand [1] was important when wavelength cost dominated the total network cost. However, modern technology now allows a very large number of wavelengths to be simultaneously transmitted on a fiber. This decreases the wavelength cost to the extent that it is no longer the dominant cost factor in the network. Instead, the dominant cost factors today are the number and cost of higher layer electronic components, such as add/drop multiplexers (ADMs) and IP routers [2–5]. A higher layer electronic component, such as an IP router, has a limited number of ports for transmitting or receiving traffic. In this work, we aim to design minimum cost WDM networks for multicast traffic grooming where the network cost is calculated based on the number of higher layer electronic ports and the number of wavelengths used.

In wavelength-routed networks, a lightpath connects its two end nodes without interrupting the optical signal at intermediate nodes and is effectively a single logical hop [6]. The authors of [7,8] investigated the multicast traffic grooming problem using lightpaths and proposed an integer linear programming (ILP) formulation with the objective of minimizing the network cost in terms of the number of ADMs and the number of wavelengths used. However, it is clearly not cost effective to use multiple one-to-one lightpaths for the inherently one-to-many requirements of multicast connections, as this would require more ADMs. A light-tree extends the lightpath concept by supporting a one-to-many connection in one single logical hop. This can substantially reduce the average packet hop distance and the total number of transceivers used [9]. In the logical layer, a light-tree is presented as a set of direct links from the source to the destination set of the light-tree. Since the transmission from the source to all the destinations takes only one hop and is done all optically, this is called a logical one hop tree (LOHT) [10,11]. These light-trees act as the conduits for upper layer traffic. Low bandwidth connections are routed by combining several light-trees and forming a larger tree to reach all the destinations.
with optical–electronic–optical (OEO) conversion will be required at the nodes connecting light-trees so that traffic can be forwarded from an upstream light-tree to downstream light-trees. Since a tree topology is natural for supporting multicast applications, considerable research has been reported on approaches for light-tree-based multicast traffic grooming [10–22]. It may be noted that, in [22], we studied the problem of optimizing the cost of multicast traffic grooming based on light-trees and proposed a light-tree-based ILP formulation to minimize network cost in terms of the number of higher layer electronic ports and the number of wavelengths used.

An optical power splitter, which splits incoming light into multiple identical copies in the optical domain, is required for a light-tree [9]. Network nodes that have these optical splitters are referred to as multicast capable nodes as they can split an incoming optical signal into multiple copies. A cost-effective way to achieve light splitting in the optical domain is by fusing fibers together; however, the power loss due to splitting needs to be compensated by deploying active amplifiers (e.g., erbium-doped fiber amplifiers, EDFA) so that the signals can still be detected by optical receivers. Unfortunately, optical amplifiers have negative side effects such as non-uniform gain over the operating waveband, gain saturation and additional noise [24]. For a node to provide full multicast capability it would have to be equipped with a large number of splitters for every wavelength of all the input fibers, and a large number of optical amplifiers would also be required. This increases the cost of the node and is also difficult to implement. The network cost may be reduced with sparse light splitting [24–28], where only a subset of nodes are multicast capable while the others are multicast incapable. The works reported in [24–28] find the minimal cost light-forest (multiple light-trees) to accommodate a multicast connection request, with the additional condition that light signals cannot be split at multicast incapable nodes.

The high cost, negative side effects and control complexity of multicast capable nodes has motivated investigation of the multicast problem in networks without multicast capable nodes. The authors of [29] considered the multicast routing problem in a WDM network with multicast incapable nodes that have a tap-and-continue (TaC) feature. The function of TaC is to tap a small amount of optical power for the local station, with the additional condition that light signals cannot be split at multicast incapable nodes.

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B. Contribution

To the best of our knowledge, no research has been done on optimal design of multicast grooming in TaC WDM networks (based on trails). In this paper, we investigate the optimal design and provisioning of WDM networks with multicast traffic grooming in multicast incapable networks where network nodes do not have multicast capability but do support the TaC mechanism, and trails instead of light-trees are used to convey multicast traffic. We propose a simple and efficient node architecture with the TaC mechanism, which can be a simple and cost-effective upgrade to existing network nodes designed for unicast traffic. This new node architecture is also expected to simplify network management and control. Using nodes of this type and for networks using trails instead of light-trees for multicasting, an ILP formulation is proposed to minimize the network cost (computed based on the number of higher layer electronic ports and the number of wavelengths used). Even though the ILP formulation can achieve an optimal result, it is not scalable to realistic size problems. Two new heuristic algorithms, called multicast trail grooming (MTG) and multiple destination trail-based grooming (MDTG), are proposed to efficiently deal with the multicast traffic grooming problem in large networks. It is noted that the proposed ILP formulation and the heuristic algorithms are not limited to networks with our proposed node architectures but may also be applied to other networks with the TaC capability. The solutions obtained from the proposed ILP formulation are used to benchmark the results obtained from the heuristic algorithms. The optimal results from the ILP formulation can also be helpful in guiding the design of the heuristic algorithms.

We compare our trail-based ILP formulation with the light-tree-based ILP formulation proposed in [22], and show that the performance of the trail-based ILP formulation is very close to that of the light-tree-based one. The trail-based ILP formulation shows somewhat higher wavelength usage due to longer routes of trails but still saves on node costs, as the nodes do not need to have multicast capability. We also show that, in a six-node test network, our proposed heuristic algorithms perform close to the trail-based ILP solution. We also investigate trail-based routing and propose a new node adding trail routing (NATR) algorithm which performs better with shorter trails than others reported earlier [29].
C. Organization

The rest of this paper is organized as follows. Section II describes the proposed node architectures. In Section III, the formal statement of the multicast traffic grooming problem in TaC networks is presented. In Section IV, we present the ILP formulation, the complexity evaluation of the ILP formulation and the performance evaluation of the proposed formulation in a small network. Section V focuses on the routing of the trails and provides a new routing algorithm. In Section VI, we describe details of our proposed heuristic algorithms. Section VII presents the numerical results of the light-tree-based ILP formulation, the proposed trail-based ILP formulation and the heuristic algorithms in a small network, and also provides performance results for the heuristic algorithms in a large network. Finally, Section VIII concludes this paper.

II. NODE ARCHITECTURES

A node architecture with the TaC mechanism was originally proposed in [29]. This taps a small amount of optical power at each destination on a trail, while forwarding the remaining power to downstream destinations. Such TaC networks can implement multicasting with potentially much lower node costs than networks with other kinds of multicast capable node (e.g., node architecture with splitter-and-delivery as in [30]). This is because TaC networks do not require signal splitting loss compensation (by optical amplifiers).

The node architecture proposed in [29] deploys only one TaC module (TCM) for each wavelength and can therefore tap at most one optical signal at each wavelength for the local station of the node. When the multicast traffic load is heavy, this architecture may cause a large blocking probability due to the lack of a TCM to tap the signal. This can be avoided by a simple extension of the node architecture of [29] as shown in Fig. 2, with multiple TCMs connected to each wavelength-routing switch (WRS). Here there are $M_i$ TCMs ($1 \leq i \leq W$ and $1 \leq M_i \leq N$, where $W$ is the number of wavelengths per fiber, $N$ is the number of the output fibers of the node) connected to the WRS on wavelength $i$. Multiple TCMs can support multiple trails of each wavelength to be tapped for the local station. The maximum number of TCMs deployed for each wavelength can be equal to the number of the output fibers $N$ (full tap capability). A TCM is illustrated in Fig. 3, where 0.5% of an input optical signal is tapped by a TAP (tapping device) for the local station, while the rest is switched to an output port through a multistage network of $1 \times 2$ switches [29].

We propose an alternative node architecture illustrated in Fig. 4, which has the full tap capability; i.e., any input signal from the fibers can be tapped. Here every output port of a WRS has a TaC deployed to tap its signal. This TaC device is simply a $1 \times 2$ coupler without using photonic switches, whose function is to tap a small fraction of the signal and forward it to the local station, while the remainder continues to go to the mux. The size of a WRS used in this new node architecture is smaller. For example, as shown in Fig. 4, if there are $A_i$ add ports and $B_i$ drop ports at the WRS on wavelength $i$, where $1 \leq i \leq W$, the size of each WRS is $(N + A_i) \times (N + M_i + B_i)$, which is smaller than the earlier value $(N + A_i) \times (2N + B_i)$ for the full tap capability if both architectures have the same number of add ports and drop ports. The number of TaC devices will be NW. The tapped signal can be received or discarded at the receiver bank, and the dropped signals (terminated at the node) are directly switched to drop ports by WRSs and are then received by the receiver bank.
The actual amount of signal to be tapped down depends on the sensitivity of the optical receivers used. A receiver with higher sensitivity can successfully detect lower power signals but would be costlier. Generally, for an OC-48 transmitter, the output power of a laser is about 1 mW, a tap device can cheaply tap 1/1000 power [31] out for the local station and the receiver sensitivity would need to be about \(-30\) dBm for reasonably priced receivers. In TaC networks with the proposed node architecture of Fig. 4, a light signal can undergo tens of tapping operations, even if each tapping takes up 1/500 of the power. This number of tapping operations is acceptable in realistic optical networks as most trails will have far fewer physical hops (number of tappings) than this value.

### III. General Problem Statement

The problem of grooming sub-wavelength multicast traffic onto wavelength channels in a TaC WDM network can be stated as follows:

1. A physical topology \(G(V,E)\) is an undirected graph denoting a network, where \(V\) is the set of network nodes and \(E\) is the set of physical optical links. We assume that there are two fibers in each optical link running in opposite directions.
2. Fibers of each link are identical and each fiber can simultaneously transmit the same number of wavelengths \(W\) each with a capacity of \(C\).
3. There are no wavelength converters in the network. This means that a trail traversing multiple links must use the same wavelength. (Note that it is straightforward to extend the optimization approach to the case of wavelength conversion, which, in fact, makes the problem simpler in terms of the number of variables and equations.)
4. All nodes are capable of optical tapping with TaC devices. A node will tap a fraction of the optical signal for the local station and will transmit the remainder to the output port. The analysis in this paper uses the node architecture in Fig. 4, but it is also applicable to other TaC node architectures, such as the one in Fig. 2.
5. A set of multicast connection requests with sub-wavelength bandwidth requirements.

Our goal is to design the network with minimal cost, in terms of the number of higher layer electronic ports and the number of wavelengths used in the network. The detailed outputs of the design process are as follows:

1. **A set of trails that constitute the logical layer.** A trail has one source node, one end node and multiple intermediate nodes. A higher layer electronic port and one add port of a WRS are used at the source node to send out the multicast traffic, and an electronic port and a drop port of a WRS is also used at the end node as the termination (also destination) of the trail; intermediate nodes configure their receiver banks to receive the tapped signals as destinations (which will take one electronic port each in the higher layer) or discard the tapped signals. Therefore, the transmission from the source to all the destinations takes only one logical hop and is done all optically, making this an LOHT in the logical layer.
2. **Multicast traffic routing.** A multicast connection request may be accommodated by multiple LOHTs in the logical layer, which are trails in the optical layer. Multiple trails are connected together in the electronic domain by higher layer electronic components. Usually, electronic packet switching is used for interconnecting LOHTs (trails) and implementing traffic forwarding from an upstream LOHT to downstream LOHTs with OEO conversion.
3. **Trail routing and wavelength assignment (RWA).** The routing and wavelength assignment of all the trails in the optical layer.

It has been shown that the unicast traffic grooming based on lightpaths is an NP-hard problem [32]. Since the unicast is a special case of the multicast with only one destination, and...
since a lightpath is a special case of a trail with only one destination node, then by generalization the multicast traffic grooming based on trails is also NP-hard.

IV. MATHEMATICAL (ILP) FORMULATION

A trail can be identified by the combination of its source node, end node and intermediate nodes along with information on whether a receiver (or a receiving electronic port) is needed to receive the tapped signal or not. There are large numbers of combinations of nodes to be included in or excluded from a trail. In ILP formulations, we should try to avoid introducing a large number of variables and constraints caused by the problem of the large number of combinations. We find that, although the number of combinations of nodes to be included in or excluded from a trail is large, the number of combinations from a node is small. In a realistic WDM mesh network, a node is usually physically adjacent to only a few nodes so the nodal degree is small. The number of connections sourcing from or terminating to a node is limited by its nodal degree. Specifically, the number of connections is less than or equal to the nodal degree times the number of wavelengths W. This implies that the number of trails that start from a node and work on a specific wavelength cannot exceed the nodal degree.

A trail is denoted by (i, w, k), where i is the source node of the trail, w denotes the wavelength used by the trail and k is the index of the trail, 1 ≤ k ≤ Deg(i), where Deg(i) is the nodal degree of node i. The information of the receiver (receiving electronic port) usage at intermediate nodes and the end node of trail (i, w, k) is denoted by a Boolean parameter T_{i,w,k}^d, which takes the value of 1 if node t uses a receiver to receive the tapped/dropped signal on trail (i, w, k), otherwise 0. Note that the end node of the trail has T_{i,w,k}^d = 1 as the end node uses a receiver to terminate the signal. There are two types of intermediate node, one of which does not use a receiver (called a forwarding node) and has T_{i,w,k}^d = 0, and the other with T_{i,w,k}^d = 1, which uses a receiver (called tap-receiving node). Accordingly, there may be four types of node on a trail: source node, end node, forwarding node and tap-receiving node.

A. The ILP Formulation

Given

\begin{align*}
|V|: & \quad \text{Number of nodes in the network.} \\
W: & \quad \text{Number of wavelengths per fiber, which is preset for optimization. Usually, the optimal value is lower than this value.} \\
w: & \quad \text{Wavelength index, starting from 1 and ending at W.} \\
C: & \quad \text{Capacity of a wavelength.} \\
Z: & \quad \text{A sufficiently large integer number.} \\
a: & \quad \text{Relative cost of a higher layer electronic port.} \\
\beta: & \quad \text{Relative cost of a wavelength in the network.} \\
m \text{ and } n: & \quad \text{Two endpoints of a physical fiber link.} \\
P_{mn}: & \quad \text{Number of fibers interconnecting node } m \text{ and node } n \text{ in the physical layer. It is zero if } m \text{ and } n \text{ are not physically adjacent to each other. In this study, we assume } P_{mn} = P_{nm} = 1 \text{ if there is a physical fiber link between nodes } m \text{ and } n. \\
R: & \quad \text{Maximal index of the multicast connection requests.} \\
r (1 \leq r \leq R): & \quad \text{A multicast connection request } r. \\
s_r: & \quad \text{The source of connection request } r. \\
D_r = \{d_1, d_2, d_3, \ldots\}: & \quad \text{The destination set of connection request } r. \\
f_r: & \quad \text{The required bandwidth of connection request } r. \\
(s_r; D_r; f_r): & \quad \text{A 3-tuple of the elements } s_r, D_r \text{ and } f_r \text{ representing the multicast connection request } r. \\
\text{Deg}(i): & \quad \text{Nodal degree of node } i. \\
\end{align*}

Variables

\begin{align*}
A_{1}^{w}: & \quad \text{Number of add ports of WRS for wavelength } w \text{ at node } t. \\
B_{1}^{w}: & \quad \text{Number of drop ports of WRS for wavelength } w \text{ at node } t. \\
A_{1}^{t}: & \quad \text{Number of transmitting electronic ports at node } t. \\
B_{1}^{t}: & \quad \text{Number of receiving electronic ports at node } t. \\
\phi: & \quad \text{Number of wavelengths to be used in the network.} \\
\alpha: & \quad \text{Relative cost of a wavelength in the network.} \\
\beta: & \quad \text{Relative cost of a higher layer electronic port.} \\
\end{align*}

This equation shows the optimization objective function of the network cost in terms of the number of transmitting electronic ports, receiving electronic ports and wavelengths used in the network. Here \( \alpha \) and \( \beta \) represent the relative costs...
of a higher layer electronic port and a wavelength channel, respectively.

Constraints on network resource variables

Equation (2) ensures that the number of trails starting from node \( t \) and working on a wavelength \( w \) is no larger than the number of add ports of the WRS for the wavelength \( w \) at node \( t \). Equation (3) ensures that the total number of add ports used in a node is no larger than the total number of transmitting electronic ports at the node. Equation (4) ensures that, for each wavelength, the number of trails terminating at node \( t \) is no larger than the number of drop ports of the WRS on that wavelength at node \( t \). It is noted that a source node may have an incoming stream; Equation (5) ensures that the number of receivers (receiving electronic ports) used at a node is no larger than the total number of receiving electronic ports at that node.

\[
\sum_k U_{i,w,k} \leq A^w_t \quad \forall t,w, \tag{2}
\]

\[
\sum_w A^w_t \leq A_t \quad \forall t, \tag{3}
\]

\[
\sum_i \sum_m M^m_{i,w,k} \leq B^w_t \quad \forall t,w, \tag{4}
\]

\[
\sum_i \sum_k T^t_{i,w,k} \leq B_t \quad \forall t. \tag{5}
\]

As the objective is to minimize \( \varphi \), Eq. (6) and the objective function (1) together ensure that \( \varphi \) is the highest index of the used wavelengths. Equation (7) ensures that \( y_w \) is set to 1 if wavelength \( w \) is used by any trail. Equations (8) and (9) ensure that trail \((i,w,k)\) is built if it uses at least one receiver.

\[
\varphi \geq w \ast y_w \quad \forall w, \tag{6}
\]

\[
y_w \geq \sum_k U_{i,w,k}/Z \quad \forall w, \tag{7}
\]

\[
U_{i,w,k} \geq \sum_t T^t_{i,w,k}/|V| \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \tag{8}
\]

\[
U_{i,w,k} \leq \sum_t T^t_{i,w,k} \quad \forall i,w,1 \leq k \leq \text{Deg}(i). \tag{9}
\]

Constraints on physical route variables

Equations (10)–(19) are the commodity-flow conservation constraints to create a physical route for trail \((i,w,k)\). Equation (10) ensures that, for source node \( i \), the number of units of the outgoing commodity is larger than that of the incoming commodity by the number of destinations of the trail. It is noted that a source node may have an incoming stream; e.g., in Fig. 1 source node \( s \) has an incoming stream from \( d_1 \). Equation (11) ensures that for each trail \((i,w,k)\), if node \( t \) is a destination of the trail, the number of units of the incoming commodity is larger than that of the outgoing commodity by 1; otherwise, they are equal. Equations (12) and (13) ensure that if link \((m,n)\) transmits the commodity of trail \((i,w,k)\), then this link is not traversed.

\[
\sum_{n \in \text{adj}(t)} F^m_{i,w,k} - \sum_{m \in \text{adj}(i)} F^{m,i}_{i,w,k} = T^t_{i,w,k} \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \tag{10}
\]

\[
\sum_{m \in \text{adj}(t)} F^m_{i,w,k} - \sum_{n \in \text{adj}(t)} F^{n,i}_{i,w,k} = T^t_{i,w,k} \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \tag{11}
\]

\[
F^m_{i,w,k} \geq M^{m,i}_{i,w,k} \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \forall m \in E, \tag{12}
\]

\[
F^m_{i,w,k} \leq |V| \cdot M^{m,i}_{i,w,k} \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \forall m \in E. \tag{13}
\]

The main difference between a trail and a light-tree is that a trail does not split the incoming optical signal, while a light-tree splits the incoming optical signal into multiple copies at certain nodes. Equations (14)–(19) ensure that trail routing does not split an incoming optical signal. Equation (14) ensures that, if trail \((i,w,k)\) is built, a node which is adjacent to source \( i \) is traversed as the first hop. Equation (15) ensures that other nodes apart from the source node have no adjacent node traversed as the first hop. Equation (16) ensures that, for each trail, the number of outgoing streams forwarded from a specific incoming stream at a node is no larger than 1, so incoming optical signals are not split. Equation (17) ensures that, if link \((t,n)\) is traversed by a trail, there should be one incoming stream to node \( t \) which is forwarded to link \((t,n)\) in the trail.

\[
\sum_{n \in \text{adj}(i)} S^t_{i,w,k} = U_{i,w,k} \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \tag{14}
\]

\[
\sum_{n \in \text{adj}(t)} S^t_{i,w,k} = 0 \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \forall t \neq i, \tag{15}
\]

\[
\sum_{n \in \text{adj}(t)} S^m_{i,w,k} = 1 \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \tag{16}
\]

\[
\sum_{m \in \text{adj}(t),t \cup t} S^m_{i,w,k} \leq T^t_{i,w,k} \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \tag{17}
\]

Equation (18) ensures that, if a node is traversed and the trail returns to the upstream node along the same link with the opposite direction, this node is a destination. This equation can avoid unnecessary links being made into a trail. Equation (19) ensures that the number of units of commodity on the incoming stream is no less than that on the outgoing stream, where the outgoing stream is directly forwarded from the incoming stream. Equation (20) ensures that the wavelength \( w \) of a fiber link \((m,n)\) can only be occupied by at most one trail.

\[
S^m_{i,w,k} \leq T^t_{i,w,k} \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \tag{18}
\]

\[
F^m_{i,w,k} \geq P^m_{i,w,k} - (1 - S^m_{i,w,k}) |V| \quad \forall i,w,1 \leq k \leq \text{Deg}(i), \tag{19}
\]

\[
\forall t, m \in \text{adj}(t), n \in \text{adj}(t), \tag{20}
\]

\[
\sum_k M^{m,i}_{i,w,k} \leq P_{mn} \quad \forall w,mn \in E. \tag{20}
\]

Constraints on traffic routing variables

Each multicast connection request is supported by at least one trail. These trails connect each other in the electronic domain to form a multicast session. As \( Q^r_{i,w,k} \) is set to \( \lambda^r_{i,w,k} \) times \( T^t_{i,w,k} \), we use Eqs. (21) and (22) to assign a value to \( Q^r_{i,w,k} \) which ensures that, if connection request \( r \) traverses
trail \((i, w, k)\), and node \(t\) uses a receiver to receive the signal on that trail, then \(Q_{r,i,w,k}^{t,1}\) will be set to 1, otherwise it will be 0.

\[
\begin{align*}
\lambda_{r,i,w,k}^t + T_{r,i,w,k}^t & \geq 2Q_{i,w,k}^{r,1} \quad \forall r,t,i,w,1 \leq k \leq \text{Deg}(i), \quad (21) \\
\lambda_{r,i,w,k}^t + T_{r,i,w,k}^t & \leq Q_{i,w,k}^{r,1} + 1 \quad \forall r,t,i,w,1 \leq k \leq \text{Deg}(i). \quad (22)
\end{align*}
\]

The following equations constrain the incoming and outgoing streams of a source node. Equation (23) ensures that at least one trail will start from the source of the connection request. Equation (24) ensures that, for every connection request, no trail that supports the connection request terminates at the source of that request. Equation (25) ensures that each destination of a connection request must have one incoming stream. Equation (26) ensures that for every connection request, except the source and destinations of the connection request, other nodes have at most one incoming stream; i.e., each intermediate node of the multicast session has one incoming stream, and other nodes which are not traversed by the session have no incoming stream.

\[
\begin{align*}
\sum_w \sum_k \lambda_{s,i,w,k}^r & \geq 1 \quad \forall r, \quad (23) \\
\sum_w \sum_k Q_{r,i,w,k}^{t,s} & = 0 \quad \forall r, \quad (24) \\
\sum_t \sum_w \sum_k Q_{i,w,k}^{t,d} & = 1 \quad \forall r,t \in D_r, \quad (25) \\
\sum_t \sum_w \sum_k Q_{i,w,k}^{t,d} & \leq 1 \quad \forall r,t \neq s_r, t \notin D_r. \quad (26)
\end{align*}
\]

Equation (27) ensures that, apart from the source node of the connection request, the starting node of the trail that supports the connection request must be the end node or a tap-receiving node of another trail that also supports that request. Equation (28) ensures that, apart from the source and destinations of the connection request, each end node or a tap-receiving node on that multicast session must be the starting node of some trails which support the connection request. Equation (29) ensures that the bandwidth used by all multicast connection requests in the trail \((i, w, k)\) must be equal to or less than the total capacity offered by that trail.

\[
\begin{align*}
\sum_w \sum_k \lambda_{t,i,w,k}^r & \leq \text{Deg}(t) \cdot W \cdot \sum_w \sum_k Q_{i,w,k}^{r,1} \quad \forall r,t \neq s_r, \quad (27) \\
\sum_k \sum_i \sum_w Q_{i,w,k}^{r,1} & \geq \sum_r \sum_t \sum_k Q_{i,w,k}^{t,d} \quad \forall r,t \neq s_r, t \notin D_r, \quad (28) \\
\sum_t f_r \cdot \lambda_{i,w,k}^r & \leq C \quad \forall i,w,1 \leq k \leq \text{Deg}(i). \quad (29)
\end{align*}
\]

### Constraints on traffic route loop-free variables

The following constraints ensure that the routing of each connection request is loop free. Equation (30) ensures that, for each connection, nodes that are not traversed by the connection have a zero value, and the source node of the connection has a zero value as well. Equation (31) ensures that, for each connection, if trail \((p,w,k)\) is traversed, where \(p\) is the source of the trail and node \(t\) on this trail uses a receiver, the hop number from the source of the request to node \(i\) is larger than that from the source to node \(p\); if two nodes are not on the same trail used by the connection request, this equation is always satisfied.

\[
\begin{align*}
Y_t^r & \leq |V| \cdot \sum_i \sum_w Q_{i,w,k}^{r,1} \quad \forall r,t, \quad (30) \\
Y_t^r & \geq Y_t^r + 1 - \left(1 - \sum_p \sum_w Q_{p,w,k}^{r,1}\right) |V| \quad \forall r,t, \forall p \neq t. \quad (31)
\end{align*}
\]

Note that if bifurcation is used then the traffic can be divided into components where each component is routed separately. This would increase the complexity and cost because of traffic bifurcation, and reassembly may introduce delay jitter [7]. In the ILP formulation proposed here, if a trail \((i, w, k)\) has been used by a connection request, all traffic of the request is conveyed by this trail, so bifurcation is not allowed.

### B. Number of Variables and Constraints

As the complexity of an ILP problem is decided by the number of variables and constraints, we count these to get an insight into the complexity of the framework. The number of variables is \(O(RW|V| |E| + gW|E|^2)|\), which grows quadratically with the number of edges in the network, where \(E\) is the set of physical links in the network and \(g\) is the maximal nodal degree. The number of constraints is \(O(RW|V||E| + gW|E|^2)|\), which also grows quadratically with the number of edges. As the size of the network grows (the number of edges also increases in connected networks), solving the ILP problem becomes very time consuming and prohibitively complicated. Therefore, the ILP-based approach cannot scale to large networks. For these, heuristic algorithms are needed to deal with the problem efficiently.

### C. An Example of the ILP Formulation

We present here an example of a TaC network that uses the ILP formulation described above for multicast grooming. The test network has six nodes as shown in Fig. 5. Ten randomly generated multicast connection requests, shown in Table I, are given as the input to the formulation. The source of a multicast connection request is randomly chosen from the network nodes; the multicast destination set size is also randomly chosen, ranging from 1 to 5. The destination nodes are randomly selected from the network nodes (excluding the source node). We assume that the capacity \(C\) of a wavelength is OC-12, and the required bandwidth of a multicast connection request is a random integer with uniform distribution from 1 to 12 (an integer \(i\) denotes a bandwidth of OC-\(i\)), e.g., the first request (4; 1, 6; 2), where node 4 is the source, nodes 1 and 6 are destinations and value 2 is the required bandwidth. To compare the performance of the proposed trail-based ILP formulation, the light-tree-based ILP formulation proposed in [22] is also examined. We used a commercial ILP solver, CPLEX [34], to solve the ILP formulations.

The ILP objective is to reduce the network cost in terms of the number of higher layer electronic components and wavelengths deployed. As explained in [7], the values of parameters \(\alpha\) and \(\beta\) depend on the network topology as well as
TABLE I
TRAIL-BASED TRAFFIC ROUTINGS OF TEN CONNECTION REQUESTS

<table>
<thead>
<tr>
<th>Index</th>
<th>Requests</th>
<th>Traffic routings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(4; 1, 6; 2)</td>
<td>(4 → 6) (6 → 1)</td>
</tr>
<tr>
<td>2</td>
<td>(6; 1, 3, 5; 12)</td>
<td>(6 → 1, 3, 5)</td>
</tr>
<tr>
<td>3</td>
<td>(6; 1, 2, 3, 4, 5; 10)</td>
<td>(6 → 2, 3, 5) (2 → 1, 4)</td>
</tr>
<tr>
<td>4</td>
<td>(5; 1, 2, 1)</td>
<td>(5 → 1, 2)</td>
</tr>
<tr>
<td>5</td>
<td>(6; 2, 3, 5; 1)</td>
<td>(6 → 2, 3, 5)</td>
</tr>
<tr>
<td>6</td>
<td>(3; 1, 2, 4, 5, 6)</td>
<td>(3 → 4, 5) (5 → 1, 2)</td>
</tr>
<tr>
<td>7</td>
<td>(1; 2, 3, 4, 5, 6; 6)</td>
<td>(1 → 2, 3, 4, 5) (4 → 6)</td>
</tr>
<tr>
<td>8</td>
<td>(6; 1; 7)</td>
<td>(6 → 1)</td>
</tr>
<tr>
<td>9</td>
<td>(2; 5; 6, 11)</td>
<td>(2 → 5, 6)</td>
</tr>
<tr>
<td>10</td>
<td>(3; 4; 5, 3)</td>
<td>(3 → 4, 5)</td>
</tr>
</tbody>
</table>

actual equipment costs. Following [7], we assume that a higher layer electronic port is about three times as expensive as a wavelength in this six-node network; i.e., the values of α and β are set to be 3 and 1, respectively. Solving the ILP problem, we obtain the optimal solution of accommodating these ten multicast connection requests.

Table I gives the traffic routings of ten multicast connection requests in the trail-based ILP formulation. In the table, a trail is denoted as an LOHT in the logical layer; e.g., (6 → 1, 3, 5) denotes a trail started from node 6 using receiving electronic ports to receive tapped/dropped traffic at nodes 1, 3, and 5. Some multicast sessions consist of several trails, and some trails are shared by several multicast connection requests whose required bandwidth is smaller than a full wavelength bandwidth. For example, trails (3 → 4, 5) and (6 → 2, 3, 5) are shared by multicast connection requests 6, 10, and 3, 5 separately. It is noted that most of the trails are rooted at the source or one destination node of a connection request. This can reduce one electronic port since an electronic port is needed for receiving the traffic anyway at each destination node; e.g., the routing of the sixth connection request consists of two trails, (3 → 4, 5) and (5 → 1, 2), and these two trails connect with each other at node 5, which is a destination of the connection request. Other connection requests, such as the first, third and seventh, behave similarly.

Table II shows the network cost, numbers of transmitting and receiving electronic ports, and wavelengths obtained by the trail-based ILP formulation and the light-tree-based ILP formulation. The trail-based ILP formulation has a slightly higher cost (about 1.1% higher) than the light-tree-based ILP formulation, and this extra cost arises from the one more wavelength used in the trail-based formulation. However, the number of required higher layer electronic ports is the same in the two formulations. Given a light-tree, a trail with the same source and destinations can be derived by visiting all nodes of the tree (start from the source and then go to every node in sequence). Since some nodes may be visited multiple times, this will lead to more links to travel. Therefore, the trail-based ILP formulation results in the same number of higher electronic ports as the light-tree-based formulation, but using more wavelengths. In Table II, the trail-based formulation and the light-tree-based case use the same number of electronic ports, but the allocations are different; i.e., nine and 20 electronic ports are used for transmitting and receiving in the trail-based case, while ten and 19 electronic ports are used in the light-tree-based case. In this example, even though the trail-based ILP formulation has a slightly higher cost than the light-tree-based one, it has the advantage that costly multicast capable nodes are not needed.

In this six-node test network, the trail-based ILP formulation has the same number of higher layer electronic ports as the light-tree-based ILP formulation. The reason is that, in such a small network, a higher layer electronic port is three times as expensive as a wavelength. As a result, increasing one wavelength is more economical than increasing one more electronic port and also one additional wavelength usage is sufficient for rerouting a light-tree to form a trail with the same number of electronic ports. However, for large networks, the cost of deploying one more wavelength in all the optical links of the network could be much more than that of increasing one more electronic port (α and β are changed accordingly). In such a case, the trail-based ILP formulation prefers the solution using more electronic ports than using more wavelengths to reduce the cost. This balanced procedure arises naturally in the optimization process. It is noted that one more wavelength could benefit many trails, as a trail usually uses only a few links in a network, especially in a large network with many optical links. This means, in most cases, that the cost of one more wavelength is distributed to multiple trails, which reduces the relative cost of a wavelength. This implies that, even in large networks, the solution with the same (or slightly greater) number of electronic ports as the light-tree-based formulation and with more wavelength usage is preferred by the trail-based case. Therefore, in small networks, the trail-based ILP formulation prefers the solutions with the same number of higher layer electronic ports as the light-tree-based ILP formulation and with a few more wavelengths; in large networks, the trail-based formulation may also prefer the solution with the same (or a slightly greater) number of electronic ports as the light-tree-based case but with a few more wavelengths. In both scenarios, the trail-based formulation would perform very closely to the light-tree-based case with the cost difference of at most a few more wavelengths.

Table III shows that the physical routes of trails are evenly distributed over the optical layer so as to minimize the number of wavelengths used (this arises naturally in the optimization process). At times, this will lead to a physical route that does not have the shortest routing in the given network, e.g., the physical routes of two trails, (6 → 1, 3, 5) and (6 → 2, 3, 5). To share the wavelength λ2 with the trail (6 → 1, 3, 5), the trail (6 → 2, 3, 5) uses the route of 6 → 3 → 5 → 4 → 1 → 2 instead of the shortest one 6 → 5 → 3 → 2 to avoid overlapping.
This phenomenon has also been found in the light-tree-based ILP formulation with the same objective [22].

From Table III, we can see that the nodes denoted with bold numbers are destinations of trails, which are either tap-receiving nodes or end nodes, where only tap-receiving nodes need tap devices to tap optical signals for local stations for this set of multicast connection requests; other end nodes can receive optical signals directly from WRSs through drop ports. If we check the resource configurations for the two different node architectures shown in Figs. 2 and 4, we should see the improvement of our newly proposed architecture. From Table III, the numbers of tappings at nodes 1–6 are one, two, two, three, and one, respectively. This implies that the numbers of TCMs deployed at nodes 1–6 are one, two, two, three, and one, respectively, for the first node architecture in Fig. 2. In this case, the number of WRS output ports has to be increased accordingly. However, if the second node architecture with the full tap capability in Fig. 4 is applied, no extra WRS output ports are needed.

In the above example, it took tens of hours to obtain the optimal results by solving the ILP formulation running on a PC with a 2.8 GHz CPU and 1024 MB RAM. The ILP approach cannot scale to large networks, so heuristic algorithms are needed. The optimal results obtained by solving the ILP formulation on the test network provide us with the following useful observations, which lead us to develop the heuristic algorithm presented in Section VI.

1) Multicast sessions may consist of multiple trails, and a trail may be shared by multiple connection requests.
2) The root of a trail is usually either the source or a destination of a connection request.
3) The routes of trails are not always the shortest. It may be desirable to distribute the optical link usage over the network so that the number of wavelengths used can be reduced.
4) A connection request with a full wavelength capacity requirement would be optimally accommodated by one trail.

D. Considering Wavelink Cost

In this section, we examine how the optimization results are affected if we replace wavelength cost with wavelink (wavelength link) cost in the ILP objective given by Eq. (1). The new objective function is the following.

\[
\text{Minimize : } \mu \sum_n (A_n + B_n) + \tau \sum_{i,u,k,m} M_{i,u,k,m}^{m,n}, \tag{32}
\]

where \(\mu\) and \(\tau\) are the relative costs of an electronic port and a wavelink, respectively. Following [7], we assume that their costs satisfy \(\mu/\tau = 50\). Now the constraints are the same as given by Eqs. (2)–(31). We use the same set of connection requests in Table I as the input to the new ILP formulation. It turns out that the optimal solution of Eq. (32) uses the same number of transmitting and receiving electronic ports as the ILP formulation with the objective function of Eq. (1), but different routings and wavelength assignments for trails, which are shown in Table IV.

From the solution, the total cost is 1475, where 29 electronic ports (the same as the previous ILP formulation) contribute 1450, and 25 wavelinks contribute the remaining 25, so the cost of electronic ports still contributes the most, as an electronic port is much more expensive than a wavelink. In Table IV, the solution uses the minimal number of wavelinks to accommodate the connection requests, which implies that the solution uses the shortest path to connect the source and destinations of a trail. This makes some links traverse many more times than other links, leading to unbalanced network load, and a large number of wavelengths to be deployed. For example, the link from node 5 to node 4 is traversed five times, so the total number of wavelengths deployed in the network is five, which is larger than the three of the previous ILP formulation. This implies an increase of two wavelengths. On the other hand, the design based on Eq. (32) saves seven wavelinks. However, since based on [7] the cost of a wavelength is 16 times more than the cost of a wavelink in this six-node network, according to the present example, using Eq. (1) will lead to lower total network cost than using Eq. (32). Finally, we remind the reader that for tractability we aim for a simple objective function where we neglect costs that are not very dominant. In this case, we have illustrated that we can neglect wavelink costs.

V. TRAIL ROUTING

Before describing our proposed heuristic trail-based grooming algorithm, we first investigate the trail routing problem.
TABLE IV
RWA OF TRAILS CONSIDERING WAVELINK COST

<table>
<thead>
<tr>
<th>Trail</th>
<th>Physical route</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 → 2, 3, 4)</td>
<td>1 → 4 → 2 → 3 (λ2)</td>
</tr>
<tr>
<td>(2 → 5, 6)</td>
<td>2 → 4 → 5 → 6 (λ4)</td>
</tr>
<tr>
<td>(3 → 4, 5)</td>
<td>3 → 5 → 4 (λ3)</td>
</tr>
<tr>
<td>(4 → 5)</td>
<td>→ 5 (λ2)</td>
</tr>
<tr>
<td>(5 → 1, 2)</td>
<td>5 → 4 → 2 → 1 (λ1)</td>
</tr>
<tr>
<td>(5 → 6)</td>
<td>5 → 6 (λ3)</td>
</tr>
<tr>
<td>(6 → 1, 2, 3, 4, 5)</td>
<td>6 → 3 → 5 → 4 → 1 → 2 (λ3)</td>
</tr>
<tr>
<td>(6 → 1)</td>
<td>6 → 5 → 4 → 1 (λ4)</td>
</tr>
<tr>
<td>(6 → 1, 3, 5)</td>
<td>6 → 3 → 5 → 4 → 1 (λ3)</td>
</tr>
</tbody>
</table>

Itself, as this would determine the performance of heuristic grooming algorithms. In [29], the problem of routing a trail in a TuC network with the minimal cost is called the multiple destination minimum cost trail (MDMCT) problem, and it has been proved to be a NP-complete problem. The multiple destination trail (MDT) algorithm was proposed to solve MDMCT [29]. There are two steps in this MDT algorithm. The first step is to use the minimum path heuristic (MPH) algorithm developed in [35] to find a Steiner tree for the multicast connection. The second step is to derive the trail by visiting each branch of the Steiner tree in sequence. Since the cost of the Steiner tree obtained by the MPH is up to twice that of the optimal one [35] and the links of the Steiner tree may be traversed twice, the MDT heuristic is a 4-approximation algorithm for the MDMCT problem.

In the MDT algorithm, the idea of visiting each branch of the Steiner tree to derive the trail may result in a longer route for the trail, because all but one of the branches of the tree would be traversed downward and upward. We instead propose a new heuristic routing algorithm, called the NATR algorithm, to efficiently route a trail with fewer edges to travel.

A. Node Adding Trail Routing (NATR) Algorithm

The idea of the NATR algorithm is to add destinations one by one to a trail, while avoiding long routes being generated. An initial trail will be given first and then the trail is subsequently developed by adding other destinations of the connection request with minimal costs. In Fig. 6, three scenarios of adding nodes are given. Figure 6(a) is the original trail. In Fig. 6(b), a destination node d is added to a node m of the trail, and the trail will be changed to s→...→m→...→d→...→m→...→e, where node m is traversed twice. Figure 6(c) shows the scenario where node d is appended to the end node e of the trail. In Fig. 6(d), destination node d is added between nodes m and n, so the trail becomes s→...→m→...→d→...→m→...→e, and the edge (m, n) is not traversed in the new trail. Accordingly, the increased costs of adding node d to the trail in the three scenarios of Figs. 6(b), 6(c) and 6(d) are the following: the sum cost of the path from m to d and the path from d to m; the cost of the path from e to d; and the sum cost of the path from m to d and the path from d to n minus the cost of link (m, n), respectively. To select a destination to add into, all destinations that are not reached by the trail are checked in the three scenarios and the destination d is selected with the criterion of the minimal adding cost. The pseudocode of the NATR algorithm is given as follows.

Node adding trail routing (NATR) algorithm

Input: A network G(V, E), and a multicast connection request (s; D)
Output: The trail T for the connection request
Algorithm:
BEGIN
1. Among all shortest paths from s to each destination in D, select the one which traverses the largest number of destinations in D as the initial trail T.
2. D = D−(destinations traversed by T) // delete destinations traversed by the initial trail
3. While D ≠ ∅ do
4. Generate a graph G′, which is a copy of G but excludes the edges of the trail T
5. Find shortest paths from every node of T to every node of D on G′, so that all paths are edge disjointed to the trail T
6. Find the node d in D, which has the lowest cost to add into the trail T
7. Add the path(s) to connect d into the trail T
8. D = D\d // delete d from D
9. End While
END

In the algorithm, the initial trail is selected from the shortest paths from the source node to each of destinations at line 1; the complexity of this step is O(|V|log|V|). The initial trail will then be developed to a feasible solution by adding destinations one by one from line 3 to 9. Since the trail cannot traverse an edge with the same direction twice [29], we generate an auxiliary graph G′ that excludes the edges of the existing trail in line 4. The destination d with the minimal adding cost will be selected (line 6), and then the path(s) will be added into the trail to form a new trail. The complexity of improving the solution is O(|V|^3log|V|) as there are at most |V| nodes to be added into and each adding operation would
have the complexity of $O(|V|^2 \log |V|)$ (node pair routing at line 5 is $O(|V|^2 \log |V|)$; selecting a node $d$ at line 6 is $O(|V|^2)$). Therefore, the overall complexity of the NATR algorithm is $O(|V|^3 \log |V|)$.

### B. Optimal Trail Routing

We use a short version of the ILP formulation in Section IV to derive the optimal trail routing, which accommodates only one multicast connection. A multicast connection request is given, and the ILP formulation is used to find the optimal routing of the trail for the connection request.

**Given**

$(s; D)$: A 2-tuple of the elements $s$ and $D$ representing a multicast connection request, where $s$ and $D = \{d_1, d_2, d_3, \ldots \}$ are the source and the destination set of the connection request, respectively.

**Variables**

$F_{mn}$: An integer commodity-flow variable. It denotes the number of units of commodity flowing on link $(m, n)$, and is also the number of destinations downstream of link $(m, n)$.

$M_{mn}$: A Boolean variable. It is 1 if the trail traverses link $(m, n)$, otherwise 0.

$S^m_{t,n}$: A Boolean variable. It is 1 when the physical routing includes links $(m, t)$ and $(t, n)$ to forward an incoming stream where $m \in \text{adj}(t) \cup \{t\}$, $n \in \text{adj}(t)$, otherwise 0. At the source node $s$, $S^s_{s,n}$ indicates whether the first physical hop $s \rightarrow n$ is traversed or not.

**Minimize:**

$$\sum_{mn \in E} M_{mn}.$$  \hspace{1cm} (33)

The objective function is to minimize the number of edges to accommodate the multicast connection request $(s; D)$.

$$\sum_n F_{sn} - \sum_m F_{ms} = |D|,$$  \hspace{1cm} (34)

$$\sum_m F_{md} - \sum_n F_{dn} = 1 \quad \forall d \in D,$$  \hspace{1cm} (35)

$$\sum_m F_{mt} = \sum_n F_{tn} \quad \forall t \neq s, t \neq D,$$  \hspace{1cm} (36)

$$F_{mn} \geq M_{mn} \quad \forall mn \in E,$$  \hspace{1cm} (37)

$$F_{mn} \leq |D| : M_{mn} \quad \forall mn \in E.$$  \hspace{1cm} (38)

Equation (34) ensures that the total number of units of the outgoing commodity is larger than that of the incoming commodity by $|D|$ at the source node $s$. Equation (35) ensures that the number of units of the incoming commodity is one larger than that of the outgoing commodity at each destination node. Equation (36) ensures that the incoming stream and the outgoing stream of an intermediate node of the trail have the same units of commodity. Equations (37) and (38) ensure that edge $(m, n)$ is used by the trail if it has commodity traversed through it; otherwise, it is not used.

$$\sum_{n \in \text{adj}(s)} S^s_{s,n} = 1,$$  \hspace{1cm} (39)

$$\sum_{n \in \text{adj}(t)} S^t_{t,n} = 0 \quad \forall t \neq s,$$  \hspace{1cm} (40)

$$\sum_{n \in \text{adj}(t) \cup \{t\}} S^m_{t,n} \leq 1 \quad \forall t, m \in \text{adj}(t) \cup \{t\},$$  \hspace{1cm} (41)

$$\sum_{m \in \text{adj}(t) \cup \{t\}} S^m_{t,n} = M_{tn} \quad \forall t, n \in \text{adj}(t),$$  \hspace{1cm} (42)

$$F_{mt} \geq F_{tn} - (1 - S^m_{t,n})|D| \quad \forall t, m \in \text{adj}(t), n \in \text{adj}(t).$$  \hspace{1cm} (43)

Equation (39) ensures that there is an adjacent node of source $s$ traversed as the first hop. Equation (40) ensures that other nodes apart from the source node have no adjacent node traversed as the first hop. Equation (41) ensures that the number of outgoing streams forwarded from a specific incoming stream is no larger than one. Equation (42) ensures that, if link $(t, n)$ is traversed, there should be one incoming stream to node $t$ that is forwarded to link $(t, n)$. Equation (43) ensures that the number of units of commodity on the incoming stream is no less than that on the outgoing stream, if these two streams are connected at the node.

In this ILP formulation, the number of variables is $O(g|E|)$ and the number of constraints is also $O(g|E|)$, where $g$ is the maximal nodal degree.

### C. Simulation Results

As discussed above, the MDT algorithm may have long trails to route, which is caused by the rerouting of the Steiner tree (obtained by MPH). The rerouting process will traverse all except one branch to reach all destinations in a trail. Therefore, the best rerouting result is to traverse all the branches downward and upward, except the longest branch (from the source to the farthest leaf node), which is traversed only downward. This is equal to twice the cost of the Steiner tree minus the path cost from the source to the farthest leaf in the tree. To evaluate our proposed trail routing algorithm, NATR, we compare the performance of the NATR algorithm, optimal trail routing derived from ILP formulation and the best result of MDT.

We compare the performance of the three algorithms for the NSFNET network with 14 nodes shown in Fig. 7. In each simulation experiment, one multicast connection request is generated as the input to each of the three methods. The number of edges of the trail is counted to compare the performance of the different methods; i.e., the number of edges per request (or per experiment as there is only one connection request in an experiment) indicates the efficiency of the routing algorithms. The source of a multicast connection request is randomly chosen from the network nodes, and the destination nodes are randomly selected from the network nodes (excluding the source node). The results are shown in Fig. 8, where the values at each destination set size are the average value of 100 simulation runs.
is referred to as Lin et al. the number of wavelengths used. It does not affect the number of electronic ports used, but does affect the results and shows significantly better performance than the optimal trail routing result. These simulation results show that our proposed NATR algorithm is closer to the optimal trail routing result. As shown in Fig. 8, for all algorithms, when the number of destinations increases, more edges are used. We can see that the optimal trail routing derived by ILP uses the smallest number of edges. Our proposed algorithm NATR uses fewer edges than the best result of MDT. The difference between NATR and the best result of MDT becomes larger as the number of destinations increases. In this case, the NATR algorithm achieves more gains by connecting destinations one by one in a trail than by rerouting a big tree as MDT does. On average, NATR has a 23% lower value than the best result of MDT, and has only an 8% higher value than the optimal trail routing result. These simulation results show that our proposed NATR algorithm is closer to the optimal results and shows significantly better performance than the MDT algorithm. In Section VII, we shall show that for traffic grooming algorithms the difference in routing strategies does not affect the number of electronic ports used, but does affect the number of wavelengths used.

VI. Heuristic Approaches

A trail in the logical layer can be denoted as an LOHT, which is referred to as \(s;D;a\), where \(s\) the source node of the trail, \(D\) is the set of nodes which use receiving electronic ports to receive the tapped/dropped traffic for local stations and \(a\) is the available bandwidth of the trail. A multicast session may be constructed by multiple trails, and they connect with each other to form a larger multicast session with OEO conversions. Motivated by the observations made from the trail-based ILP optimization approach in Section IV, we propose a heuristic algorithm, called MTG, to efficiently deal with the multicast traffic grooming problem in large TaC networks. The idea of the algorithm is to groom traffic to multiple small trials (with a few destinations) to form a larger multicast session. We observe that this indeed increases the resource utilization, leading to a lower network cost. The pseudocode of the MTG algorithm is given as follows.

Multicast trail grooming (MTG) algorithm

**Input:**
A network \(G(V,E)\) with capacity \(C\) of each wavelength, and a set of multicast requests.

**Output:**
(a) A set of trails in the physical layer, (b) the traffic routing \(T_i\) of each multicast connection request, (c) RWA of trails in the physical layer and (d) number of electronic ports and wavelengths used and network cost.

**Algorithm:**

BEGIN
1. Call **NATR algorithm** to construct a trail for each connection request whose required bandwidth is of a full wavelength capacity. If there are multiple such connection requests then these can be done in any order.
2. Sort the remaining connection requests in ascending order of destination set size and label them as \(t_1, t_2, \ldots, t_n\), and denote the \(i\)th connection request as \(t_i = (s_i;D_i;a_i)\)
3. For \(i = 1\) to \(n\)
   // select existing trails to groom connection request \((s_i;D_i;a_i)\), \(Ex\) contains all existing trails; each is denoted as an LOHT \(s_j;D_j;a_j\), \(a_j\) is the available bandwidth of trail \(j\), \(T_i\) contains all trails used to support request \(i\)
4. While \(D_j \neq \emptyset\) do
   5. Select the existing trail \(j\) in \(Ex\), which can honor the maximum number of nodes in \(D_j\), and with \(D_j \subseteq D_i\) & & \(a_j \geq f_i\). If the trail cannot be found, break
   6. If \(s_j \neq s_i \& \& s_j \notin D_i \& \& s_j \notin T_i\)
5. \(D_i = D_i \cup \{s_j\}\) //add \(s_j\) into \(D_i\)
8. \(a_j = a_j - f_i\)
9. \(D_j = D_j \setminus D_i\) //delete \(D_j\) from \(D_i\)
10. \(T_i = T_i \cup \{\text{trail}\}\)
11. End While
   // build a new trail for destinations that are not accommodated
12. If \(D_i \neq \emptyset\)
13. Call **NATR algorithm** to build a new trail from \(s_i\) to \(D_i\), set the available bandwidth of the trail as \(C - f_i\), save the trail into \(T_i\)
14. End For
END

In the MTG algorithm, we first construct a trail for every connection request with a full wavelength bandwidth demand, as this is the optimal way to support these connection requests. Since the complexity of constructing a trail is \(O(|V|^3 \log |V|)\), the complexity of this step is \(O(R|V|^3 \log |V|)\). In line 2, the
remaining connection requests are sorted in ascending order of destination set size. The algorithm gives preference to smaller connection requests because smaller connection requests are more likely to be shared by other connection requests. The loop from line 4 to line 11 is to select existing trails to groom the connection request. Using the index $j$ for existing trails and index $i$ for connection requests, the destination set $D_j$ of the selected trail $j$ has to be a subset of the connection request destinations $D_i$, and the available bandwidth $a_j$ has to be no less than the required bandwidth $f_i$. In line 6, if the source node of the selected trail is not in the coverage of the multicast session, then this root node will be added as an unreached node to the destination set of the request. Lines 8 and 9 are to update the available bandwidth of the selected trail and the remaining destinations of the connection request. The complexity of the loop is $O(R^2|V|^2)$, as the upper bound of the number of existing trails is $R|V|$ and the number of trails selected is at most $|V|$. In the last step, the NATR algorithm is called if necessary, to build a new trail from the source node to those destinations that are not accommodated. The available bandwidth of the new trail is set to $C - f_i$. The complexity of this step is $O(|V|^3 \log |V|)$. The total complexity of the MTG algorithm is therefore $O(R^2|V|^2 + R|V|^3 \log |V|)$.

To examine how different trail routing algorithms influence the overall grooming performance, we also consider another heuristic algorithm called the MDT-based grooming algorithm (MDTG). This algorithm is very similar to MTG, except that the trail routing algorithm is different. In lines 1 and 13, the MDT algorithm is called instead of NATR. Please refer to [29] for details of the MDT algorithm. The performance comparison of MTG and MDTG is presented in Section VII.

To reduce the total number of wavelengths used, the link cost would be changed during the operation. If an edge has been traversed, the cost of the edge will be increased by 1. With this method, the routings of trails are more evenly distributed in the optical layer and should help in reducing the number of wavelengths used.

**VII. Numerical Results**

In this section, the performance of the four methods, the light-tree-based ILP formulation, the trail-based ILP formulation, MTG and MDTG, are compared in terms of cost, total numbers of higher layer electronic ports, transmitting electronic ports, receiving electronic ports, wavelengths and wavelinks in the six-node network of Fig. 5. We then study the performance of the two heuristic algorithms in a larger network, NSFNET of Fig. 7.

**A. Results of the Six-Node Network**

Ten multicast connection requests are randomly generated in each experiment. The source, the multicast destination set size and the destination nodes are randomly generated as before in Section IV. The required bandwidth of a connection request is still a random integer from 1 to $C$, where $C$ equals OC-12. The results are shown in Figs. 9–14, where the value at each instance is the average over 20 simulation experiments each involving ten requests. Again, following [7], we assume that the values of $\alpha$ and $\beta$ are 3 and 1, respectively.

In Fig. 9, the network costs of the four methods are compared. Given the numbers of higher layer electronic ports and wavelengths, network cost can be calculated by Eq. (1). The light-tree-based ILP formulation has the best performance, followed by the trail-based ILP formulation, MTG and MDTG. The cost of the trail-based ILP formulation is 0.8% higher on average than that of the light-tree-based one; this is due to the longer routes of trails, leading to more wavelength usage. Specifically, the two ILP formulations result in the same number of higher layer electronic ports (shown in Fig. 10), but the trail-based ILP formulation uses more wavelinks (refer to Fig. 14), which leads to more wavelengths (refer to Fig. 13). The trail-based ILP formulation performs very closely to the light-tree-based formulation. However, the former approach will generally be much less costly, as in that case the network nodes do not need multicast capability. MDTG costs about 1% more than MTG since MDT used for trail routing in MDTG.
needs more wavelinks and wavelengths than NTAR used in MTG (refer to Figs. 14 and 13, respectively). The cost value of MTG is quite close to the optimal value derived by the trail-based ILP formulation. It is about 8% higher than the ILP result averaged over 20 instances.

The number of higher layer electronic ports is the summation of the numbers of transmitting electronic ports and receiving electronic ports. In Fig. 10, the number of higher layer electronic ports of the trail-based ILP formulation is exactly the same as that of the light-tree-based ILP formulation. This is because the solution obtained by the light-tree-based ILP formulation can also be achieved by the trail-based ILP formulation in a different form with some additional cost of wavelength usage. MTG and MDTG have the same transmitting and receiving electronic port usages (refer to Figs. 11 and 12 separately) because they use the same grooming procedure and are only different in the use of the trail routing algorithm. The number of electronic ports of MTG (or MDTG) is about 8% higher than that of either ILP formulation.

In Fig. 11, the numbers of transmitting electronic ports are compared. The ILP formulation based on the light-tree has the largest value, followed by the trail-based ILP formulation and MTG and MDTG, where MTG and MDTG have the same values. It is interesting to see that the trail-based ILP formulation uses fewer ports for transmitting than the light-tree-based ILP formulation, while both the formulations use the same number of higher layer electronic ports. The reason for this may be that, among the solutions that use the same number of electronic ports, the trail-based ILP formulation tries to select the solution that minimizes the usage of wavelengths. Therefore, longer trails may be preferred as they can travel to more destinations; otherwise, more short trails have to be built, which may increase the possibility that multiple trails traverse the same links, and this will increase the usage of wavelengths. Therefore, fewer trails would be built, which use fewer transmitting electronic ports. It is noted that, even though the light-tree-based ILP formulation uses more transmitting electronic ports, the wavelength usage is the lowest, as shown in Fig. 13. This is because, with multicast capable nodes, light-trees can be more efficiently routed to
reduce wavelength usage than when trails are used. In Fig. 11, we also see that MTG (or MDTG) has the lowest value. The reason may be that, in MTG (or MDTG), sub-wavelength connection requests are accommodated in ascending order of destination set size. This would increase both the sharing of trails and the sharing of transmitting electronic ports, reducing their usages.

In Fig. 12, the numbers of receiving electronic ports are compared. MTG and MDTG have the same value, which is the largest, followed by the trail-based ILP formulation and the light-tree-based ILP formulation. Note that the light-tree-based ILP formulation and the trail-based ILP formulation use the same number of higher layer electronic ports to accommodate connection requests. Therefore, the summations of the number of transmitting electronic ports in Fig. 11 and the number of receiving electronic ports in Fig. 12 for two different formulations should have the same value. From Figs. 11 and 12, it is clear that the values of the trail-based ILP formulation and MTG (or MDTG) differ greatly in the numbers of transmitting and receiving ports. MTG (or MDTG) uses slightly fewer transmitting electronic ports than the trail-based ILP formulation in Fig. 11, but uses many more receiving electronic ports in Fig. 12, resulting in higher overall electronic port usage. The reason may be that the sharing of trails can improve the utilization of transmitting ports, but MTG (or MDTG) uses more receiving ports, as a trail has only one source node but multiple destination nodes, and the trail-based ILP method can balance the utilization of transmitting and receiving electronic ports more efficiently to obtain the optimal result, but with a very high cost of computation complexity.

In Fig. 13, the wavelength usages are compared. The value of the light-tree-based ILP formulation is lowest, followed by those of the trail-based ILP formulation, MTG and MDTG. The light-tree-based ILP formulation achieves the best performance at the cost of deploying multicast capable devices at network nodes. The average difference of two ILP formulations is less than one wavelength. It is noted that the cost of wavelengths contributes much less (2%–4%) than the cost of higher layer electronic ports (96%–98%) which has the same value in the two ILP formulations. In the figure, MDTG has the highest value, as its trail routing algorithm (MDT) use the same number of higher layer electronic ports, but MTG (or MDTG) uses more receiving ports, as a trail has only one source node but multiple destination nodes, and the trail-based ILP method can balance the utilization of transmitting and receiving electronic ports more efficiently to obtain the optimal result, but with a very high cost of computation complexity.

In this section, we consider a larger network, i.e., NSFNET with 14 nodes (Fig. 7). Multicast connection requests are randomly generated for three different scenarios, the multicast destination set size ranging from 1 to 13, 1 to 9 and 7 to 13. Therefore, the average destination set sizes of three scenarios are 7, 5 and 10, respectively. We assume that the bandwidth required by a multicast connection request is still randomly chosen between 1 and C, where C is equal to OC-48. 100 multicast connection requests are generated in each experiment and 25 experiments are simulated to obtain the average value as shown in Tables V and VI for MTG and MDTG. Following [7], we assume that the values of \(a\) and \(b\) are 6 and 5, respectively.

As shown in Tables V and VI, MTG and MDTG have very similar network costs with slight differences in wavelength usage under different ranges. The two methods still have the same transmitting and receiving electronic port usages (denoted as \(A\) and \(B\), respectively) under different traffic scenarios due to the same grooming procedure of the two methods. Due to the difference of the routing algorithms used, the two methods use different numbers of wavelengths (denoted by \(\phi\)) and wavelinks. It turns out that MDTG uses more wavelengths and wavelinks than MTG, resulting in a slight higher cost. In Table V, the three scenarios have similar numbers of transmitting electronic ports, but significantly different usage of receiving electronic ports, wavelengths \(\phi\) and wavelinks. This is because the three scenarios have the same number of connection requests and the same distribution of required bandwidth, but different average size of destination sets (7, 5 and 10). That is, the effective loads of the network are different for the three scenarios, which are in the proportion of 7:5:10. We can verify this proportion from the usages of the receiving electronic ports for the three scenarios: more destination nodes result in more usage of the receiving electronic ports. The numbers of transmitting electronic ports for the three scenarios are about the same. This is because they have the same number of connection requests and the sharing of trails contributes more to the receiving electronic port utilization than to the transmitting electronic port utilization. The total numbers of higher layer electronic ports are different mainly due to the different usage of receiving electronic ports.

<table>
<thead>
<tr>
<th>TABLE V</th>
<th>RESULTS OF MTG</th>
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<tbody>
<tr>
<td></td>
<td>Cost</td>
</tr>
<tr>
<td>1–13</td>
<td>3479.84</td>
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<tr>
<td>1–9</td>
<td>2918.04</td>
</tr>
<tr>
<td>7–13</td>
<td>4921.96</td>
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</table>

<table>
<thead>
<tr>
<th>TABLE VI</th>
<th>RESULTS OF MDTG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost</td>
</tr>
<tr>
<td>1–13</td>
<td>3508.24</td>
</tr>
<tr>
<td>1–9</td>
<td>2942.04</td>
</tr>
<tr>
<td>7–13</td>
<td>4963.16</td>
</tr>
</tbody>
</table>

B. Results of the NSFNET Network
The overall trend is that larger multicast size leads to higher usage of electronic ports. The numbers of wavelengths and wavelinks are highly related to the effective load of the network; i.e., a larger destination set size may result in a heavier network load, where more wavelinks and wavelengths are needed to build trails to accommodate the traffic. These observations can also be found in Table VI for MDTG.

We have also verified the performance of MTG and MDTG for different destination set sizes under an even larger problem size, where each experiment involves 1000 multicast requests, and 100 experiments are simulated to obtain the average value as shown in Figs. 15–17. In Fig. 15, MTG and MDTG have almost the same cost. This is because both schemes have the same electronic port usage (refer to Fig. 16), which contributes the most to the network cost, while the difference in wavelength usages (refer to Fig. 17) only causes a slight cost difference.

Figure 16 shows that the two methods use the same number of transmitting and receiving electronic ports. This occurs because the two methods use the same grooming procedure. When the destination set size increases, the transmitting port usage of the two methods remains almost the same, but the receiving port usage increases, resulting in an increase in the total number of higher layer electronic ports used. These results are consistent with those in Tables V and VI. Notice that the number of electronic ports increases in a nonlinear way with the destination set size. From destination set size 2 to 6, it is almost linear. However, when the destination set size is larger than 6, the number of required electronic ports increases relative to the destination set size at a rate slower than linear. This is because, with larger destination set sizes, the algorithms can benefit more from grooming gains.

In Fig. 17, wavelinks and wavelengths are compared. Because of the lower efficiency of MDT used in MDTG, the numbers of wavelinks and wavelengths required by MDTG are higher than those required by MTG. The wavelength and wavelink usages of the two methods increase when the destination set size increases; this is because a larger destination set size would lead to a larger multicast session consuming more resources. It is noted that, when the destination set size increases, the difference in the number of required wavelengths and wavelinks between the two methods also increases. This is because, when the multicast destination set size increases, a larger light-tree would be derived by the MDT of MDTG, so even more wavelinks are required when rerouting the light-tree to be a trail, while the NATR of MTG achieves a larger wavelength gain. Since higher layer electronic ports are dominant in the network cost, and MTG and MDTG have the same port consumption, the two methods perform very closely to each other, though MTG is slightly better than MDTG in terms of lower wavelength requirement.

VIII. CONCLUSIONS

In this paper, we have considered the multicast traffic grooming problem in TaC WDM networks. We have proposed a new low-complexity node architecture with the TaC mechanism, which can be readily upgraded from the existing unicast nodes. We have also proposed a trail-based ILP formulation for multicast traffic grooming in TaC networks, with an objective of minimizing the cost associated with the number of
higher layer electronic ports and the number of wavelengths used. The results reveal that small trails can connect each other to form a larger multicast session, which improves the utilization of the resources and reduces the cost of the network. The trail routing is also investigated. We have proposed a new algorithm with polynomial complexity, NATR, to route a trail with shorter length than others; this gives near-optimal performance. We have also developed two heuristic multicast traffic grooming algorithms based on trails—MTG and MDTG, which aim to construct large multicast sessions with multiple small trails, and increase the sharing of the trails. We have compared the performance of the proposed trail-based ILP formulation, the light-tree-based ILP formulation, MTG and MDTG in a small test network. Results have demonstrated that (a) the trail-based ILP formulation performs very closely to the light-tree-based ILP formulation with slightly higher wavelength costs, (b) the trail-based network design saves on the overall network cost, as it does not need to deploy multicast capable devices, and (c) the two heuristic algorithms MTG and MDTG perform almost as well as the optimal results obtained from the trail-based ILP, with MTG performing better than MDTG. To demonstrate the scalability of our heuristic algorithms, we have studied their performance in the NSFNET network. Our results have shown that the MTG and MDTG can scale to a network of practical size with a very large number of connection requests.

**References**


