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Adaptive Regional Feedback Control of Robotic Manipulator with Uncertain Kinematics and Depth Information

X. Li and C. C. Cheah

Abstract—While much progress has been achieved in task-space control of robot, existing task-space sensory feedback control methods fail when the sensor is out of working range. In this paper, we propose an adaptive regional feedback control strategy that enables the robot to start from an initial position outside the field of view and leave the field of view during the movement. The robot kinematics is partitioned into a known internal portion and an unknown external portion. Cartesian-space feedback is used for region reaching control of the known portion and vision feedback is used for tracking control of the unknown portion. The dual feedback information is integrated into a unified controller without designing multiple controllers and switching between them. We shall show that the adaptive controller can transit smoothly from Cartesian-space feedback to vision feedback in the presence of uncertainties in robot dynamics, kinematics and depth information. Experimental results are presented to illustrate the performance of the proposed control method.

I. INTRODUCTION

The basic idea of task-space control is to formulate a control scheme using Cartesian-space error or visual error directly to eliminate the requirement of solving the inverse kinematics. While much progress has been achieved in task-space control of robot [1]–[6], most task-space control schemes require the exact knowledge of the Jacobian matrix from joint space to task space. Unfortunately, it is difficult to derive the Jacobian matrix accurately especially when the manipulator grasps several unknown tools. In vision based control, it is also difficult to obtain the image Jacobian matrix [7]–[9] in the presence of uncertain camera parameters.

To overcome the problem of uncertain kinematics, Cheah *et al.* [10], [11] proposed approximate Jacobian control methods for setpoint control of robots. Dixon [12] developed an amplitude-limited regulator for robotic manipulators with uncertain kinematics and dynamics. Ozawa *et al.* [13] proposed an adaptive setpoint controller which employed sensory information only for kinematics adaptation. To deal with tracking control problem with uncertain kinematics, an adaptive Jacobian controller was proposed in [14]. The results were also extended to include actuator parameters and redundant robots in [15]. A neural-network technique was employed in [16] to estimate the approximate Jacobian matrix. In [17], an adaptive controller was proposed for free-floating space manipulator with uncertainties in both kinematics and dynamics. To avoid singularities associated with the Euler angles representation, an adaptive tracking controller based on the unit quaternion representation was

developed for robotic manipulators with uncertain kinematic and dynamic models [18]. To cope with uncertain depth problem, Liu *et al.* [9] and Wang *et al.* [19], [20] introduced vision based controllers with a depth-independent interaction matrix. Cheah *et al.* [21] presented an adaptive Jacobian setpoint controller with concurrent adaptation to uncertain depth information and kinematics.

However, the aforementioned controllers have assumed that the sensory feedback information is available throughout the entire task. While sensory information is important to improve the endpoint accuracy in the presence of uncertainty [11], [14], it is difficult to ensure that the feedback can be used for the entire task due to limited sensing zone. Therefore, existing task-space sensory feedback control methods fail when the sensor is out of working range. In [22], a dual task-space control method that only required vision feedback in the ending stage was proposed, which was able to drive the robot from outside to inside the field of view, but exact kinematic parameters were assumed to be known.

In general, the kinematics of robotic manipulator is fixed and can be obtained with sufficient accuracy, but it is difficult to obtain the exact lengths and grasping angles of the tools, and it is also difficult to obtain the exact camera parameters such as the depth information. To solve the problem of limited field of view with uncertainties, we propose an adaptive task-space controller with regional feedback in this paper. The main idea is to partition the robot kinematics into a known internal portion (manipulator kinematic parameters) and an unknown external portion (tool kinematic parameters). Cartesian-space feedback is used for region reaching control of the known portion and vision feedback is used for tracking control of the unknown portion. Unlike conventional task-space control methods which employ single task-space information for the entire task, the proposed method integrates dual feedback information into a unified controller. It can transit smoothly from Cartesian-space feedback to vision feedback subject to the uncertainties of robot kinematics, dynamics and camera's depth information. The proposed controller was implemented on an industrial robot and experimental results were presented to illustrate the performance of the controller.

II. ROBOT KINEMATICS AND DYNAMICS

Consider a robot system with camera(s) fixed in the work space. Let $r \in \mathfrak{R}^p$ denotes a position of the end effector in Cartesian space as [3] [23]:

$$r = h(q), \quad (1)$$

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where $\mathbf{h}(\cdot) \in \mathbb{R}^n \rightarrow \mathbb{R}^p$ is generally a non-linear transformation describing the relation between joint space and Cartesian space, $\mathbf{q} = [q_1, \dots, q_n]^T \in \mathbb{R}^n$ is a vector of joint angles of the manipulator. The velocity of the end effector $\dot{\mathbf{r}}$ is related to joint-space velocity $\dot{\mathbf{q}}$ as:

$$\dot{\mathbf{r}} = \mathbf{J}_r(\mathbf{q})\dot{\mathbf{q}}, \quad (2)$$

where $\mathbf{J}_r(\mathbf{q}) \in \mathbb{R}^{p \times n}$ is the Jacobian matrix from joint space to Cartesian space.

When the end effector grasps a tool, the position of tool is dependent on the position of end effector. Let $\mathbf{r}_t \in \mathbb{R}^p$ denotes the position of tool in Cartesian space and then the velocity of the tool $\dot{\mathbf{r}}_t$ is related to the velocity of the end effector $\dot{\mathbf{r}}$ as:

$$\dot{\mathbf{r}}_t = \dot{\mathbf{r}} + \mathbf{J}_t(\mathbf{q})\dot{\mathbf{q}} = \mathbf{J}_r(\mathbf{q})\dot{\mathbf{q}} + \mathbf{J}_t(\mathbf{q})\dot{\mathbf{q}}, \quad (3)$$

where $\mathbf{J}_t(\mathbf{q}) \in \mathbb{R}^{p \times n}$ denotes the tool Jacobian matrix.

The pinhole camera model is chosen to represent the mapping from Cartesian space to image space. Let $\mathbf{x}_i = [x_{i_h}, x_{i_v}]^T \in \mathbb{R}^2$ denotes a feature point, while x_{i_h} represents the horizontal coordinate and x_{i_v} the vertical coordinate. Then $\mathbf{x} = [x_1, \dots, x_m]^T \in \mathbb{R}^{2m}$ denotes a vector of image features, where m is the number of image features. The vector of rates of change of the image features is related to the velocities in Cartesian space as [9]:

$$\dot{\mathbf{x}} = \mathbf{Z}^{-1}(\mathbf{q})\mathbf{L}(\mathbf{x})\dot{\mathbf{r}}_t, \quad (4)$$

where $\mathbf{Z}(\mathbf{q}) \in \mathbb{R}^{2m \times 2m}$ is a diagonal matrix that contains the depth information of the feature points with respect to the camera image frame and $\mathbf{L}(\mathbf{x}) \in \mathbb{R}^{2m \times p}$ is a Jacobian matrix. The overall matrix $\mathbf{Z}^{-1}(\mathbf{q})\mathbf{L}(\mathbf{x})$ is called the image Jacobian matrix.

From equation (3) and (4), one has

$$\dot{\mathbf{x}} = \mathbf{Z}^{-1}(\mathbf{q})\mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \quad (5)$$

where $\mathbf{J}(\mathbf{q}) = \mathbf{L}(\mathbf{x})[\mathbf{J}_r(\mathbf{q}) + \mathbf{J}_t(\mathbf{q})] \in \mathbb{R}^{2m \times n}$. If the end effector is chosen as the image features, $\dot{\mathbf{r}}_t$ is replaced by $\dot{\mathbf{r}}$ in equation (4) and $\mathbf{J}(\mathbf{q}) = \mathbf{L}(\mathbf{x})\mathbf{J}_r(\mathbf{q})$ in equation (5).

Both $\mathbf{Z}(\mathbf{q})\dot{\mathbf{x}}$ and $\mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$ are linear in sets of kinematic parameters $\boldsymbol{\theta}_z = [\theta_{z_1}, \dots, \theta_{z_o}]^T \in \mathbb{R}^o$ and $\boldsymbol{\theta}_k = [\theta_{k_1}, \dots, \theta_{k_t}]^T \in \mathbb{R}^t$, such as camera intrinsic and extrinsic parameters. Hence, $\mathbf{Z}(\mathbf{q})\dot{\mathbf{x}}$ and $\mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$ can be expressed as:

$$\mathbf{Z}(\mathbf{q})\dot{\mathbf{x}} = \mathbf{Y}_z(\mathbf{q}, \dot{\mathbf{x}})\boldsymbol{\theta}_z, \quad (6)$$

$$\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{Y}_k(\mathbf{q}, \dot{\mathbf{q}})\boldsymbol{\theta}_k, \quad (7)$$

where $\mathbf{Y}_z(\mathbf{q}, \dot{\mathbf{x}}) \in \mathbb{R}^{2m \times o}$ is called the depth regressor matrix and $\mathbf{Y}_k(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{2m \times t}$ is called the kinematics regressor matrix.

The dynamics of the robotic manipulator is given as [3] [23]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \left(\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})\right)\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}, \quad (8)$$

where $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n \times n}$ is an inertia matrix which is symmetric, positive definite for all $\mathbf{q} \in \mathbb{R}^n$. The vector $\mathbf{g}(\mathbf{q}) \in \mathbb{R}^n$ denotes a gravitational force, $\boldsymbol{\tau} \in \mathbb{R}^n$ denotes the control input, and $\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n \times n}$ is skew-symmetric. In addition,

the dynamic model as described by equation (8) is linear in a set of physical parameters $\boldsymbol{\theta}_d = [\theta_{d_1}, \dots, \theta_{d_l}]^T \in \mathbb{R}^l$ as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \left(\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})\right)\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\theta}_d, \quad (9)$$

where $\mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \in \mathbb{R}^{n \times l}$ is called the dynamic regressor matrix.

III. REGION FUNCTIONS AND POTENTIAL ENERGY

The entire robot workspace is split into two regions: a Cartesian-space region for region reaching control and an image-space region for adaptive tracking control. With the regional feedback, the robot could smoothly transit from Cartesian space to image space and employ vision information only after it enters the image-space region.

A. Image-space Region

The image-space region is specified as a set of external regions and task-oriented regions.

1) *External Image-Space Regions*: The external image-space regions are introduced to match the field of view as:

$$f_{E_i}(\mathbf{x}_i) = \frac{(x_{i_h} - x_{E_i h})^{n_{E_i}}}{a_{i_h}^{n_{E_i}}} + \frac{(x_{i_v} - x_{E_i v})^{n_{E_i}}}{a_{i_v}^{n_{E_i}}} - 1 \leq 0, \quad (10)$$

where $\mathbf{x}_{E_i} = [x_{E_i h}, x_{E_i v}]^T \in \mathbb{R}^2$ represent reference positions inside the regions $f_{E_i}(\mathbf{x}_i) \leq 0$, and a_{i_h}, a_{i_v} are positive constants and n_{E_i} are the orders of the region functions which are also even integers. Note that the regions $f_{E_i}(\mathbf{x}_i) \leq 0$ are fixed, and the robot employs visual feedback after it enters the regions where $f_{E_i}(\mathbf{x}_i) \leq 0$.

To construct the potential energy for the external region functions $f_{E_i}(\mathbf{x}_i)$, we introduce a set of reference regions inside the external regions as:

$$f_{E_{r_i}}(\mathbf{x}_i) = \frac{(x_{i_h} - x_{E_i h})^{n_{E_i}}}{(\kappa_{E_i} a_{i_h})^{n_{E_i}}} + \frac{(x_{i_v} - x_{E_i v})^{n_{E_i}}}{(\kappa_{E_i} a_{i_v})^{n_{E_i}}} - 1 \leq 0, \quad (11)$$

where κ_{E_i} are positive constants and $\kappa_{E_i} < 1$. By using $f_{E_i}(\mathbf{x}_i)$ and $f_{E_{r_i}}(\mathbf{x}_i)$, the potential energy $P_{E_i}(\mathbf{x}_i)$ are introduced as:

$$P_{E_i}(\mathbf{x}_i) = \frac{k_{x_{E_i}}}{N^2} \{ \min[0, [\min(0, f_{E_i}(\mathbf{x}_i))]^N - (\kappa_{E_i}^{n_{E_i}} - 1)^N] \}^N, \quad (12)$$

where $k_{x_{E_i}}$ are positive constants, and $N \geq 4$ is a constant integer so that $P_{E_i}(\mathbf{x}_i) \in C^2$. From equation (12), it can be seen that $P_{E_i}(\mathbf{x}_i)$ are smooth and lower bounded by zero.

2) *Task-oriented Image-Space Regions*: Note that the gradient of the potential energy $P_{E_i}(\mathbf{x}_i)$ described by equation (12) reduces to zero where $f_{E_{r_i}}(\mathbf{x}_i) \leq 0$. Next, a sets of task-oriented regions are introduced in image space to ensure that the robot moves towards the desired position after it enters the external reference regions where $f_{E_{r_i}}(\mathbf{x}_i) \leq 0$. Let $\mathbf{x}_d = [x_{d1}, \dots, x_{dm}]^T \in \mathbb{R}^{2m}$ be the desired trajectory, where $\mathbf{x}_{di} = [x_{di_h}, x_{di_v}] \in \mathbb{R}^2$ is the desired trajectory for the i^{th} feature point. The task-oriented regions are defined to enclose the desired trajectory as:

$$f_{T_i}(\mathbf{x}_i) = \frac{(x_{i_h} - x_{di_h})^2}{(x_{bi_h} - x_{di_h})^2} + \frac{(x_{i_v} - x_{di_v})^2}{(x_{bi_v} - x_{di_v})^2} - 1 \leq 0, \quad (13)$$

where $\mathbf{x}_{bi} = [x_{bi_h}, x_{bi_v}]^T \in \mathbb{R}^2$ denote the boundary of the region. Since the desired position $\mathbf{x}_{di}(t)$ is time-varying

and thus not necessary the geometric center, the boundary positions are divided into four parts.

By using the task-oriented regions in equation (13), the corresponding potential energy functions $P_{T_i}(\mathbf{x}_i)$ are specified as follows:

$$P_{T_i}(\mathbf{x}_i) = \frac{k_{T_i}}{N} \{1 - [\min(0, f_{T_i}(\mathbf{x}_i))]^N\}, \quad (14)$$

where k_{T_i} are positive constants.

3) *Overall Image-Space Potential Energy*: Note that the top contour of potential energy $P_{E_i}(\mathbf{x}_i)$ is fixed which can match the field of view, but its bottom is flat. Whereas the potential energy $P_{T_i}(\mathbf{x}_i)$ drives the robot towards the desired position but its top contour is not fixed and thus cannot match the field of view. Therefore, the overall image-space potential energy $P_O(\mathbf{x})$ is defined as the summation of $P_{T_i}(\mathbf{x}_i)$ and $P_{E_i}(\mathbf{x}_i)$ to combine the advantages together, and the overall potential energy function is specified as:

$$P_O(\mathbf{x}) = k_p \alpha_x \sum_{i=1}^m [P_{T_i}(\mathbf{x}_i) + P_{E_i}(\mathbf{x}_i)]. \quad (15)$$

where α_x and k_p are positive constants. As \mathbf{x}_{di} varies, the bottom of potential energy $P_O(\mathbf{x})$ changes while the top contour remains the same. Since the top contour of $P_O(\mathbf{x})$ corresponds to the external region functions $f_{E_i}(\mathbf{x}_i)$ while its bottom is the desired position, the potential energy $P_O(\mathbf{x})$ can match the field of view and ensure the convergence of robot movement.

Partial differentiating $P_O(\mathbf{x})$ with respect to \mathbf{x} yields:

$$\left(\frac{\partial P_O(\mathbf{x})}{\partial \mathbf{x}}\right)^T = k_p \alpha_x \sum_{i=1}^m \left[\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T + \left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T \right] \triangleq k_p \alpha_x \Delta \varepsilon_x, \quad (16)$$

where $\Delta \varepsilon_x$ is the image-space region error which is continuous. It drives the robot towards the desired trajectory and it is activated automatically after the robot enters the image-space region.

In equation (16), the partial derivative $\left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T$ is given as:

$$\begin{aligned} \left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T &= k_{x_{E_i}} \{ \min\{0, [\min(0, f_{E_i}(\mathbf{x}_i))]^N - \\ & (\kappa_{E_i}^{n_{E_i}} - 1)^N\} \}^{N-1} [\min(0, f_{E_i}(\mathbf{x}_i))]^{N-1} \left(\frac{\partial f_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T \end{aligned} \quad (17)$$

and the partial derivative $\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T$ is given as:

$$\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T = -k_{T_i} [\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1} \left(\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T. \quad (18)$$

B. Cartesian-space Region

Since the image-space region error is zero when the robot is outside the image-space region, an additional control term defined in Cartesian space is introduced to enable the robot to move to the image-space region and hence eliminate the need of vision when robot is not within the image-space region.

The Cartesian-space region is defined by a set of inequality functions as:

$$\begin{aligned} f_{C_1}(r_1) &= \frac{(r_1 - r_{c_1})^2}{(r_{b_1} - r_{c_1})^2} - 1 \geq 0, \\ &\dots \\ f_{C_p}(r_p) &= \frac{(r_p - r_{c_p})^2}{(r_{b_p} - r_{c_p})^2} - 1 \geq 0, \end{aligned} \quad (19)$$

where $\mathbf{r} = [r_1, \dots, r_p]^T \in \mathbb{R}^p$ denotes the position of robot in Cartesian space. Since the objective is to bring the robot into the image-space region, only the position of the robot is sufficient. The vector $\mathbf{r}_b = [r_{b_1}, \dots, r_{b_p}]^T \in \mathbb{R}^p$ represents boundary positions in individual coordinate, and the vector $\mathbf{r}_c = [r_{c_1}, \dots, r_{c_p}]^T \in \mathbb{R}^p$ denotes reference positions. The total potential energy in Cartesian space is defined as:

$$P_C(\mathbf{r}) = \sum_{i=1}^p P_i(r_i) = \sum_{i=1}^p \frac{k_p \alpha_r}{N} [\max(0, f_{C_i}(r_i))]^N, \quad (20)$$

where α_r is a positive constant.

Partial differentiating the above potential energy function (20) with respect to \mathbf{r} , we have:

$$\begin{aligned} \frac{\partial P_C(\mathbf{r})}{\partial \mathbf{r}} &= k_p \alpha_r \sum_{i=1}^p [\max(0, f_{C_i}(r_i))]^{N-1} \left(\frac{\partial f_{C_i}(r_i)}{\partial \mathbf{r}}\right)^T \\ &\triangleq k_p \alpha_r \Delta \varepsilon_r, \end{aligned} \quad (21)$$

where $\Delta \varepsilon_r$ is the Cartesian-space region error which is continuous since the potential energy $P_C(\mathbf{r})$ is smooth. Note that $\Delta \varepsilon_r$ naturally reduces to zero once the robot leaves the Cartesian-space region where $f_{C_i}(r_i) < 0$, ($i = 1, \dots, p$).

Remark. Both the external image-space region and the Cartesian-space region are static because the reference positions \mathbf{x}_{E_i} and \mathbf{r}_c are constants. The Cartesian-space region is slightly overlapped with the external image-space region so that the robot does not get stuck when it transits from outside to inside field of view. This can be easily ensured by placing a set of landmarks that represent the boundaries of the Cartesian-space region to be within the fields of view of the cameras. The locations of the landmarks are then obtained in Cartesian space and therefore camera calibration is not required. Since the field of view is fixed, it is a one-time setup that does not vary with the desired motion.

IV. ADAPTIVE TASK-SPACE CONTROL WITH REGIONAL FEEDBACK

The main idea is to partition the robot kinematics into a known internal portion (manipulator kinematic parameters) and an unknown external portion (tool kinematic parameters), while the known portion is specified in Cartesian space and the unknown portion is specified in image space. The partition method is also applicable for other manipulator systems whose kinematics are partially known. In that case, we consider the tool and uncertain part of manipulator kinematics together as the unknown external portion, while the other part of manipulator kinematics as the known internal portion.

In the presence of unknown tool and uncalibrated camera, the estimated image-space velocity of the tool $\dot{\hat{\mathbf{x}}}$ is obtained as:

$$\dot{\hat{\mathbf{x}}} = \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \dot{\mathbf{q}}, \quad (22)$$

where $\hat{\boldsymbol{\theta}}_z$ are the approximate parameters of $\boldsymbol{\theta}_z$, and $\hat{\boldsymbol{\theta}}_k$ are the approximate parameters of $\boldsymbol{\theta}_k$, and $\hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z)$ is the approximate depth matrix, and $\hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)$ represents the approximate matrix of $\mathbf{J}(\mathbf{q})$.

Since the desired position is only specified in the region function $f_{T_i}(\mathbf{x}_i)$, the desired velocity is specified as:

$$\dot{\mathbf{x}}_{di} = \mathbf{0}, \quad \text{if } f_{Er_i}(\mathbf{x}_i) \geq 0. \quad (23)$$

That is, the desired velocity is zero when the robot is outside the external reference image-space regions. To satisfy equation (23), the desired trajectory $\mathbf{x}_{di}(t)$ can be specified as $\mathbf{x}_{di}(t) = \mathbf{x}_{ci} + w_i \mathbf{x}_{vi}(t)$, where $\mathbf{x}_{ci} = [x_{ci_h}, x_{ci_v}]^T$ are the constant parts, and $\mathbf{x}_{vi}(t) = [x_{vi_h}, x_{vi_v}]^T$ are the time-varying parts, and w_i are weight factors [24]. If the robot is outside the external reference regions, $w_i = 0$ and thus $\mathbf{x}_{di}(t) = \mathbf{x}_{ci}$ and $\dot{\mathbf{x}}_{di} = \mathbf{0}$, then equation (23) is satisfied. After it enters the external reference regions, it is also inside the task-oriented regions such that $f_{T_i}(\mathbf{x}_i) \leq 0$, w_i smoothly increases to 1, and hence $\mathbf{x}_{di}(t) = \mathbf{x}_{ci} + \mathbf{x}_{vi}(t)$ which is the actual trajectory.

Next, a reference vector is introduced as:

$$\dot{\mathbf{x}}_{ai} = [\dot{x}_{di_h} \frac{x_{bi_h} - x_{i_h}}{x_{bi_h} - x_{di_h}}, \dot{x}_{di_v} \frac{x_{bi_v} - x_{i_v}}{x_{bi_v} - x_{di_v}}]^T, \quad (24)$$

and $\dot{\mathbf{x}}_a = [\dot{x}_{a1}, \dots, \dot{x}_{am}]^T \in \mathbb{R}^{2m}$.

By using the region errors, the sliding vector is proposed as:

$$\hat{\mathbf{s}}_q = \dot{\mathbf{q}} - \hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \dot{\mathbf{x}}_a + \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r, \quad (25)$$

where $\hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)$ is the pseudo-inverse matrix of $\hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)$. Let $\mathbf{A} \in \mathbb{R}^{2m \times 2m}$ denotes a matrix as:

$$\mathbf{A} = \begin{bmatrix} \frac{\partial w_1}{\partial \hat{x}_{i_h}} (x_{v1_h} \frac{x_{b1_h} - x_{i_h}}{x_{b1_h} - x_{d1_h}}) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{\partial w_m}{\partial \hat{x}_{m_v}} (x_{vm_v} \frac{x_{bm_v} - x_{m_v}}{x_{bm_v} - x_{dm_v}}) \end{bmatrix},$$

where $\hat{\mathbf{x}}_i = [\hat{x}_{i_h}, \hat{x}_{i_v}]^T$ is the estimated position of the feature point. Then the sliding vector $\hat{\mathbf{s}}_q$ can be rewritten as:

$$\begin{aligned} \hat{\mathbf{s}}_q &= [\mathbf{I} - \hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \mathbf{A} \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)] \dot{\mathbf{q}} \\ &\quad - \hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \dot{\mathbf{x}}_f + \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x \\ &\quad + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r, \end{aligned} \quad (26)$$

where $\mathbf{I} \in \mathbb{R}^{n \times n}$ is an identity matrix, and $\dot{\mathbf{x}}_f = [w_1 \dot{x}_{v1_h} \frac{x_{b1_h} - x_{1_h}}{x_{b1_h} - x_{d1_h}}, \dots, w_m \dot{x}_{vm_v} \frac{x_{bm_v} - x_{m_v}}{x_{bm_v} - x_{dm_v}}]^T$.

Next, a new sliding vector is defined as:

$$\begin{aligned} \hat{\mathbf{s}} &= \dot{\mathbf{q}} - \dot{\hat{\mathbf{q}}}_r \\ &= \dot{\mathbf{q}} - [\mathbf{I} - \hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \mathbf{A} \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)]^{-1} \\ &\quad \times [\hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \dot{\mathbf{x}}_f - \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x \\ &\quad - \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r], \end{aligned} \quad (27)$$

and $\hat{\mathbf{s}}_q = [\mathbf{I} - \hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \mathbf{A} \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)] \hat{\mathbf{s}}$.

The task-space adaptive controller with the regional feedback is proposed as:

$$\begin{aligned} \boldsymbol{\tau} &= -[\mathbf{I} - \hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \mathbf{A} \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)]^T \\ &\quad \times [k_p \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x + k_p \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r] \\ &\quad - \mathbf{K}_s \hat{\mathbf{s}} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r) \hat{\boldsymbol{\theta}}_d, \end{aligned} \quad (28)$$

where \mathbf{K}_s is a positive definite matrix, and the estimated parameters $\hat{\boldsymbol{\theta}}_d$, $\hat{\boldsymbol{\theta}}_k$ and $\hat{\boldsymbol{\theta}}_z$ are updated using the following update laws:

$$\dot{\hat{\boldsymbol{\theta}}}_d = -\mathbf{L}_d \mathbf{Y}_d^T(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r) \hat{\mathbf{s}}, \quad (29)$$

$$\dot{\hat{\boldsymbol{\theta}}}_k = k_p \alpha_x \mathbf{L}_k \mathbf{Y}_k^T(\mathbf{q}, \dot{\mathbf{q}}) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x, \quad (30)$$

$$\dot{\hat{\boldsymbol{\theta}}}_z = -k_p \alpha_x \mathbf{L}_z \mathbf{Y}_z^T(\mathbf{q}, \dot{\mathbf{x}}) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x, \quad (31)$$

where \mathbf{L}_d , \mathbf{L}_k and \mathbf{L}_z are symmetric positive definite matrices. The task-space adaptive controller $\boldsymbol{\tau}$ is continuous since the region errors $\Delta \varepsilon_x$ and $\Delta \varepsilon_r$ are continuous.

Using equation (27), the robot dynamics is written as:

$$\mathbf{M}(\mathbf{q}) \dot{\hat{\mathbf{s}}} + (\frac{1}{2} \dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})) \hat{\mathbf{s}} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r) \boldsymbol{\theta}_d = \boldsymbol{\tau}. \quad (32)$$

The closed-loop equation of the system is obtained by substituting equation (28) into equation (32) to give:

$$\begin{aligned} &\mathbf{M}(\mathbf{q}) \dot{\hat{\mathbf{s}}} + (\frac{1}{2} \dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})) \hat{\mathbf{s}} \\ &+ [\mathbf{I} - \hat{\mathbf{J}}^+(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \mathbf{A} \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)]^T \times \\ &[k_p \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x + k_p \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r] \\ &+ \mathbf{K}_s \hat{\mathbf{s}} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r) \Delta \boldsymbol{\theta}_d = \mathbf{0}, \end{aligned} \quad (33)$$

where $\Delta \boldsymbol{\theta}_d = \boldsymbol{\theta}_d - \hat{\boldsymbol{\theta}}_d$.

We can now state the following theorem:

Theorem: *The adaptive control law (28), the update laws (29)-(31) for the robot system (8) guarantee the global convergence of the tracking errors. That is $\mathbf{x}(t) \rightarrow \mathbf{x}_d(t)$, $\dot{\mathbf{x}}(t) \rightarrow \dot{\mathbf{x}}_d(t)$ as $t \rightarrow \infty$.*

Proof: A Lyapunov-like function candidate is proposed as:

$$\begin{aligned} V &= \frac{1}{2} \hat{\mathbf{s}}^T \mathbf{M}(\mathbf{q}) \hat{\mathbf{s}} + P_O(\mathbf{x}) + P_C(\mathbf{r}) + \frac{1}{2} \Delta \boldsymbol{\theta}_d^T \mathbf{L}_d^{-1} \Delta \boldsymbol{\theta}_d \\ &\quad + \frac{1}{2} \Delta \boldsymbol{\theta}_k^T \mathbf{L}_k^{-1} \Delta \boldsymbol{\theta}_k + \frac{1}{2} \Delta \boldsymbol{\theta}_z^T \mathbf{L}_z^{-1} \Delta \boldsymbol{\theta}_z, \end{aligned} \quad (34)$$

where $\Delta \boldsymbol{\theta}_k = \boldsymbol{\theta}_k - \hat{\boldsymbol{\theta}}_k$ and $\Delta \boldsymbol{\theta}_z = \boldsymbol{\theta}_z - \hat{\boldsymbol{\theta}}_z$.

Differentiating equation (34) with respect to time and substituting equations (27), (29) and (33) into it, we have:

$$\begin{aligned} \dot{V} &= -\hat{\mathbf{s}}^T \mathbf{K}_s \hat{\mathbf{s}} + k_p \alpha_x (\dot{\mathbf{x}} - \dot{\hat{\mathbf{x}}})^T \Delta \varepsilon_x \\ &\quad - k_p [\alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r + \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x]^T \\ &\quad \times [\alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r + \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x] \\ &\quad - \dot{\hat{\boldsymbol{\theta}}}_k^T \mathbf{L}_k^{-1} \Delta \boldsymbol{\theta}_k - \dot{\hat{\boldsymbol{\theta}}}_z^T \mathbf{L}_z^{-1} \Delta \boldsymbol{\theta}_z + k_p \alpha_x \dot{\mathbf{x}}_a^T \sum_{i=1}^m (\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \\ &\quad + k_p \alpha_r \dot{\mathbf{x}}_a^T \hat{\mathbf{Z}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \hat{\mathbf{J}}^+T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r. \end{aligned} \quad (35)$$

From equations (23) and (24), we have:

$$\dot{\mathbf{x}}_{ai} = \mathbf{0}, \quad \text{if } f_{Er_i}(\mathbf{x}_i) \geq 0. \quad (36)$$

Equation (36) indicates that $\dot{\mathbf{x}}_{ai}$ is nonzero where $f_{Er_i}(\mathbf{x}_i) < 0$. Since $f_{Er_i}(\mathbf{x}_i) < 0$ indicates that $f_{C_i}(r_i) < 0$ where the Cartesian-space region error $\Delta \varepsilon_r$ reduces to zero, $\dot{\mathbf{x}}_{ai}$ and $\Delta \varepsilon_r$ cannot be nonzero at the same time. In addition, since the gradient of potential energy $P_{E_i}(\mathbf{x}_i)$ reduces to zero where $f_{Er_i}(\mathbf{x}_i) < 0$, $\dot{\mathbf{x}}_{ai}$ and $(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T$ cannot be nonzero at the same time. Therefore, the last two terms in the equation (35) reduce to zero. In addition, from equation (5), it is given that $\mathbf{Z}(\mathbf{q}) \dot{\mathbf{x}} = \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}$, then we have: $(\dot{\mathbf{x}} - \dot{\hat{\mathbf{x}}})^T = -\Delta \boldsymbol{\theta}_z^T \mathbf{Y}_z^T(\mathbf{q}, \dot{\mathbf{x}}) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) + \Delta \boldsymbol{\theta}_k^T \mathbf{Y}_k^T(\mathbf{q}, \dot{\mathbf{q}}) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z)$.

Next, substituting equations (30), (31) into equation (35), we have:

$$\begin{aligned} \dot{V} = & -\hat{s}^T \mathbf{K}_s \hat{s} - \\ & k_p [\alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r + \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x]^T \\ & \times [\alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r + \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x]. \end{aligned} \quad (37)$$

Since $V > 0$ and $\dot{V} \leq 0$, V is bounded. Hence, \hat{s} , $\Delta \boldsymbol{\theta}_d$, $\Delta \boldsymbol{\theta}_k$, $\Delta \boldsymbol{\theta}_z$, $P_C(\mathbf{r})$ and $P_O(\mathbf{x})$ are bounded. Hence, $f_{E_i}(\mathbf{x}_i)$, $f_{T_i}(\mathbf{x}_i)$ and $f_{C_i}(\mathbf{r}_i)$ are also bounded. Therefore, \mathbf{x} and \mathbf{r} are bounded. Hence, $\frac{\partial f_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}$, $\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}$ and $\frac{\partial f_{C_i}(\mathbf{r}_i)}{\partial \mathbf{r}}$ are also bounded. Therefore, $\Delta \varepsilon_x$ and $\Delta \varepsilon_r$ are bounded. Since x is bounded, \dot{x}_d is bounded if \ddot{x}_d is bounded. Hence, $\dot{\mathbf{q}}_r$ is bounded. From equation (27), $\dot{\mathbf{q}}$ is bounded because \hat{s} is bounded. The boundedness of $\dot{\mathbf{q}}$ guarantees the boundedness of $\dot{\mathbf{x}}$ and $\dot{\mathbf{r}}$ since both $\mathbf{J}(\mathbf{q})$ and $\mathbf{J}_r(\mathbf{q})$ are trigonometric functions of \mathbf{q} . Therefore, $\Delta \dot{\varepsilon}_x$ and $\Delta \dot{\varepsilon}_r$ are bounded, and $\dot{\mathbf{q}}_r$ is also bounded. From the closed-loop equation (33), we can conclude that $\dot{\hat{s}}$ is bounded. In addition, $\ddot{\mathbf{q}}$ is also bounded because $\dot{\hat{s}} = \ddot{\mathbf{q}} - \dot{\mathbf{q}}_r$. Thus, \dot{V} is bounded since \hat{s} , $\dot{\hat{s}}$, $\Delta \dot{\varepsilon}_x$, $\Delta \varepsilon_x$, $\Delta \dot{\varepsilon}_r$, $\Delta \varepsilon_r$ are bounded. Therefore, \dot{V} is uniformly continuous. Applying Barbalat's lemma [25], we have $\dot{V} \rightarrow 0$ which also indicates:

$$\alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r + \alpha_x \hat{\mathbf{J}}^T(\mathbf{q}, \hat{\boldsymbol{\theta}}_k) \hat{\mathbf{Z}}^{-1}(\mathbf{q}, \hat{\boldsymbol{\theta}}_z) \Delta \varepsilon_x \rightarrow \mathbf{0}. \quad (38)$$

The error $\Delta \varepsilon_x$ is activated when the tool is within the image-space region, while the error $\Delta \varepsilon_r$ is nonzero when the end effector is inside the Cartesian-space region. If both the end effector and the tool are located outside the image-space region, $\Delta \varepsilon_r \neq \mathbf{0}$, $\Delta \varepsilon_x = \mathbf{0}$, which contracts with equation (38) since $\mathbf{J}_r(\mathbf{q})$ is non-singular. If the end effector is inside the Cartesian-space region while the tool is within the image-space region, $\Delta \varepsilon_r \neq \mathbf{0}$, $\Delta \varepsilon_x \neq \mathbf{0}$. Since the gradient of overall potential energy is not zero, the end effector can not stay in the overlapped region. Therefore, both the tool and the end effector can only settle down within the image-space region, then $\Delta \varepsilon_r = \mathbf{0}$, and from equation (38), since $\hat{\mathbf{J}}(\mathbf{q}, \hat{\boldsymbol{\theta}}_k)$ is non-singular, $\Delta \varepsilon_x = \mathbf{0}$. From equation (16), $\Delta \varepsilon_x = \mathbf{0}$ can only be satisfied where $f_{T_i}(\mathbf{x}_i) \leq 0$ and $\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} = \mathbf{0}$. That is, $\mathbf{x} \rightarrow \mathbf{x}_d$ as $t \rightarrow \infty$. In addition, from equation (5), it is obtained that:

$$\ddot{\mathbf{x}} = \mathbf{Z}^{-1}(\mathbf{q}) \mathbf{J}(\mathbf{q}) \ddot{\mathbf{q}} + \dot{\mathbf{Z}}^{-1}(\mathbf{q}) \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}} + \mathbf{Z}^{-1}(\mathbf{q}) \dot{\mathbf{J}}(\mathbf{q}) \dot{\mathbf{q}}. \quad (39)$$

Therefore, $\ddot{\mathbf{x}}$ is bounded since $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are bounded, and $\Delta \ddot{\mathbf{x}}$ is bounded if $\ddot{\mathbf{x}}_d$ is bounded. Since $\Delta \mathbf{x} \rightarrow \mathbf{0}$ and $\Delta \ddot{\mathbf{x}}$ is bounded, we have $\Delta \dot{\mathbf{x}} \rightarrow \mathbf{0}$ as $t \rightarrow \infty$. ■

V. EXPERIMENT

The experimental setup consists of a Sony SCARA robot and a PSD camera (C5949) manufactured by Hamamatsu as shown in Figure 1. The proposed controller was implemented on the first two links of the robot. The lengths of the first and the second links are $l_1 = 0.35$ m, $l_2 = 0.37$ m respectively.

The PSD camera was used to measure the position of tool-tip and output the coordinate data in the unit of voltage [26] within $-5 \sim 5$ V. The size of image plane is 0.01 m \times 0.01 m while the focal length of the camera $f = 0.025$ m.

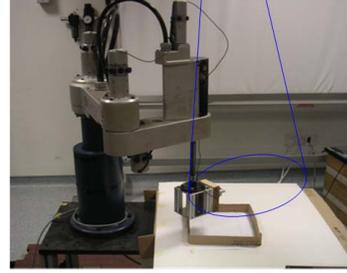


Fig. 1. Experimental setup

In this experiment, the camera was tilt by 6.6° from the vertical direction (see Figure 1), and the image plane was not parallel to the evolving plane. Thus, the approximate depth $\hat{Z}(t)$ was time-varying.

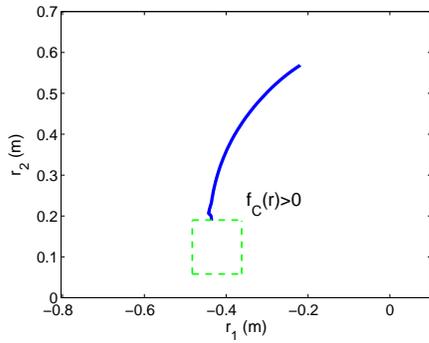
In the experiment, the controller described by equations (28) to (31) was implemented on the robot. The desired trajectory was specified as: $x_{d_h} = 3.12 + 0.7 \cos(0.4t)w$ and $x_{d_v} = 1.57 + 0.7 \sin(0.4t)w$ where the w is the weight factor. The control parameters were set as: $k_{xE} = 0.01$, $k_T = 0.1$, $\mathbf{K}_s = \text{diag}\{0.0005, 0.0005\}$, $\alpha_x = 1$, $\alpha_r = 1$, $k_p = 1$, $\mathbf{L}_d = \text{diag}\{0.1, 0.1\}$, $\mathbf{L}_z = \text{diag}\{0.7, 0.7\}$, $\mathbf{L}_k = \text{diag}\{0.5, 0.5\}$. The initial depth was estimated as $\hat{Z}(0) = 1.3$ m, and the initial length of the tool was estimated as $\hat{l}_0(0) = 0.07$ m. The experimental results are shown in Figure 2(a)-2(d). As seen from Figure 2(a)-2(b), the end-effector started from an initial position outside the field of view, and then transited inside the image-space region to track the desired trajectory. Figure 2(c) shows the convergence of tracking errors, which demonstrates the correct realization of the task, regardless of the uncertain kinematics and depth information.

VI. CONCLUSION

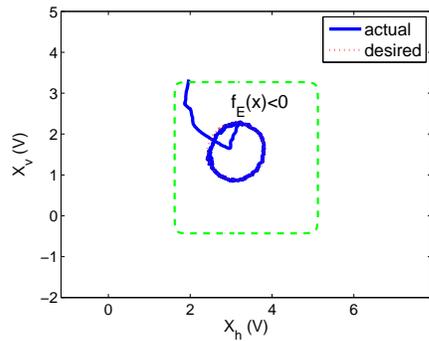
In this paper, a new adaptive controller that allows the use of sensory feedback only at the ending stage, has been presented. It has been shown that the proposed task-space controller can transit smoothly from Cartesian-space feedback at the initial stage to vision feedback when the robot is near the desired position, in the presence of uncertain robot dynamics, kinematics and depth information. Experiment results have been presented to illustrate the performance of the proposed controller.

REFERENCES

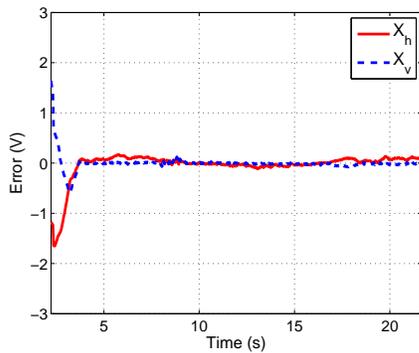
- [1] M. Takegaki and S. Arimoto, "A new feedback method for dynamic control of manipulators," *ASME J. of Dynamic Systems, Measurement and Control*, Vol. 102, pp. 119-125, 1981.



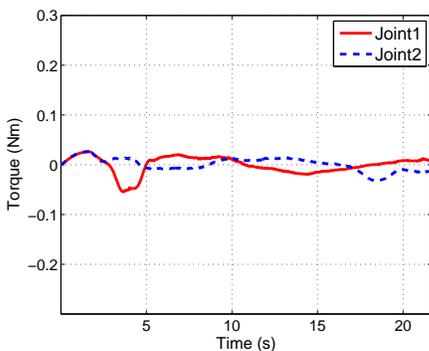
(a) Path of tool tip in Cartesian space



(b) Path of end effector in image space



(c) Tracking errors in image space



(d) Control input

Fig. 2. In the presence of uncertain kinematics and depth information, the end effector can transit from Cartesian space to image space, and converge to the desired trajectory.

- [2] R. Kelly, "Regulation of manipulators in generic task space: An energy shaping plus damping injection approach," *IEEE Trans. on Robotics and Automation*, Vol. 15, pp. 381-386, 1999.
- [3] S. Arimoto, *Control Theory of Non-Linear Mechanical Systems*. Oxford University Press, 1996.
- [4] S. Arimoto and F. Miyazaki, "Stability and robustness of PID feedback control for robot manipulators of sensory capability," *International Symposium on Robotics Research*, pp. 783-799, 1983.
- [5] J. J. E. Slotine and W. Li, "On the adaptive control of robot manipulators," *Int. J. Robot. Res.*, No. 6, pp. 49-59, 1987.
- [6] G. Niemeyer and J. J. E. Slotine, "Performance in adaptive manipulator control," *International Journal of Robotics Research*, Vol. 10, No. 2, pp. 149-161, 1991.
- [7] L. E. Weiss, A. C. Sanderson and C. P. Neuman, "Dynamic sensor-based control of robots with visual feedback," *IEEE Trans. on Robotics and Automation*, vol. RA-3, no. 5, pp. 404-417, 1987.
- [8] B. Espiau, F. Chaumette and P. Rives, "A New Approach to Visual Servoing in Robotics," *IEEE Trans. on Robotics and Automation*, vol. 8, no. 3, pp. 313-326, 1992.
- [9] Y. H. Liu, H. Wang, C. Wang and K. K. Lam, "Uncalibrated Visual Servoing of Robots Using a Depth-Independent Interaction Matrix," *IEEE Transactions on Robotics*, Vol. 22, No. 4, pp. 804-817, 2006.
- [10] C. C. Cheah, S. Kawamura and S. Arimoto, "Feedback control for robotic manipulator with an uncertain Jacobian matrix," *Journal of Robotic Systems*, Vol. 16, No. 2, pp. 19-134, 1999.
- [11] C. C. Cheah, M. Hirano, S. Kawamura and S. Arimoto, "Approximate Jacobian Control for Robots with Uncertain Kinematics and Dynamics," *IEEE Transactions on Robotics and Automation*, Vol. 19, No. 4, pp. 692-702, 2003.
- [12] W. E. Dixon, "Adaptive regulation of amplitude limited robot manipulators with uncertain kinematics and dynamics," *IEEE Transactions on Automatic Control*, Vol. 52, No. 3, pp. 488-493, 2007.
- [13] R. Ozawa and Y. Oobayashi, "Adaptive task space PD control via implicit use of visual information," *The 9th International Symposium on Robot Control*, (Gifu, Japan), pp. 209-214, 2009
- [14] C. C. Cheah, C. Liu and J. J. E. Slotine, "Adaptive tracking controls for robots with unknown kinematics and dynamic properties," *The International Journal of Robotics Research*, Vol. 25, No. 3, pp. 283-296, 2006.
- [15] C. C. Cheah, C. Liu and J. J. E. Slotine, "Adaptive Jacobian Tracking Control of Robots with Uncertainties in Kinematic, Dynamic and Actuator Models," *IEEE Transactions on Automatic Control*, Vol. 51, No. 6, pp. 1024-1029, 2006.
- [16] L. Cheng, Z. Hou and M. Tan, "Adaptive neural network tracking control for manipulators with uncertain kinematics, dynamics and actuator model," *Automatica*, Vol. 45, No. 10, pp. 2312-2318, 2009.
- [17] H. Wang and Y. Xie, "Passivity based adaptive Jacobian tracking for free-floating space manipulators without using spacecraft acceleration," *Automatica*, Vol. 45, pp. 1510-1517, 2009.
- [18] D. Braganza, W. E. Dixon, D. M. Dawson and B. Xian, "Tracking control for robot manipulators with kinematic and dynamic uncertainty," *International Journal of Robotics and Automation*, Vol. 23, No. 2, pp. 117-126, 2008.
- [19] H. Wang, Y. H. Liu and D. Zhou, "Dynamic Visual Tracking for Manipulators Using an Uncalibrated Fixed Camera," *IEEE Transactions on Robotics*, Vol. 23, No. 3, pp. 610-617, 2007.
- [20] H. Wang, Y. H. Liu and D. Zhou, "Adaptive Visual Servoing Using Point and Line Features With an Uncalibrated Eye-in-Hand Camera," *IEEE Transactions on Robotics*, Vol. 24, No. 4, pp. 843-857, 2008.
- [21] C. C. Cheah, C. Liu and J. J. E. Slotine, "Adaptive Jacobian vision based control for robots with uncertain depth information," *Automatica*, Vol. 46, pp. 1228-1233, 2010.
- [22] C. C. Cheah and X. Li, "Reach then see: a new adaptive controller for robot manipulator based on dual task-space information," *Proceedings of the 2010 IEEE International Conference on Robotics and Automation*, (Anchorage, USA), pp. 5155-5160, 2010.
- [23] F. L. Lewis, C. T. Abdallah, and D. M. Dawson, *Control of Robot End-effectors*. New York: Macmillan Publishing Company, 1993.
- [24] C. C. Cheah and X. Li, "Singularity-robust task-space tracking control of robot," *2011 IEEE International Conference on Robotics and Automation*, (Shanghai, China), pp. 5819-5824, 2011.
- [25] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, New Jersey: Prentice Hall, 1991.
- [26] <http://www.hamamatsu.com>