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<td><strong>Author(s)</strong></td>
<td>Chen, Jing-Ling; Cabello, Adán; Xu, Zhen-Peng; Su, Hong-Yi; Wu, Chunfeng; Kwek, L. C.</td>
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Hardy’s paradox for high-dimensional systems

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Hardy’s proof is considered the simplest proof of nonlocality. Here we introduce an equally simple proof that (i) has Hardy’s as a particular case, (ii) shows that the probability of nonlocal events grows with the dimension of the local systems, and (iii) is always equivalent to the violation of a tight Bell inequality. Our proof has all the features of Hardy’s and adds the only ingredient of the Einstein-Podolsky-Rosen scenario missing in Hardy’s proof: It applies to measurements with an arbitrarily large number of outcomes.

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I. INTRODUCTION

Nonlocality, namely, the impossibility of describing correlations in terms of local hidden variables [1], is a fundamental property of nature. Hardy’s proof [2,3], in any of its forms [4–7], provides a simple way to show that quantum correlations cannot be explained with local theories. Hardy’s proof is usually considered “the simplest form of Bell’s theorem” [8].

However, if one wants to study nonlocality in a systematic way, one must define the local polytope [9] corresponding to any possible scenario (i.e., for any given number of parties, settings, and outcomes) and check whether quantum correlations violate the inequalities defining the facets of the corresponding local polytope. These inequalities are the so-called tight Bell inequalities. In this sense, Hardy’s proof has another remarkable property: It is equivalent to a violation of a tight Bell inequality, the Clauser-Horne-Shimony-Holt inequality [10]. This was observed in [5].

Hardy’s proof requires two observers, each with two measurements, each with two possible outcomes. The proof has been extended to the case of more than two measurements [11,12] and more than two outcomes [13–15]. However, none of these extensions is equivalent to the violation of a tight Bell inequality. Hardy-like proofs can also be applied to contextuality [16].

Hardy’s paradox brings together two features that no other proof of nonlocality has: (i) It proves Bell’s theorem under the condition proposed by Einstein, Podolsky, and Rosen (EPR) that one party’s measurement outcome allows this party to predict with certainty the other party’s measurement outcome [17] and (ii) it is equivalent to a violation of the condition that exactly separates local from nonlocal correlations for the 2-2-2 scenario (i.e., the tight Bell inequality for the scenario with two parties, two settings, and two outcomes). However, Hardy’s paradox has a drawback: The EPR scenario is not 2-2-2 but 2-2-n with n arbitrarily large.

The aim of this work is to introduce an alternative paradox that keeps all the virtues of Hardy’s but has the only ingredient of the EPR scenario that is missing in Hardy’s paradox: It applies to measurements with an arbitrary number of outcomes. The alternative paradox shows that the maximum probability of nonlocal events, which has a limit of 0.09 in Hardy’s paradox (and in previously proposed extensions of Hardy’s paradox), actually grows with the number of possible outcomes, tending asymptotically to a limit that is more than four times higher than the one in Hardy’s paradox. Moreover, we show that, for any given number n of outcomes, the alternative paradox is equivalent to a violation of the condition that exactly separates local from nonlocal correlations for the 2-2-n scenario. Arguably, all these features make this paradox of fundamental importance.

II. ALTERNATIVE FORMULATION OF HARDY’S PARADOX

Let us consider two observers: Alice, who can measure either A1 or A2 on her subsystem, and Bob, who can measure B1 or B2 on his. Suppose that each of these measurements has d outcomes that we will number as 0, 1, 2, . . . , d − 1. Let us denote as $P(A_2 < B_1)$ the joint conditional probability that the result of $A_2$ is strictly smaller than the result of $B_1$, that is,

$$P(A_2 < B_1) = \sum_{m,n} P(A_2 = m, B_1 = n), \quad (1)$$

with $m, n \in \{0, 1, \ldots, d-1\}$. Explicitly, for $d = 2$, $P(A_2 < B_1) = P(A_2 = 0, B_1 = 1)$; for $d = 3$, $P(A_2 < B_1) = P(A_2 = 0, B_1 = 1) + P(A_2 = 0, B_1 = 2) + P(A_2 = 1, B_1 = 2)$; etc.

Then the proof follows from the fact that, according to quantum theory, there are two-quit entangled states and local measurements satisfying, simultaneously, the following conditions:

$$P(A_2 < B_1) = 0, \quad (2a)$$
$$P(B_1 < A_2) = 0, \quad (2b)$$
$$P(A_1 < B_2) = 0, \quad (2c)$$
$$P(A_2 < B_2) > 0. \quad (2d)$$

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Therefore, if events $A_2 < B_1$, $B_1 < A_1$, and $A_1 < B_2$ never happen, then, in any local theory, event $A_2 < B_2$ must never happen either. However, this is in contradiction with (2d). If $d = 2$, the proof is exactly Hardy’s [2,3].

III. BEYOND HARDY’S LIMIT

Let us define
\[
P_{\text{Hardy}} = \max P(A_2 < B_2)
\]
satisfying conditions (2a)–(2c). For $d = 2$, 
\[
P_{\text{Hardy}}^{(d=2)} = \frac{5\sqrt{5} - 11}{2} \approx 0.09,
\]
which is achieved with two-qubit systems [2,3]. In previous extensions of Hardy’s paradox to two-qutrit systems [13–15], (4) is also the maximum probability of events that cannot be explained by local theories. For example, the extension considered in Ref. [13] is based on the following four probabilities: $P(A_1 = 0, B_1 = 0) = 0$, $P(A_1 \neq 0, B_2 = 0) = 0$, $P(A_2 = 0, B_1 \neq 0) = 0$, and $P(A_2 = 0, B_2 = 0) = P_{\text{KC}} > 0$. Reference [14] proves that, for two-qutrit systems, max $P_{\text{KC}}$ equals (4) and conjectures that max $P_{\text{KC}}$ is always (4) for arbitrary dimension. Reference [15] provides a proof of this conjecture.

Interestingly, in the proof presented in the previous section, $P_{\text{Hardy}}$ equals Hardy’s limit (4) for $d = 2$, but this is no longer true for higher-dimensional systems. To show this, we will consider pure states satisfying the three conditions (2a)–(2c). An arbitrary two-qubit pure state can be written as
\[
|\Psi\rangle = \sum_{i=0}^{d-1} \sum_{j=0}^{d-1} h_{ij} |i\rangle_A |j\rangle_B, \tag{5}
\]
where the basis states $|i\rangle_A, |j\rangle_B \in \{0,1,\ldots,d-1\}$ and $h_{ij}$ are coefficients satisfying the normalization condition $\sum_{i,j} |h_{ij}|^2 = 1$.

The coefficients $h_{ij}$ completely determine the state $|\Psi\rangle$. We can associate any two-qubit state $|\Psi\rangle$ with a coefficient-matrix $H = (h_{ij})_{i,j=0,\ldots,d-1}$, where $i,j = 0, \ldots, d-1$ and $h_{ij}$ is the $i$th row and the $j$th column element of the $d \times d$ matrix $H$. The connection between the coefficient matrix $H$ and the two reduced density matrices of $|\Psi\rangle$ is
\[
\rho_A = tr_B(|\Psi\rangle\langle \Psi|) = HH^\dagger, \tag{6a}
\]
\[
\rho_B = tr_A(|\Psi\rangle\langle \Psi|) = H^T (H^T)^\dagger, \tag{6b}
\]
where $T$ denotes the matrix transpose and $H^\dagger$ is the Hermitian conjugate matrix of $H$.

The probability $P(A_i = m, B_j = n)$ can be calculated as
\[
P(A_i = m, B_j = n) = tr\left( (\hat{P}_{A_i}^m \otimes \hat{P}_{B_j}^n) \rho \right), \tag{7}
\]
where $\hat{P}_{A_i}^m$ and $\hat{P}_{B_j}^n$ are projectors and $\rho = |\Psi\rangle\langle \Psi|$.

Explicitly, the projectors are given by
\[
\hat{P}_{A_i}^m = U_i |m\rangle\langle m|U_i^\dagger, \tag{8a}
\]
\[
\hat{P}_{B_j}^n = V_j |n\rangle\langle n|V_j^\dagger, \tag{8b}
\]
\[
\hat{P}_{A_i}^m = U_i |m\rangle\langle m|U_i^\dagger, \tag{8c}
\]
\[
\hat{P}_{B_j}^n = V_j |n\rangle\langle n|V_j^\dagger, \tag{8d}
\]
where $U_i$, $V_1$, $U_2$, and $V_3$ are, in general, SU($d$) unitary matrices. Hereafter we denote by $P_{\text{Hardy}}^{\text{opt}}$ the optimal value of $P_{\text{Hardy}}$ by ranging over all unitary matrices and the state $|\Psi\rangle$.

To calculate $P_{\text{Hardy}}^{\text{opt}}$, it is sufficient to choose the settings $A_1$ and $B_1$ as the standard bases, i.e., taking $U_1 = V_1 = 1$, where $\mathbb{1}$ is the identity matrix. Evidently, the condition (2b) leads to $h_{ij} = 0$ for $i > j$. This implies that the matrix $H$ is an upper-triangular matrix.

In Table I we list the optimal values of $P_{\text{Hardy}}^{\text{opt}}$ for $d = 2, \ldots, 7$. The corresponding optimal Hardy states $H_{\text{Hardy}}^{\text{opt}}$ are explicitly given in Appendix A.

The calculations for $d > 7$ are beyond our computers capability. However, we observe that $H_{\text{opt}}$, written in the representation of $H$, have reflection symmetry with respect to the antidiagonal line, that is, $h_{ij} = h_{d-1-i,d-1-j}$. We use this to calculate approximately the maximum probability for nonlocal events $P_{\text{Hardy}}^{\text{opt}}$ by using a special class of states $H_{\text{opt}}$. The explicit form of states $H_{\text{opt}}$ is given in Appendix B. This allows us to go beyond $d = 7$ and compute $P_{\text{Hardy}}^{\text{opt}}$ from $d = 2$ to 28 000. In Fig. 1 we have plotted $P_{\text{Hardy}}^{\text{opt}}$ from $d = 2$ to 1000, showing that $P_{\text{Hardy}}^{\text{opt}}$ increases with the dimension. Values for higher dimensions are given in Appendix B.

In Table I we also compare the $P_{\text{Hardy}}$ for the optimal states and the approximate optimal states. This allows us to speculate that the asymptotic limit may be a little higher than the one showed in Fig. 1. However, the limit 1/2 can never be surpassed since $P(A_2 > B_2)$ is always bigger than $P(A_2 < B_2)$, as observed in the numerical computations. At

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{d} & \textbf{$P_{\text{Hardy}}^{\text{opt}}$} & \textbf{$P_{\text{Hardy}}^{\text{opt}}$} & \textbf{Error rates} \\
\hline
2 & 0.090170 & 0.088889 & 0.014207 \\
3 & 0.141327 & 0.138426 & 0.020527 \\
4 & 0.176512 & 0.171533 & 0.028208 \\
5 & 0.203057 & 0.195869 & 0.035399 \\
6 & 0.224221 & 0.214825 & 0.041905 \\
7 & 0.241728 & 0.230172 & 0.047807 \\
\hline
\end{tabular}
\caption{Values of $P_{\text{Hardy}}^{\text{opt}}$ and $P_{\text{Hardy}}^{\text{opt}}$ for $d = 2, \ldots, 7$.}
\end{table}
acting on Bob (the second part), then and (2c), we have

to 1. To show this we use the generalized concurrence or degree
asymptotic limit.

\[
\begin{array}{c|c|c}
 d & \text{Optimal states} & \text{Approximate states} \\
\hline
 2 & 0.763932 & 0.825885 \\
 3 & 0.793888 & 0.845942 \\
 4 & 0.813483 & 0.861735 \\
 5 & 0.827702 & 0.874459 \\
 6 & 0.838679 & 0.884926 \\
 7 & 0.847510 & 0.893695 \\
\end{array}
\]

this point, we do not know whether or not 1/2 may be the asymptotic limit.

IV. DEGREE OF ENTANGLEMENT

Hardy’s proof does not work for maximally entangled states. The same is true for the proof introduced here. However, in our proof, as \( d \) increases, the degree of entanglement tends to 1. To show this we use the generalized concurrence or degree of entanglement [18] for two-qudit systems given by

\[
C = \sqrt{\frac{d}{d-1} \left[ 1 - \text{tr}(\rho^{A_1}_B) \right]} = \sqrt{\frac{d}{d-1} \left[ 1 - \text{tr}(\rho^{B_1}_A) \right]}.
\]  

In Table II we list \( C \) for the optimal Hardy states and the approximate Hardy states. From Table II we observe that, for \( d = 2 \), the optimal Hardy state occurs at \( C^{\text{opt}} \approx 0.763932 \) and this value increases to \( C^{\text{app}} \approx 0.827702 \) when \( d = 5 \). For a fixed \( d \), the corresponding \( C^{\text{app}} \) is larger than that of \( C^{\text{opt}} \) and it also increases with the dimension \( d \). For \( d = 800 \), \( C^{\text{app}} \approx 0.998062 \) and tends to 1 as \( d \) grows.

Finally, we can prove that the proof cannot work for two-qudit maximally entangled states

\[
|\Psi\rangle_{\text{MES}} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |j\rangle_A |j\rangle_B.
\]  

Proof. Here \( \text{tr}((\hat{F}^n_A \otimes \hat{F}^n_B)|\Psi\rangle\langle\Psi|) \) can be expressed as

\[
\text{tr}(|m\rangle\langle m| \otimes |n\rangle\langle n|)\text{(U}^T_1 \otimes \text{V}^T_1) |\Psi\rangle\langle\Psi|(U_1 \otimes \text{V}_1).
\]

We will use

\[
H_{\text{MES}} \mapsto |\Psi\rangle_{\text{MES}}, \quad H' \mapsto (U_1^T \otimes \text{V}^T_1)|\Psi\rangle_{\text{MES}}.
\]  

Taking into account that, (i) given a pure state \( H \mapsto |\Psi\rangle_{AB} \) and a local action \( U \) acting on Alice (the first part) and \( V \) acting on Bob (the second part), then

\[
H' \mapsto (U \otimes V)|\Psi\rangle_{AB} = UV^{T},
\]  

(ii) Eq. (2b) requires \( H' \) to be an upper-triangular matrix, and (iii) \( H_{\text{MES}} = \frac{1}{\sqrt{d}}1 \), we have the solution

\[
U_1^T \text{V}^T_1 = D_1,
\]  

where \( D_1 = \text{diag}(e^{i\theta_0}, e^{i\theta_1}, \ldots, e^{i\theta_{d-1}}) \). Similarly, from (2a) and (2c), we have

\[
U_1 \text{V}^T_1 = D_2, \quad U_2 \text{V}^T_1 = D_3,
\]

which directly leads to \( P(A_2 < B_2) = 0 \) for \( |\Psi\rangle_{\text{MES}} \). This ends the proof.

V. CONNECTION TO TIGHT BELL INEQUALITIES

As can be easily seen, for any \( d \), our proof can be transformed into the following Bell inequality:

\[
P(A_2 < B_1) + P(B_1 < A_1) - P(A_1 < B_2) - P(A_2 < B_2) \geq 0,
\]

where LHV indicates that the bound is satisfied by local hidden variable theories. The interesting point is that, for any \( d \), inequality (17) is equivalent to Zhou and Gill’s version [19] of the Collins-Gisin-Linden-Massar-Popescu inequalities (plural because there is a different inequality for each \( d \)) [20], which are tight Bell inequalities for any \( d \) [21]. This feature distinguishes our proof from any previously proposed nonlocality proof having Hardy’s as a particular case.

VI. CONCLUSION

We have introduced a simple proof of nonlocality for pairs of systems of arbitrary dimension that has all the features of the celebrated proof by Hardy but applies to many other scenarios, including the scenario originally considered by Einstein, Podolsky, and Rosen in which measurements have an arbitrarily large number of outcomes [22].

As in the case of Hardy’s paradox, an experimental test of our paradox consists of observing that the probabilities of three events are zero while the probability of a fourth event is not zero. The fact that in our proof the value of this fourth probability is larger than in Hardy’s (since it grows with the dimension of the physical system from the value it has in Hardy’s proof) makes it more adequate for experimental observation of Hardy-like nonlocality and for applications based on this type of nonlocality.

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APPENDIX A: OPTIMAL HARDY STATES

The optimal Hardy states $H_d$ for $d = 2, \ldots, 7$ are

\[
H_2 = \begin{pmatrix}
0.618034 & 0.485868 \\
0 & 0.618034 \\
\end{pmatrix},
\]

\[
H_3 = \begin{pmatrix}
0.498328 & 0.316483 & 0.329301 \\
0 & 0.441108 & 0.316483 \\
0 & 0 & 0.498328 \\
\end{pmatrix},
\]

\[
H_4 = \begin{pmatrix}
0.429796 & 0.262169 & 0.224332 & 0.249934 \\
0 & 0.376201 & 0.217224 & 0.224332 \\
0 & 0 & 0.376021 & 0.262169 \\
0 & 0 & 0 & 0.429796 \\
\end{pmatrix},
\]

\[
H_5 = \begin{pmatrix}
0.383613 & 0.230044 & 0.189636 & 0.175427 & 0.201533 \\
0 & 0.334102 & 0.185035 & 0.157012 & 0.175427 \\
0 & 0 & 0.33072 & 0.185035 & 0.189636 \\
0 & 0 & 0 & 0.334102 & 0.230044 \\
0 & 0 & 0 & 0 & 0.383613 \\
\end{pmatrix},
\]

\[
H_6 = \begin{pmatrix}
0.349686 & 0.207877 & 0.16845 & 0.150559 & 0.144455 & 0.16883 \\
0 & 0.303795 & 0.165105 & 0.134967 & 0.125208 & 0.144455 \\
0 & 0 & 0.29972 & 0.16666 & 0.134967 & 0.150559 \\
0 & 0 & 0 & 0.29972 & 0.165105 & 0.16845 \\
0 & 0 & 0 & 0 & 0.303795 & 0.207877 \\
0 & 0 & 0 & 0 & 0 & 0.349686 \\
\end{pmatrix},
\]

\[
H_7 = \begin{pmatrix}
0.323377 & 0.191279 & 0.153539 & 0.135037 & 0.12545 & 0.122887 & 0.145233 \\
0 & 0.280442 & 0.150851 & 0.121193 & 0.108665 & 0.104707 & 0.122887 \\
0 & 0 & 0.276282 & 0.145271 & 0.117498 & 0.108665 & 0.12545 \\
0 & 0 & 0 & 0.275414 & 0.145271 & 0.121193 & 0.135037 \\
0 & 0 & 0 & 0 & 0.276282 & 0.150851 & 0.153539 \\
0 & 0 & 0 & 0 & 0 & 0.280442 & 0.191279 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.323377 \\
\end{pmatrix}.
\]

APPENDIX B: APPROXIMATE OPTIMAL HARDY STATES

The form of $H_d$ for $d = 2, \ldots, 7$ suggests to define the approximate optimal Hardy states as follows:

\[
H_d^{\text{app}} = \begin{pmatrix}
\alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_{d-2} & \alpha_{d-1} & \alpha_d \\
\alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_{d-2} & \alpha_{d-1} & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_d \\
\end{pmatrix},
\]

where

\[
\alpha_r = \frac{\beta_r}{\sqrt{d+1-r}}, \quad r = 1, 2, \ldots, d,
\]

with $\beta_r > 0$ satisfying the following relations:

\[
\beta_1 : \beta_2 : \beta_3 : \cdots : \beta_d = 1 : \frac{1}{2} : \frac{1}{3} : \cdots : \frac{1}{d},
\]

\[
\sum_{r=1}^{d} \beta_r^2 = 1.
\]

In Table III we have listed $P_{\text{Hardy}}^{\text{app}}$ up to $d = 28000$.

<table>
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<tr>
<th>$d$</th>
<th>$P_{\text{Hardy}}^{\text{app}}$</th>
<th>$d$</th>
<th>$P_{\text{Hardy}}^{\text{app}}$</th>
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<th>$P_{\text{Hardy}}^{\text{app}}$</th>
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<td>2</td>
<td>0.088889</td>
<td>300</td>
<td>0.405106</td>
<td>2000</td>
<td>0.414711</td>
<td>10000</td>
<td>0.416300</td>
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<td>10</td>
<td>0.263168</td>
<td>400</td>
<td>0.407749</td>
<td>2200</td>
<td>0.414885</td>
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<td>0.416339</td>
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<tr>
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<td>30</td>
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<td>1800</td>
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<td>9000</td>
<td>0.416254</td>
<td>28000</td>
<td>0.416575</td>
</tr>
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</table>
[22] A subtlety here is that, albeit both are infinite, the outcomes in Hardy’s scenario presented here are discrete variables, while those in the original EPR scenario are continuous ones.