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An efficient modified flanges only method for plate girder bending resistance calculation

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Abstract
The very popular “Flange Only” method suggested in the BS 5950 for calculation of bending resistance of plate girder with Class 4 slender web is studied and compared with the more complicated effective width method adopted by the EC3. It is shown that within all practical range of web depth to thickness ratio, that the flanges only method is conservative but sometime may be inefficient. Based on the study of these two methods, a very simple modification factor, which can be obtained conveniently by hand calculation, is proposed to improve the efficiency of the flanges only method. It is shown that the proposed modified flanges only method, while still always remains conservative, could be able to estimate the bending resistance of the girder more accurately than the original flanges only method. In most cases, the bending resistances predicted by the proposed method are within the range of 94% to 97% of those predicted by the more complicated effective width method.

Key Words: Plate girder, Class 4 slender web, bending resistance, flanges only method, effective width method
1. Introduction

The introduction of the Eurocodes is undoubtedly the biggest change in design concept and practices in the European Union and many other countries [1]. With the Europe wide implementation of the Eurocodes since 2010, conflicting national standards are gradually withdrawn in the European Union and other countries which committed to adopt the Eurocodes such as Singapore [2]. Regarding design of steel structures, while both the Eurocode 3 (EC3) Part 1-1 [3] and the BS 5950 Part 1 [4] share the same origin and are both based on the limit states design concept [3-6], there are significantly changes in different aspects of the design practices, including notations, theoretical background, structural analysis requirements and calculation procedures [7]. Hence, many design aids were written to help practical engineers and engineering students to uptake the new design standard [8-12]. In general, when comparing with other national standards like the British Standard (BS) [4], the Eurocodes gives better harmonisation of treatment while more preferences and emphases are given to the use of appropriate (and sometime more complicated) mechanical models and comprehensive analysis procedure. Regarding the design of plate girder which is one of the most commonly encountered plated structures, unlike the BS [4], no explicit section is devoted in the corresponding EC3 Part 1-5 [13] to describe the detailed design requirements and procedures. Instead, in EC3 Part 1-5 [13] for plated structures design, only the main design principles and a set of rules for some common plated structures are presented and some design formulae are given in the accompanied informative annexes. Hence, practicing engineers often need referring to some design manuals such as reference [14] for design calculations and design examples. Toward this end, one example for such situation is the calculation of bending resistance of a plate girder without longitudinal stiffener when only the web of the girder is a Class 4 slender section. While both the BS [4]
and the EC3 [13] are, in fact essentially based on the same design principle, in the BS it is stated explicitly (Clause 3.6.2.4 of reference [4]) how the effective width could be calculated and a sub-section (Section 4.4 of reference [4]) is devoted to describe the design requirement of plate girder structures. Furthermore, a very popular simplified “Flanges only” method [4, 15] is also described to allow practicing engineers to quickly estimate a conservative bending resistance of the plate girder. However, in EC3 [13] (Section 4.3 of reference [13]), only the essential design principle are described and they are supplemented by a footnote which indicates that in order to use the “Effective width” (or sometimes called the “Effective modulus” method) method suggested by EC3, an iterative procedure is required to obtain the effective cross section and then the bending resistance. In order to fully understand the theory behind the effective width method and the actual design and calculation steps needed, a practicing engineer who is new to the EC3 may need to refer to some detailed manuals such as reference [14] for the design rule (Section 2.4.2.2 of reference [14]) and for calculation details (Example 2.4.2 of reference [14]). For a structural engineer who is switching from the BS design to the EC3 design, it is interesting and important to note that while the BS allows the use of the more exact effective width method stated in EC3, EC3 does not state (nor in guidelines such as reference [14]) that whether the well accepted the flanges only method mentioned in the BS is acceptable or not. Since when portioning the dimensions of a plate girder, estimating its bending resistance is often an essential initial step towards an efficient design, it would be much useful if a comparison between the bending resistances predicted by the flanges only method and that by the effective width method could be made. The main objective of this paper is to carry out such a study to compare the bending resistances predicted by these two methods for a plate girder with a Class 4 web without longitudinal stiffener. It will be shown that while in strict
In the mathematical sense, the flanges only method does not always give a conservative bending resistance when comparing with the effective width method, it is indeed conservative within virtually all practical aspect ratios covered in usual plate girder design. However, in some cases the flanges only method appears to be too conservative and is not efficient. As a result, another contribution of this paper is to suggest a simple but practical alternative procedure to increase the efficiency of the flanges only method.

In the next section, the calculation steps for predicting the bending resistance of a plate girder with a Class 4 web without longitudinal stiffener based on the BS’s flanges only method and the more exact EC3’s effective width method are described. They are then followed by a comprehensive analytical study on these two methods. Based on the study results, a simple modification factor will be suggested to improve the efficiency of the flanges only method while the resulting bending resistance is still be conservative when comparing with the more exact but tedious effective width method. A calculation example will be given to demonstrate the results obtained. Finally, conclusions of the works presented will be given.

2. The BS’s “Flanges only” method and the EC3’s “Effective width” method

2.1 Notations and assumptions

Since in this paper, both the design methods based on the BS [4] and the EC3 [3, 13] are referred, in order to avoid ambiguity, all the symbols used (except those newly defined in this paper) will be based on the EC3 notations. In addition, in order to simplify the discussion, it is assumed that the plate girder under concern is symmetrical about its major axis and subjected to pure bending only. The flanges of the girder are not in Class 4 according to EC3.
classification. Furthermore, the yield strength of the flanges and web are the same (i.e. no hybrid cross section is allowed) and no longitudinal stiffener is applied.

2.2 The BS’s “Flanges only” method [Clause 4.4.4.2(b) of Reference [4]]

In the BS’s flanges only method, it is assumed that the web is designed for shear only and all the bending resistance of the plate girder (Fig. 1) is provided by the flanges only. Note that since it is assumed that the whole web contribution is ignored, the centroid of the section $G$ remains at the middle of the web. By ignoring the contribution from the weld between the web and the flanges (since the leg length of the welding $l_w<<b_f$ and $h_w$), the bending resistance based on the flanges only method, $M_{f,Rd}$, can be expressed as

$$M_{f,Rd} = 2b_f t_f (h_w/2+t_f/2)f_y = b_f t_f (h_w+t_f)f_y = A_f (h_w+t_f)f_y$$  \hspace{1cm} (1a)$$

where in Eqn. 1a, $f_y$ is the yield strength of the section and $A_f=b_f t_f$ is the area of the single flange. Eqn. (1a) can be rewritten in the form of

$$M_{f,Rd} = A_f h_w (1+t_f/h_w)f_y = A_f h_w (1+\xi)f_y = W_{pl,f} f_y$$  \hspace{1cm} (1b)$$

$$W_{pl,f} = A_f h_w (1+\xi)$$  \hspace{1cm} (1c)$$

where $W_{pl,f}$ is the section plastic modulus contributed by the flanges only. Since usually $t_f<h_w$, the dimensionless parameter $\xi = t_f/h_w$ should be within the range $0<\xi<0.2$. The BS [4] allows to take $M_{f,Rd}$ as a conservative estimate of the section bending resistance $M_{c,Rd}$. Obviously, Eqns. 1a to 1c are very simple and $M_{f,Rd}$ could be conveniently obtained by hand calculation. It can be seen that one important assumption of the flanges only method is that despite the web is Class 4 slender, both the compression and tension flanges are assumed to be yielded to provide the bending resistance at the ultimate limit state (ULS).
2.3 The EC3’s “Effective width” method [Section 4.3 of Reference [13]]

In the EC3’s effective width method, it is assumed that since the web is Class 4 slender, part of the compressive web is subjected to local buckling and could no longer able to contribute to the effective section of the plate girder (Fig. 2). In this case, only the remaining effective section will act like an equivalent Class 3 section to provide the bending resistance at the ULS.

As part of the compressive web is ineffective, under the action of a sagging moment, the centroid of the effective area will shift down from $G$ to its new position $G'$ (Fig. 2). Hence, at the ULS the top extreme fibre will be yielded while the bottom extreme fibre will remain elastic. From reference [14], in the effective width method, the following assumptions are made in order to compute the bending resistance of the equivalent Class 3 section:

(1) A linear strain distribution is assumed for the web.

(2) The ULS is reached when $f_y$ is reached at the centroid of the compressive flange while stress at the centroid of the tension flange will be less than $f_y$.

In order to compute the corresponding effective section properties, EC3 requires the following iterative calculations steps [3, 13, 14].

(i) Assuming a linear stress distribution, compute the stress ratio $\psi$ such that

$$\psi = \frac{\sigma_1}{\sigma_2} = \frac{-(b_t + t_f)}{(b_c + t_f)}$$

(2)

In Eqn. 2, $\sigma_1$ and $\sigma_2$ are the direct stress at the centroids of top and bottom flanges, respectively. Note that in Eqn. 2, it is assumed that the section is subjected to sagging moment and compressive stress is taken as positive. To start the calculation, one could assume that initially the section is fully effective so that $b_t/b_c=1$ and hence $\psi=-1$.

(ii) Use Table 4.1 of reference [13] and the value of $\psi$ to determine the buckling factor $k_\alpha$

(iii) From $k_\alpha$, compute the reduction factor $\rho$ from Eqn. 4.2 of reference [13] such that
where \( \varepsilon = \sqrt{\frac{235}{f_y}} \).

(iv) The effective breath \( b_{\text{eff}} \) is then taken as

\[
(b_{\text{eff}} = \rho \cdot b_c = b_{e1} + x + b_{e2})
\]

\[b_{e1} = 0.4b_{\text{eff}} - 0.4\rho \cdot b_c, \quad b_{e2} = 0.6b_{\text{eff}} - 0.6\rho \cdot b_c, \quad x = (1-\rho)b_c\]  \hspace{1cm} (3c)

(v) Based on the values of \( \rho, b_c, b_{e1} \) and \( b_{e2} \), compute the location of the new centroid of the effective section \( G' \) and the corresponding shift in centroid location, \( \Delta G \) (Fig. 3).

(vi) Based on \( \Delta G \), recomputed the values of \( b_t \) and \( b_c \) and go back to step (i). Iterate the above calculation steps until the change of \( \Delta G \) is negligible and the values of \( b_t \) and \( b_c \) are converged.

(vii) Compute the effective section’s second moment of area \( I_{\text{eff}} \) based on the converged values of \( b_t, b_c, \Delta G \) and other section dimensions.

(viii) The effective section modulus of the section \( W_{\text{eff}} \), and the bending resistance \( M_{c,Rd} \) are finally calculated as

\[
W_{\text{eff}} = I_{\text{eff}}/d_c = I_{\text{eff}}/(h_w/2 + t_f/2 + \Delta G)
\]

\[M_{c,Rd} = W_{\text{eff}} f_y\]  \hspace{1cm} (4b)

Obviously, the above calculation steps are not convenient for hand calculation and most often a spreadsheet programme is preferred. Hence, in reference [14], it is mentioned that EC3 does allow a simplified approaches for I-section (and box section in bending only) that end at two steps only. In the first step, it is only required to compute the values of \( \rho \) and \( \Delta G \) once based on the initial assumption of \( b_t = b_c \). In the second step, based on the values of \( \rho \) and \( \Delta G \) obtained, the values of \( I_{\text{eff}}, d_c \) and \( W_{\text{eff}} \) are computed.
If one investigate more closely the above effective width method (or carry out a complete calculation once), it could be easily found that the most clumsy step is the calculation of \( I_{\text{eff}} \), which involve calculations of the second moment of areas of a few separated rectangles. In order to avoid calculation errors, it is preferable to carry out this step using a pre-defined spreadsheet.

3. Comparing the flanges only method with the effective width method

In order to compare the bending resistances predicted by these two methods, from Eqns. (1) and (4), one only needs to consider the ratio

\[
R = \frac{W_{\text{eff}}}{W_{\text{pl,fl}}}
\]

(5)

In case that \( R > 1 \), then the flanges only method is conservative and could be used as a safe substitute for the more complex effective width method. While \( W_{\text{pl,fl}} \) could be easily obtained from Eqn. 1(a), Section 2.3 indicates that \( W_{\text{eff}} \) can only be obtained after some calculations involving the computation of \( I_{\text{eff}} \) of the effective cross section.

3.1 Calculation of the factor \( \rho \)

In order to evaluate \( R \) (Eqn. 5) and to determine whether the flanges only method is conservative or not, it is first necessary to express \( W_{\text{eff}} \) in a form similar to Eqn. 1c. Toward this end, the two-step approach suggested by EC3 [3, 13, 14] is adopted here. As it is assumed that initially \( b_t = b_c = h_w/2 \) so that \( \psi = -1 \) and from Table 4.1 of reference [13], \( k_6 = 23.9 \), Eqn. (3) can be written as

\[
\lambda_p = \frac{h_w}{138.84}, \quad \rho = \frac{\lambda_p - 0.055 \times 2}{(\lambda_p)^2} = \frac{\lambda_p - 0.11}{(\lambda_p)^2}
\]

(6)
A plot of $\rho$ against $h_w/(t_w \cdot \varepsilon)$ is shown in Fig. 4. It can be seen that $\rho$ decreases as $h_w/(t_w \cdot \varepsilon)$ is larger than the Class 4 slender limit of 124 and $\rho=0.2$ when $h_w/(t_w \cdot \varepsilon)=680$. It should be stressed that Fig. 4 is generated based on the assumption that $b_t=b_c$ and hence the stress ratio $\psi=-1$ and $k_\sigma=23.9$ remain constant. In practice, if the full iteration steps (Section 2.3) are carried out, when $h_w/(t_w \cdot \varepsilon)$ increases $\psi$ will also be increased (e.g. changes from -1 to -0.9). The values of $k_\sigma$ will then be decreased. The net outcome is that $\bar{\lambda}_p$ will be increased at a faster rate and eventually $\rho$ will be decreased more rapidly as shown in Fig. 4. Hence, the two-step simplified method suggested in [14] should be used with caution whenever $h_w/(t_w \cdot \varepsilon)$ is very large. In this paper, it is assume that the ratio $h_w/(t_w \cdot \varepsilon)$ for the plate girder is less than 400. Note that in BS5950 [4], a limiting value of $h_w/(t_w \cdot \varepsilon)$ approximately equal to 250 is imposed for the design of plate girder without longitudinally stiffener.

3.2 Parametric form of $W_{eff}$

Now for the effective section modulus calculation, from Eqns. 3b and 3c, when the two-step simplified method is used

$$b_{eff}=\rho b_c=\rho h_w/2$$  \hspace{1cm} (7a)

$$b_{e1}=0.4b_{eff}=0.2\rho h_w, \quad b_{e2}=0.6b_{eff}=0.3\rho h_w$$  \hspace{1cm} (7b)

$$x=(1-\rho)h_w/2 = F_1(\rho)h_w$$  \hspace{1cm} (7c)

where

$$F_1(\rho)=(1-\rho)/2$$  \hspace{1cm} (7d)

and the length $r$ in Fig. 3 can be expressed as

$$r= b_{e2}+x/2=0.3\rho h_w+(1-\rho)h_w/4= h_w(1+\rho/5)/4=F_2(\rho)h_w$$  \hspace{1cm} (8a)

where

$$F_2(\rho)=(1+\rho/5)/4$$  \hspace{1cm} (8b)

The total area of the gross section $A_G$ can be conveniently expressed as
where \( \mu = \frac{A_w}{A_f} \) and \( F_3(\mu) = (2+\mu) \) \( F_3(\mu) = (2+\mu) \) (9a)

In Eqn. 9, \( A_w = h_w t_w \) is the area of the web and the practical range for the parameter \( \mu \) considered in this paper is \( 0 < \mu \leq 5 \). From Fig. 3, the reduction of area, \( \Delta A \), due to the ineffective section in the compression web is given by

\[
\Delta A = F_1(\rho) h_w t_w = h_w t_w (1-\rho)/2 = A_w (1-\rho)/2 = A_f (1-\rho)/2 = A_f F_4(\rho, \mu)
\] (10a)

where \( F_4(\rho, \mu) = (1-\rho) \mu/2 \) (10b)

In order to calculate the distance between the gross section centroid \( G \) and the effective cross section centroid \( G' \), first note that if \( A_{eff} \) is the effective area of the section, then

\[
A_0 = A_{eff} + \Delta A
\] (11a)

or \( A_{eff} = A_0 - \Delta A = A_f F_3(\mu) - A_f F_4(\rho, \mu) = A_f (F_3(\mu) - F_4(\rho, \mu)) \) (11b)

Since the first moment of area of \( A_0 \) above \( G \) is zero and it is equal to the sum of the first moment of areas of \( A_{eff} \) and \( \Delta A \) above \( G \), one could write

\[
-A_{eff} \Delta G + r \Delta A = 0
\] (12a)

or \( \Delta G = r \Delta A/A_{eff} \) (12b)

By using Eqns. 8, 10 and 11, Eqn.2b could be simplified to the form

\[
\Delta G = h_w \left[ \frac{\mu(1+\rho/5)(1-\rho)}{4(\mu(1+\rho)+4)} \right] = h_w F_5(\mu, \rho)
\] (12c)

Hence, the distance between \( G' \) and the centroid of the compression flange, \( d_c \), is equal to

\[
d_c = \frac{h_w}{2} + \frac{t_w}{2} + \Delta G = h_w \left( \frac{1+\xi}{2} + F_5(\mu, \rho) \right) = h_w F_6(\mu, \rho)
\] (13)

\[
F_6(\mu, \rho) = \frac{1+\xi}{2} + \left[ \frac{\mu(1+\rho/5)(1-\rho)}{4(\mu(1+\rho)+4)} \right]
\]
The second moment of area of the effective cross section with respect to \( G' \) can be obtained by subtracting the second moment of area of the ineffective area from the second moment of area of the gross section so that

\[
l_{\text{eff}} = l_g + A_g (\Delta G)^2 - \frac{x^3 t_w}{12} - \Delta A (r + \Delta G)^2
\]  

(14)

In Eqn. 14, \( l_g \) is the second moment of area of the gross section with respect to \( G \) and it can be expressed as

\[
l_g = \frac{h_w^3 t_w}{12} + 2 \left[ \frac{b_f t_f^3}{12} + A_f \left( \frac{h_w + t_f}{2} \right)^2 \right]
\]  

(15a)

Eqn. 15a can be simplified to the form of

\[
l_g = A_f h_w^2 F_f(\xi, \mu)
\]

\[
F_f(\xi, \mu) = \left[ \frac{\mu}{12} + \frac{\xi^2}{6} + \frac{(1 + \xi)^2}{2} \right]
\]  

(15b)

Combining Eqn. 15 (for \( l_g \)), Eqn. 9 (for \( A_g \)), Eqn. 12 (for \( \Delta G \)), Eqn. 7 (for \( x \)), Eqn. 10 (for \( \Delta A \)) and Eqn. 8 (for \( r \)) and simplify the results, \( l_{\text{eff}} \) can be expressed in the form

\[
l_{\text{eff}} = A_f h_w^2 \left( F_f + F_3 F_5^2 - \frac{\mu F_f^3}{12} - F_4 (F_2 + F_5)^2 \right)
\]  

(16a)

Thus, the effective section modulus \( W_{\text{eff}} \) is given by

\[
W_{\text{eff}} = \frac{l_{\text{eff}}}{d_c} = \frac{A_f h_w^2 \left( F_f + F_3 F_5^2 - \frac{\mu F_f^3}{12} - F_4 (F_2 + F_5)^2 \right)}{h_w F_6} = \frac{A_f h_w \left( F_f + F_3 F_5^2 - \frac{\mu F_f^3}{12} - F_4 (F_2 + F_5)^2 \right)}{F_6}
\]  

(16b)

By using Eqns. 1c and 16b, the ratio \( R \) defined in Eqn. 5 can now be expressed as

\[
R(\rho, \mu, \xi) = \frac{W_{\text{eff}}}{W_{\text{pl,ft}}} = \frac{F_f + F_3 F_5^2 - \frac{\mu F_f^3}{12} - F_4 (F_2 + F_5)^2}{F_6 (1 + \xi)}
\]  

(17)
Since $R$ is now expressed in terms of the parameters $\rho$, $\mu$ and $\xi$ only, whether the flanges only method is conservative or not could be verified by checking whether $R$ is greater than unity for the valid ranges of $\rho$, $\mu$ and $\xi$ and this can be easily done by using a simple spreadsheet programme. Toward this end, in this study the value of $R$ is checked for the following ranges of $\rho$, $\mu$ and $\xi$

Range for $\rho$: $0 \leq \rho \leq 1$ \hspace{1cm} (18a)

Range for $\mu$: $0 \leq \mu \leq 5$ \hspace{1cm} (18b)

Range for $\xi$: $0 \leq \xi \leq 0.2$ \hspace{1cm} (18c)

It is eventually found that the ratio $R$ is strictly greater than unity if

$$\rho \geq 0.2$$ \hspace{1cm} (19)

for any values of $\mu$ and $\xi$ within the ranges given in Eqns. 18b and 18c. From Fig. 4, such condition is equivalent to a very slender girder with $h_w/(t_w \cdot \varepsilon) \geq 680$. However, as discussed in Section 3.1, the two-step method tends to underestimate the ineffective area, especially for high value of $h_w/(t_w \cdot \varepsilon)$. Hence, it is more reasonable to use the safer condition of

$$h_w/(t_w \cdot \varepsilon) \leq 400 \quad \text{or} \quad \rho \geq 0.333$$ \hspace{1cm} (20)

as the condition that the BS’s flanges only method is guaranteed to be more conservative and safe to employ when comparing with the EC3’s effective width method. Fig. 5 shows a typical plot of the variation of $R$ against $\xi$ for different values of $\mu$ when $\rho=0.7433$ (equivalent to a value of $h_w/(t_w \cdot \varepsilon)=170$) which clearly shows that $R$ is always greater than unity. Furthermore, similar plots for different values of $\rho$ could be easily generated by using Eqn. 17 and they are all found to contain curves that are all above the line $R=1$. 

12
4. The modified flanges only method

While it is shown that $R$ is strictly greater than unity for the range of $1 \geq \rho \geq 0.333$, Fig. 5 shows that the flanges only method sometime could be too conservative in the sense that for a high value of $\mu$, $R$ could be greater than 1.3. That is, the flanges only method could underestimate the bending resistance by more than 30% and such situation is more obvious as $\rho$ increases. Hence, a more efficient design could be achieved by increasing the bending resistance predicted by the flanges only method. Note that direct calculation of $R$ using Eqn. 17 has little practical value as it is even more tedious than the direct calculation of the effective modulus. However, if one examines Fig. 5 more closely it could be found that for a given value of $\rho$ (or $h_{w}/(t_{w} \cdot \varepsilon)$) and $\mu$, the variation of $R$ with respect to the variation of $\xi$ is small. This observation suggests that, for a given value of $\rho$ (or $h_{w}/(t_{w} \cdot \varepsilon)$), a reasonable and not too conservative lower bound of $R$ could be obtained by plotting the minimum value of $R$ from all $\xi$ against $\mu$. Such a plot for the range of $124 \leq h_{w}/(t_{w} \cdot \varepsilon) \leq 400$ is shown in Fig. 6. From Fig. 6, it can be seen that for a given value of $\rho$ (or $h_{w}/(t_{w} \cdot \varepsilon)$), the minimum value of $R$ is almost a linear function of $\mu$ and increases as $\mu$ increases. On the other hand, in order to find out the influence of $h_{w}/(t_{w} \cdot \varepsilon)$, a plot of the minimum value of $R$ from all $\xi$ against $h_{w}/(t_{w} \cdot \varepsilon)$ for different values of $\mu$ is shown in Fig. 7. From Fig. 7, it can be seen that the minimum value of $R$ decreases in a quadratic manner as $h_{w}/(t_{w} \cdot \varepsilon)$ increases. Based on the above observations, a simple way to improve the efficiency of the flanges only method is to introduce a modification factor $\overline{R}(\mu, h_{w}/t_{w} \cdot \varepsilon)$ which depends on $\mu$ and $h_{w}/(t_{w} \cdot \varepsilon)$ only such that the estimated bending resistance $\overline{M}_{f,hd}$ (c.f. Eqn. 1b) is given by

$$\overline{M}_{f,hd} = \overline{R}(\mu, h_{w}/t_{w} \cdot \varepsilon) A_{f} h_{w}(1+\xi) f_{y} = \overline{R}(\mu, h_{w}/t_{w} \cdot \varepsilon) W_{pl,fl} f_{y}$$ (21)
where in Eqn. 21, the modification factor should take a simple form of

\[
\bar{R}(\mu, \frac{h_w}{t_w\varepsilon}) = 1 + \mu \left( g_1 + \frac{400 - \frac{h_w}{t_w\varepsilon}}{g_2} \right)^2
\]  

(22)

After some trials and tests, it is found that if one takes \(g_1 = 0.028\), \(g_2 = 950\), Eqn. 22 shall always remain conservative (i.e. \(\bar{R}(\mu, \frac{h_w}{t_w\varepsilon}) \leq R\)) whenever \(\rho \geq 0.333\) while the modified bending resistance calculated shall be very close (almost always greater than 92%) to the bending resistance predicted by the effective width method (Eqns. 4b and 16b). As a result, the following final form for the modification factor \(\bar{R}(\mu, \frac{h_w}{t_w\varepsilon})\) is suggested for the modified flanges only method.

\[
\bar{R}(\mu, \frac{h_w}{t_w\varepsilon}) = 1 + \mu \left( 0.028 + \frac{400 - \frac{h_w}{t_w\varepsilon}}{950} \right)^2, \quad \mu = \frac{A_w}{A_f}
\]

(23a)

\[
M_{c,Rd} = W_{eff}f_y \approx M_{f,Rd} = \bar{R}(\mu, \frac{h_w}{t_w\varepsilon}) A_f h_w (1 + \varepsilon) f_y
\]

(23b)

5. Calculation example

In this section, a calculation example is given to demonstrate the efficiency of the modified flanges only method. A symmetric plate girder shown in Fig. 8 is constructed using grade 355 steel. For this case, since \(\varepsilon = 0.814\), with \(t_w = 10\)mm, the minimum value of \(h_w\) which makes the web a Class 4 slender section is \(10 \times 0.814 \times 124 = 1009.36\)mm. Hence, in this example, the depth of the web \(h_w\) is set to vary from 1010mm to 3000mm, which is corresponding to a range of \(h_w/(t_w\varepsilon)\) from 124 to 369. By using Eqns. 1, 4 and 23, the bending resistance
predicted by the BS’s flanges only method ($M_{f,Rd}$), the EC3’s effective width method ($M_{c,Rd}$) and the modified flanges only method $\overline{M}_{f,Rd}$ are respectively computed for different values of $h_w$ and are plotted in Fig. 9. From Fig. 9, it can be seen that $M_{c,Rd}$ is always the largest but it is clumsy to calculate while $\overline{M}_{f,Rd}$ is larger than $M_{f,Rd}$ and is close to the value of $M_{c,Rd}$. Fig. 10 compares the performance of the flanges only method and the modified flanges only method, it can be seen that while the original flanges only method could only able to predict a bending resistances of approximately equal to 86% of that by the effective width method, the modified method could yield bending resistances in the range of 94% to 97% of that by the effective width method with very simple calculations.

6. Conclusions

In this paper, both the simple BS’s flanges only method and the more complicated EC3’s effective width method for calculating the bending resistance of a plate girder with Class 4 slender web without longitudinal stiffener are reviewed. A detailed analytical study and comparison are conducted for these two methods. It is found that within virtually all practical range of the web depth to thickness ratio, the flanges only method is deemed to be conservative when comparing with the effective width method. Hence, it could be safely used in the prediction of bending resistance during the design plate girder according to the Eurocodes 3. However, sometime the original form of the flanges only method could be too conservative, especially when the ratio between the web area and the flange area is high. In order to increase the efficiency of the flanges only method, a simple modification factor that is suitable for hand calculation is suggested while the essential condition of conservative prediction is strictly preserved. Finally, a calculation example is given to show that the
suggested modified method could result in better bending resistance prediction which is very close to that predicted by the more complicated effective width method.

Reference


[12] Steel Building Design: Worked examples for students, in accordance with Eurocodes, The Edited by M. E. Brettle (2008), Steel Construction Institute, UK.


Figure 1: A symmetrical plate girder with Class 4 web and with no longitudinal stiffener

Figure 2: Equivalent effective Class 3 section for the EC3’s effective width method
Figure 3: Calculation of effective cross section for a symmetric plate girder with Class 4 slender web.

\[ \rho \text{ vs } h_w/(t_w c) \]

Figure 4: A plot of \( \rho \) against \( h_w/(t_w c) \)
Figure 5: A plot of $R$ against $\xi$ for different values of $\mu$ when $\rho=0.7433$ (equivalent to $h_w/(t_w \cdot \epsilon)=170$)

Figure 6: A plot of the minimum value of $R$ from all $\xi$ against $\mu$ for different values of $h_w/(t_w \cdot \epsilon)$
Figure 7: A plot of the minimum value of $R$ from all $\xi$ against $h_w/(t_w\varepsilon)$ for different values of $\mu$

Figure 8: A calculation example
Figure 9: Bending resistance predicted by different methods

Fig. 10. Comparison of bending resistance predicted by different methods