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A Hierarchical Approach to the Multi-Vehicle SLAM Problem

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Abstract—In this paper we present a novel hierarchical solution to the Multi-Vehicle SLAM (MVSLAM) problem by extending the recently developed random finite set (RFS) based SLAM filter framework. Instead of fusing control and measurement data at each time step, we introduce a RFS Single-Vehicle SLAM based sub-mapping process, where each robot periodically produces a local sub-map of its vicinity and communicates the resultant sub-map along with the sequence of applied control commands for further fusion into a higher level MVSLAM algorithm, reducing the required network bandwidth and computational power at the fusion node. Our solution is based on the factorization of MVSLAM posterior into a product of the vehicle trajectories posterior and the landmark map posterior conditioned on the vehicle trajectory. The landmarks and the measurements are modelled as RFSs, instead of using data from exteroceptive sensors, measurements are derived from the produced local sub-maps. The vehicle trajectories posterior is estimated using a Rao-Blackwellised particle filter, while the landmark map posterior is estimated using a Gaussian mixture (GM) probability hypothesis density (PHD) filter.

I. INTRODUCTION

Almost all existing Multi-Vehicle SLAM (MVSLAM) solutions found in the robotics literature [1] [2] [3] [4] [5] [6] [7] have been developed by extending the conventional vector based Single-Vehicle SLAM algorithms such as extended Kalman filter based SLAM (EKF-SLAM) [8], FastSLAM [9] and sparse extended information filter based SLAM (SEIF-SLAM) [10]. All these Single-Vehicle SLAM algorithms solve the SLAM problem by propagating a posterior probability density of a vector consisting of the landmark map augmented with the vehicle state in time. As a result, all such algorithms require solving certain additional sub-problems such as data association, clutter filtering and map management outside the Bayesian SLAM recursion in order to produce a consistent solution. In addition, landmark detection uncertainty or data association uncertainty are not taken into account. As a result, all existing MVSLAM solutions inherit these problems.

In order to address these issues in the conventional Single-Vehicle SLAM solutions, random finite set (RFS) modelling was adopted into SLAM. The very first RFS based SLAM solution was presented by Mullane et al. in [11], where they modelled the measurements and augmented vehicle-landmark map state as RFSs. The augmented vehicle-landmark map

state was propagated in time using a probability hypothesis density (PHD) filter [12] [13] from which the state of the vehicle and the landmark locations were jointly estimated. Further improving their original solution, in [14] [15] Mullane et al. proposed a Rao-Blackwellised PHD filtering solution by factorizing the full SLAM posterior into a product of the vehicle trajectory posterior and the landmark map posterior conditioned on the vehicle trajectory. This solution addressed the map management, landmark detection uncertainty and false measurements (clutter) in a single filtering step by representing the landmark map and measurements as RFSs and modelling the landmark map transition model more natural and appropriate manner. Moreover This approach catered propagation of vehicle trajectory posterior using a particle filter and the landmark map posterior conditioned on the vehicle trajectory using a Gaussian mixture (GM) implementation [16] of a PHD filter.

In this paper, we present a novel hierarchical solution to the Multi-Vehicle SLAM problem by introducing a RFS Single-Vehicle SLAM based sub-mapping process. Instead of communicating the control and measurement data from each vehicle at each time step, the resultant sub-maps along with the sequence of applied control commands from the sub-mapping process are communicated into a higher level MVSLAM algorithm in order to jointly propagate the landmark map and the vehicle trajectories. The landmark map and the measurements are modelled as RFSs and the full MVSLAM posterior is evaluated by factorizing into a product of the vehicle trajectories posterior and the landmark map posterior conditioned on the vehicle trajectories. The vehicle trajectories are propagated using a Rao-Blackwellised particle filter and the landmark map conditioned on the vehicle trajectories is propagated using a GM implementation of a PHD filter [16].

II. THE RANDOM FINITE SET MULTI-VEHICLE SLAM PROBLEM

Let the landmark map be denoted by the set $M_k = \{m_{k,1}, m_{k,2}, ..., m_{k,l_k}\}$ at time k, where l_k denotes the number of landmarks present in the map. Let the time sequence of poses history of each vehicle be denoted by $X_{1:k}^{(r)} =$

$[X_1^{(r)}, X_2^{(r)}, ..., X_k^{(r)}]^T$, where $X_k^{(r)}$ denotes the pose of vehicle r, at time k. Let the time sequence of sets of range measurements obtained using range and bearing sensors mounted on each vehicle be denoted by $Z_{1:k}^{(r)} = [Z_1^{(r)}, Z_2^{(r)}, ..., Z_k^{(r)}]$, where $Z_k^{(r)} = \{z_{k,1}^{(r)}, z_{k,2}^{(r)}, ..., z_{k,n_k^{(r)}}^{(r)}\}$ denotes the measurement set received from vehicle r at time k, while $n_k^{(r)}$ denotes the number of measurements. Let $U_{1:k}^{(r)} = [U_1^{(r)}, U_2^{(r)}, ..., U_k^{(r)}]^T$ denote the time sequence of control commands applied to each vehicle $r, (r = 1, 2, ..., R)$ upto time k, where $U_k^{(r)}$ denotes the control command applied at time k.

Using this information, we are required to evaluate the posterior probability distribution,

$$p_{k|k}(M_k, X_{1:k}^{(1,...,R)} | Z_{1:k}^{(1,...,R)}, U_{1:k}^{(1,...,R)}, X_0^{(1,...,R)}) \quad (1)$$

in order to solve the MVSLAM problem. In (1), the abbreviations $X_{1:k}^{(1,...,R)}$, $Z_{1:k}^{(1,...,R)}$ and $U_{1:k}^{(1,...,R)}$ are given by,

$$X_{1:k}^{(1,...,R)} = (X_{1:k}^{(1)}, ..., X_{1:k}^{(R)}) \quad (2)$$

$$Z_{1:k}^{(1,...,R)} = (Z_{1:k}^{(1)}, ..., Z_{1:k}^{(R)}) \quad (3)$$

$$U_{1:k}^{(1,...,R)} = (U_{1:k}^{(1)}, ..., U_{1:k}^{(R)}) \quad (4)$$

and the abbreviation $X_0^{(1,...,R)}$ denotes the set of initial vehicle positions with respect to the global reference frame.

In a centralized approach, control commands and measurements (sub-map) from each vehicle are communicated into a dedicated fusion node at each time step. The Multi-Vehicle SLAM problem can be solved by factorizing the posterior density (1) as a product of the vehicle trajectories posterior and the landmark map posterior conditioned on the vehicle trajectories. The measurements, landmark map and the landmark map transition are modelled using RFSs. Then, the vehicle trajectories posterior can be propagated using a Rao-Blackwellised particle filter, while the landmark map posterior conditioned on the vehicle trajectories can be propagated using a gaussian mixture (GM) implementation of the Probability Hypothesis density (PHD) filter.

Although such an approach solves the MVSLAM problem in a statistically principled manner, in practice, centralized solutions introduce several drawbacks, including exponentially increasing communication bandwidth and the computational requirements which are most common. In this paper, we address these issues by presenting a novel hierarchical MVS-LAM (HMVSLAM) solution, by extending the recently developed RFS SLAM filter framework.

III. FORMULATION OF THE PROPOSED RFS HMVSLAM SOLUTION

Instead of fusing the control and observation information from all the vehicles at each time step, an approximate solution to the Multi-Vehicle SLAM problem can be obtained by fusing the local sub-maps produced by individual vehicle performing RFS Single-Vehicle SLAM (Fig.1). In this approach, each participating vehicle performs RFS Single-Vehicle SLAM by observing the landmarks in the vicinity, and periodically the
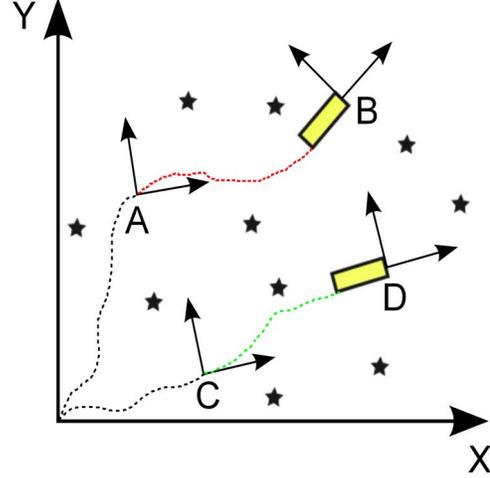

Fig. 1. RFS Single-Vehicle SLAM based sub-mapping process with two vehicles. At points A and C each vehicle initiates the RFS Single-Vehicle SLAM process with a new local frame of reference and reach points B and D respectively. Local sub-maps produced, along with the sequence of control commands are communicated into the MVSLAM algorithm. This process repeats from points B and D. Black stars denote the observed landmarks during the process.

resultant local sub-maps and the vehicle trajectories (along with the sequence of applied control commands and the origin of the current local frame of reference with respect to the global frame of reference) are communicated into a higher level fusion node, where the proposed RFS MVSLAM algorithm is run at a much lower frequency. Once communicated, each vehicle resets its local sub-map and starts a new RFS Single-Vehicle SLAM process considering the current vehicle position as the origin of the new local frame of reference. Once the fusion process is over, the updated origin of the current local frame of reference can be communicated back into each vehicle, and then each vehicle can transform their local sub-map and the vehicle trajectory into the global frame of reference if required for higher level processes. Note that the proposed approach doesn't suffer from the data incest problem since the local sub-map and the vehicle trajectory is reset after each higher level fusion step. This procedure is illustrated in Fig.2.

A. Random Finite Set Single-Vehicle SLAM filter

During each time step k to $k+1$ of the MVSLAM algorithm, each vehicle r performs RFS Single-Vehicle SLAM [14] [15] and evaluates the Single-Vehicle SLAM posterior probability density $p_{l|l}(M_l^{(r)}, Y_{1:l}^{(r)} | Z_{1:l}^{(r)}, U_{1:l}^{(r)}, Y_0^{(r)})$, which can be factorized as a product of the vehicle trajectory posterior and the landmark map posterior conditioned on the vehicle trajectory as follows.

$$\begin{aligned} &p_{l|l}(M_l^{(r)}, Y_{1:l}^{(r)} | Z_{1:l}^{(r)}, U_{1:l}^{(r)}, Y_0^{(r)}) \\ &= p_{l|l}(M_l^{(r)} | Z_{1:l}^{(1)}, Y_{0:l}^{(r)}) \times p_{l|l}(Y_{1:l}^{(r)} | Z_{1:l}^{(r)}, U_{1:l}^{(r)}, Y_0^{(r)}) \end{aligned} \quad (5)$$

where, $Y_0^{(r)}$ is the origin of the local frame of reference and the corresponding location with respect to the global frame of
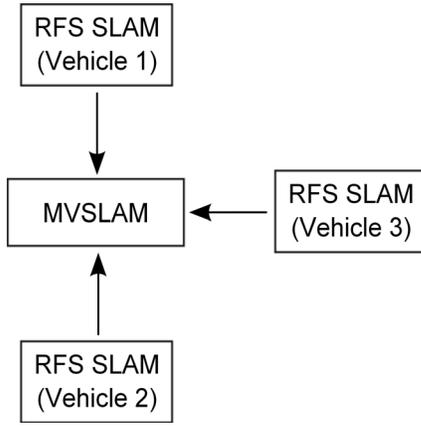
Fig. 2. The HMVSLAM algorithm for a three vehicle scenario. Each participating vehicle produces a local sub-map and a vehicle trajectory, which are used as inputs in the upper level MVSLAM filter along with the sequene of applied control commands.

reference is $X_{k-1}^{(r)}$. The landmark map and the measurements are represented as RFSs and the landmark map posterior conditioned on the vehicle trajectory is evaluated using a GM implementation of a PHD filter [16], while the vehicle trajectory posterior is propagated using a Rao-Blackwellised particle filter.

Then the resultant PHD of the RFS $M_l^{(r)}$, which represents a portion of the whole environment, along with the sequence of control commands applied into each vehicle $U_{1:l}^{(r)}$ are communicated into the higher level MVSLAM algorithm to be used as observations and control commands. Note that the actual pose of a vehicle during the sub-mapping process can be easily obtained by coordinate transformation since the origin of the local frame of reference is known.

B. Random Finite Set MVSLAM filter

For illustration purpose, let's consider the MVSLAM filter for the case of two vehicles, although it can be extended into a larger number using the proposed method. We are required to evaluate the posterior density,

$$p_{k|k}(M_k, X_{1:k}^{(1)}, X_{1:k}^{(2)}|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, U_{1:k}^{(1)}, U_{1:k}^{(2)}, X_0^{(1)}, X_0^{(2)})$$
(6)

where unlike in the centralized MVSLAM filter (1), in (6), the RFSs $Z_k^{(1)}$ and $Z_k^{(2)}$ represent the landmark locations estimated during the RFS Single-Vehicle SLAM based sub-mapping process. Using the resultant RFSs, $M_l^{(1)}$ and $M_l^{(2)}$ the observation RFSs can be obtained using the following transformation,

$$Z_k^{(1)} = M_l^{(1)}$$
(7)

$$Z_k^{(2)} = M_l^{(2)}$$
(8)

Note that the estimates of the RFSs $M_l^{(1)}$ and $M_l^{(2)}$ are independent of the estimated vehicle trajectories in the global frame of reference, yielding an independent observation at each time a sub-map is produced. Similar to (1), the RFS M_k denote the resultant landmark map (from herein after the

resultant landmark map is referred to as landmark map or global landmark map) and $X_{1:k}^{(1)}$ and $X_{1:k}^{(2)}$ represent the vehicle trajectories with respect to the global frame of reference. The control commands are derived from the sequence of control commands applied in each vehicle performing RFS Single-Vehicle SLAM as,

$$U_k^{(1)} = U_{1:l}^{(1)}$$
(9)

$$U_k^{(2)} = U_{1:l}^{(2)}$$
(10)

We proceed with factoizing the MVSLAM posterior (6) into a product of the vehicle trajectories posterior and the global landmark map posterior conditioned on the vehicle trajectories as follows.

$$p_{k|k}(M_k, X_{1:k}^{(1)}, X_{1:k}^{(2)}|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, U_{1:k}^{(1)}, U_{1:k}^{(2)}, X_0^{(1)}, X_0^{(2)})$$
$$= p_{k|k}(M_k|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)})$$
$$\times p_{k|k}(X_{1:k}^{(1)}, X_{1:k}^{(2)}|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, U_{1:k}^{(1)}, U_{1:k}^{(2)}, X_0^{(1)}, X_0^{(2)})$$
(11)

The vehicle trajectories posterior can be propagated using a Rao-Blackwellised particle filter. By modelling the dynamics of the measurements and the map using finite set statistics (FISST), the global landmark map posterior can be propagated using a GM-PHD filter [16].

1) RFS Landmark Map Transition Model: Let the RFS representing the global landmark map at time $k-1$ be denoted by M_{k-1}, then the predicted global landmark map at time k is given by,

$$M_k = \Gamma_k(X_k^{(1)}, X_k^{(2)}) \cup \left[\bigcup_{\zeta_{k-1} \in M_{k-1}} \Upsilon(\zeta_{k-1}) \right]$$
(12)

where the RFS $\Gamma_k(X_k^{(1)}, X_k^{(2)})$ denotes the newly appearing landmarks due the sub-mapping process of each vehicle between time $k-1$ and k, while the Bernoulli RFS $\Upsilon(\zeta_{k-1})$ denotes the predicted state of the landmark $\zeta_{k-1} \in M_k$ in the global landmark map.

2) RFS Landmark Measurement Model: Let M_k denote the predicted global landmark map, while $C_k^{(r)}$ denote the RFS representing the false measurements (clutter) present in the sub-map received from the rth vehicle at time k. Then corresponding measurements can be represented by the RFS,

$$Z_k^{(r)} = C_k^{(r)} \cup \left[\bigcup_{\zeta_k \in M_k} \Theta_k^{(r)}(\zeta_k) \right]$$
(13)

where $\Theta_k^{(r)}(\zeta_k)$ is a Bernoulli RFS representing the measurement corresponds to the landmark $\zeta_k \in M_k$. Since the measurements are not directly obtained from a exteroceptive sensor, the number of false alarms are much smaller than that of during the sub-mapping process.

IV. GLOBAL LANDMARK MAP POSTERIOR ESTIMATION

The global landmark map posterior conditioned on the vehicle trajectories can be propagated using a GM implementation

of a PHD filter [12] [16]. Let the global landmark map posterior at time k be denoted by,

$$p_{k|k}(M_k|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)}) \qquad (14)$$

then corresponding PHD is given by,

$$
\begin{aligned}
&v_{k|k}(\zeta_k|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)}) \\
&= \int p_{k|k}(M_k \cup \zeta_k|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)})\delta M_k
\end{aligned} \qquad (15)
$$

and the total number of landmarks in the global map in a region S is given by,

$$N_{k|k} = \int_S v_{k|k}(\zeta_k|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)})d\zeta_k \qquad (16)$$

The PHD of the global landmark map posterior (15) can be obtained using a recursive prediction and update procedure as follows.

A. Landmark Map Prediction

The predicted global landmark map posterior at time k is given by,

$$
\begin{aligned}
&p_{k|k-1}(M_k|Z_{1:k-1}^{(1)}, Z_{1:k-1}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)}) \\
&= \int f_M(M_k|M_{k-1}, X_k^{(1)}, X_k^{(2)}) \\
&\times p_{k-1}(M_{k-1}|Z_{1:k-1}^{(1)}, Z_{1:k-1}^{(2)}, X_{0:k-1}^{(1)}, X_{0:k-1}^{(2)})\delta M_{k-1}
\end{aligned} \qquad (17)
$$

and the corresponding PHD can be obtained using the PHD of the global landmark map posterior at time $k-1$ as,

$$
\begin{aligned}
&v_{k|k-1}(\zeta_k|Z_{1:k-1}^{(1)}, Z_{1:k-1}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)}) \\
&= b_k(\zeta_k|X_k^{(1)}, X_k^{(2)}) \\
&+ P_S v_{k-1|k-1}(\zeta_k|Z_{1:k-1}^{(1)}, Z_{1:k-1}^{(2)}, X_{0:k-1}^{(1)}, X_{0:k-1}^{(2)})
\end{aligned} \qquad (18)
$$

where $b_k(\zeta_k|X_k^{(1)}, X_k^{(2)})$ denotes the intensity of the newly appearing landmarks due to the local sub-mapping process and $P_S = P_S(\zeta_{k-1})$ is the probability of survival of the landmark ζ_{k-1} in the global map from time step $k-1$ to k, which is often considered as a constant.

B. Landmark Map Update

The global landmark map measurement update step is given by,

$$
\begin{aligned}
&p_{k|k}(M_k|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)}) \\
&= g_k(Z_k^{(1)}, Z_k^{(2)}|M_k, X_k^{(1)}, X_k^{(2)}) \\
&\times \frac{p_{k|k-1}(M_k|Z_{1:k-1}^{(1)}, Z_{1:k-1}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)})}{l_{k|k-1}(Z_k^{(1)}, Z_k^{(2)}|Z_{1:k-1}^{(1)}, Z_{1:k-1}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)})}
\end{aligned} \qquad (19)
$$

where the denominator in (19) is the normalization constant. Assuming that the number of false measurements (false landmarks) present in a local sub-map produced by each vehicle is Poisson distributed at an average of $\lambda^{(r)}$, and their physical

distribution given by $c^{(r)}(z^{(r)})$, the corresponding updated PHD can be obtained [17] as follows,

$$
\begin{aligned}
&v_{k|k}(\zeta_k|Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)}) \\
&= (1 - P_D^{(1)})(1 - P_D^{(2)})v_{k|k-1}(\zeta_k) \\
&+ \left[\sum_{\mathcal{P} \boxminus_2 Z_k} \omega_{\mathcal{P}} \sum_{W \in \mathcal{P}} \rho_W(\zeta_k)\right] v_{k|k-1}(\zeta_k)
\end{aligned} \qquad (20)
$$

where the abbreviation $v_{k|k-1}(\zeta_k)$ represents the PHD of the predicted global landmark map posterior (18).

The summation in (20) is taken over all so called "binary partitions" $\mathcal{P}$ of $Z_k = Z_k^{(1)} \cup Z_k^{(2)}$ (see [17] for more information). The notation, "$\mathcal{P} \boxminus_2 Z_k$" stands for "$\mathcal{P}$ partitions Z_k into binary cells W", where $W \in \mathcal{P}$ has one of the following forms,

$$W = \left\{z_k^{(1)}\right\}, \ W = \left\{z_k^{(2)}\right\}, \ W = \left\{z_k^{(1)}, z_k^{(2)}\right\} \qquad (21)$$

The values of $\rho_W(\zeta_k)$ and $\omega_{\mathcal{P}}$ are given by,

$$
\rho_W(\zeta_k)
$$
$$
= \begin{cases}
\dfrac{P_D^{(1)} l_{z^{(1)}}^{(1)}(\zeta_k)(1 - P_D^{(2)})}{1 + v_{k|k-1}[P_D^{(1)} l_{z^{(1)}}^{(1)}(1 - P_D^{(2)})]} & if \ W = \{z_k^{(1)}\} \\[3mm]
\dfrac{(1 - P_D^{(1)}) l_{z^{(2)}}^{(2)}(\zeta_k) P_D^{(2)}}{1 + v_{k|k-1}[(1 - P_D^{(1)}) l_{z^{(2)}}^{(2)} P_D^{(2)}]} & if \ W = \{z_k^{(2)}\} \\[3mm]
\dfrac{P_D^{(1)} l_{z^{(1)}}^{(1)}(\zeta_k) P_D^{(2)} l_{z^{(2)}}^{(2)}(\zeta_k)}{1 + v_{k|k-1}[P_D^{(1)} l_{z^{(1)}}^{(1)} P_D^{(2)} l_{z^{(2)}}^{(2)}]} & if \ W = \{z_k^{(1)}, z_k^{(2)}\}
\end{cases} \qquad (22)
$$

and,

$$\omega_{\mathcal{P}} = \frac{\prod_{W \in \mathcal{P}} d_W}{\sum_{\mathcal{Q} \boxminus_2 Z_k} \prod_{W \in \mathcal{Q}} d_W} \qquad (23)$$

where,

$$l_{z^{(1)}}^{(1)}(\zeta_k) = \frac{g_k^{(1)}(z_k^{(1)}|\zeta_k, X_k^{(1)})}{\lambda_k^{(1)} c_k^{(1)}(z_k^{(1)})} \qquad (24)$$

$$l_{z_k^{(2)}}^{(2)}(\zeta_k) = \frac{g_k^{(2)}(z_k^{(2)}|\zeta_k, X_k^{(2)})}{\lambda_k^{(2)} c_k^{(2)}(z_k^{(2)})} \qquad (25)$$

and,

$$
d_W = \begin{cases}
1 + v_{k|k-1}[P_D^{(1)} l_{z^{(1)}}^{(1)}(1 - P_D^{(2)})] & if \ W = \{z_k^{(1)}\} \\
1 + v_{k|k-1}[(1 - P_D^{(1)}) l_{z^{(1)}}^{(1)} P_D^{(2)}] & if \ W = \{z_k^{(2)}\} \\
1 + v_{k|k-1}[P_D^{(1)} l_{z^{(1)}}^{(1)} P_D^{(2)} l_{z^{(2)}}^{(2)}] & if \ W = \{z_k^{(1)}, z_k^{(2)}\}
\end{cases} \qquad (26)
$$

For any function $h(\zeta_k)$, $v_{k|k-1}[h(\zeta_k)]$ is given by,

$$v_{k|k-1}[h(\zeta_k)] = \int h(\zeta_k) v_{k|k-1}(\zeta_k) d\zeta_k \qquad (27)$$

The function $g_k^{(r)}(z^{(r)})$ represents the measurement likelihood and $P_D^{(r)} = P_D^{(r)}(\zeta_k|X_k^{(r)})$ denotes the probability of detection of a landmark ζ_k, by the rth vehicle during the RFS Single-Vehicle SLAM based sub-mapping process and is considered as a constant.

C. The PHD filter implementation

The PHD filter equations (18) and (20) can be implemented using a GM-PHD filter [16]. If the PHD of the global landmark map posterior at time $k-1$ and the intensity of the newly appearing landmarks in the global map due to the sub-mapping process of each vehicle can be represented by a mixture of Gaussians, then using standard results of Gaussian functions it can be shown that the PHD update (20) results in a mixture of Gaussians [16]. Hence the locations of the landmarks and the number of landmarks can be obtained in order to solve the MVSLAM problem.

V. JOINT VEHICLE TRAJECTORIES AND GLOBAL LANDMARK MAP POSTERIOR ESTIMATION

Since the global landmark map posterior is conditioned on the vehicle trajectories, they can be jointly estimated. Assume that the vehicle trajectories posterior can be propagated using a Rao-Blackwellised particle filter, represented by a set of weighted particles denoted by Ω_k as follows,

$$\Omega_k = \left\{ w_k^{[i]}, X_{1:k}^{(1),[i]}, X_{1:k}^{(2),[i]} \right\}_{i=1}^{N_s} \tag{28}$$

where $w_k^{[i]}$ is the weight of the ith particle and N_s denotes number of particles used. Assume that the weighted set of particles Ω_{k-1} represent the joint vehicle trajectories posterior at time $k-1$. Then at time k, a new joint vehicle pose is sampled from each particle as follows,

$$(X_k^{(1),[i]}, X_k^{(2),[i]}) \\ \sim f_X(X_k^{(1)}, X_k^{(2)} | X_{k-1}^{(1),[i]}, X_{k-1}^{(2),[i]}, U_k^{(1)}, U_k^{(2)}) \tag{29}$$

Using equations (9) and (10), we can rewrite (29) as,

$$(X_k^{(1),[i]}, X_k^{(2),[i]}) \\ \sim f_X(X_k^{(1)}, X_k^{(2)} | X_{k-1}^{(1),[i]}, X_{k-1}^{(2),[i]}, U_{1:l}^{(1)}, U_{1:l}^{(2)}) \tag{30}$$

Note that, although (30) is given as a joint transition model, due to the conditional independence between two vehicles' motion, each vehicle pose can be sampled using their individual motion model as follows,

$$X_k^{(r),[i]} \sim f_X^{(r)}(X_k^{(r)} | X_{k-1}^{(r),[i]}, U_{1:l}^{(r)}) \tag{31}$$

where, $r = 1, 2$ denotes the vehicle number. The new vehicle poses, $(X_k^{(1),[i]}, X_k^{(2),[i]})$, are then added to the set of particles Ω_{k-1}, creating a temporary set of particles distributed according to the proposal distribution.

It can be shown that the particle weight is given by,

$$w_k^{[i]} \\ \propto l_{k|k-1}(Z_k^{(1)}, Z_k^{(2)} | Z_{1:k-1}^{(1)}, Z_{1:k-1}^{(2)}, X_{0:k}^{(1),[i]}, X_{0:k}^{(2),[i]}) \tag{32}$$

Assuming that the number of landmarks present in M_k is Poisson distributed, by extending the approach proposed by

Mullane et al. in [14] [15], it can be shown that,

$$l_{k|k-1}(Z_k^{(1)}, Z_k^{(2)} | Z_{1:k-1}^{(1)}, Z_{1:k-1}^{(2)}, X_{0:k}^{(1)}, X_{0:k}^{(2)}) \\ = \prod_{z^{(1)} \in Z_k^{(1)}} \lambda^{(1)} c^{(1)}(z^{(1)}) \prod_{z^{(2)} \in Z_k^{(2)}} \lambda^{(2)} c^{(2)}(z^{(2)}) \\ \times \exp(N_{k|k} - N_{k|k-1} - \lambda^{(1)} - \lambda^{(2)}) \tag{33}$$

where $N_{k|k-1}$ and N_k respectively denote the number of landmarks present in the global landmark map before and after the measurement update. Hence, the joint global landmark map and vehicle trajectories posterior at time k can be represented by,

$$\left\{ w_k^{[i]}, X_k^{(1),[i]}, X_k^{(2),[i]}, v_{k|k}(\zeta_k | Z_{1:k}^{(1)}, Z_{1:k}^{(2)}, X_{0:k}^{(1),[i]}, X_{0:k}^{(2),[i]}) \right\}_{j=1}^{N_s} \tag{34}$$

VI. SIMULATION RESULTS

Performance of the proposed hierarchical MVSLAM algorithm is evaluated using a simulation. Two vehicles with identical control and sensor parameters (summarized in Table.I) are driven on two different vehicle trajectories on a simulation environment consisting of 53 randomly placed landmarks (Fig. 3). Each vehicle performs RFS Single-Vehicle SLAM based sub-mapping starting from a local frame of reference at the beginning of each time step of the RFS MVSLAM algorithm. Resultant sub-map and the sequence of control commands applied by each vehicle during a time step are used as control commands and observations in the proposed RFS MVSLAM algorithm.

It is assumed that the number of false measurements generated by the exteroceptive sensor mounted on each vehicle is Poisson distributed with an average of 3 and posesses a uniform spatial distribution in the sensor FOV. The probability of detection of a feature during the sub-mapping process is assumed to be constant and set to 0.99 and the probability of survival of a landmark from the current time step to the next is also assumed to be constant and set to 0.95. The control commands (heading and velocity) are updated at 20 Hz, and the observation update occurs at 5 Hz.

In the RFS MVSLAM filter, it is assumed that the number of false landmarks present in the sub-maps produced by each vehicle is Poisson distributed with an average of 1 and posesses a uniform spatial distribution of the entire sub-map. The probability of detection of a landmark due the sub-mapping is assumed to be constant and set to 1 and the probability of survival of a landmark in the global landmark map from current time step to the next is assumed to be constant and set to 0.9. The MVSLAM filter runs at 0.5 Hz.

Vehicle position and orientation estimation errors are shown in Fig.5 and Fig.**??** repetively. It is apparent that the estimated positional and orientation error values are acceptably smaller during the loop closure. Moreover it is clear that an improved vehicle positional accuracy can be obtained due to the overlapping regions of the vehicle trajectories.

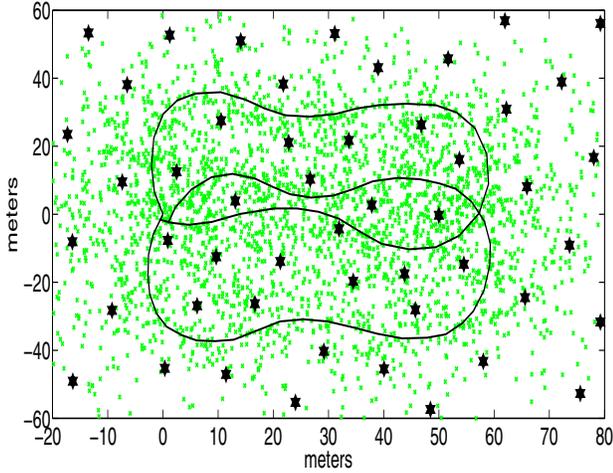| Vehicle Parameters		Values
Velocity	V	2m/s
Sensor FOV	Range (r)	0 - 30m
	Bearing (b)	$-\pi$ - $+\pi$
Control Noise	Velociy (σ_v)	0.3m/s
	Steering Angle (σ_a)	2^0
Measurement Noise	Range (σ_r)	0.3m
	Bearing (σ_b)	0.5^0

Fig. 3. Ground truth of the vehicle trajectories (in black) with randomly placed landmarks (black stars) superimposed on the actual clutter (in green) produced by each vehicle's exteroceptive sensor. The number of landmarks present in the simulation enviornment is 53.
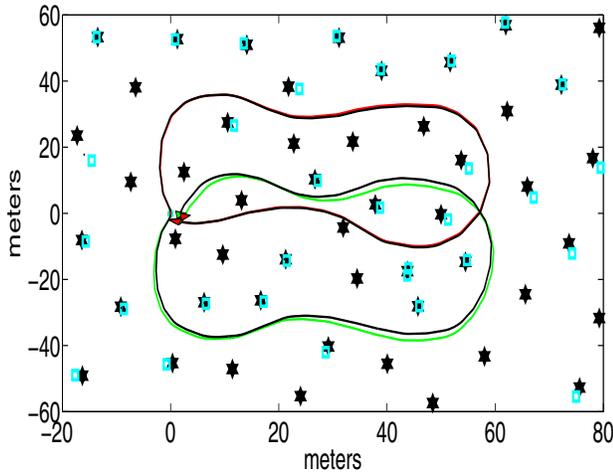

Fig. 4. A comparison of estimated vehicle trajectories of vehicle 1 (in red) and 2 (in green), superimposed on their ground truths (in black) with estimated features (cyan squares) and actual features (black stars) obtained from a sample run of the RFS MVSLAM algorithm. Due to the heavy random sensor clutter encoutered by vehicle 2 in the beginning, the estimated vehicle trajectory contains a slightly higher estimation error compared to that of vehicle 1.

VII. CONCLUSION

In this paper we have presented a noval hierarchical solution to the Multi-Vehicle SLAM problem by introducing a RFS
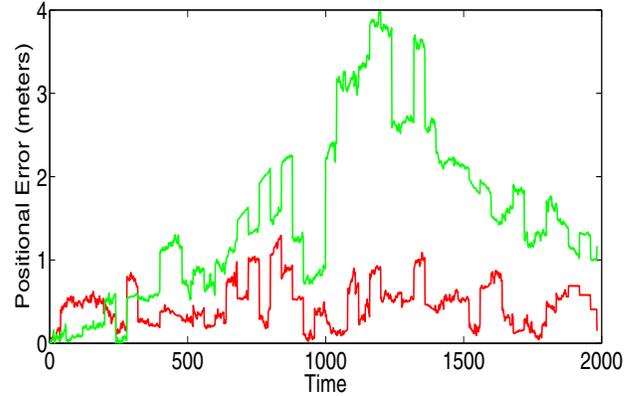

Fig. 5. Positional error of each vehicle obtained from a sample run.

Single-Vehicle SLAM based sub-mapping process. Instead of fusing the control and measurement information at each time step, resultant sub-maps along with the sequences of applied control commands during the sub-mapping process are fused at a higher level RFS based MVSLAM algorithm with a much lower frequency, resulting reduced network bandwidth and computational power. The MVSLAM posterior is evaluated by factorizing into a product of the vehicle trajectories posterior and the landmark map posterior conditioned on the vehicle trajectories. The landmark map and the measurements are modelled as RFSs, yielding a Bayesian Hierarchical MVSLAM algorithm with inbuilt map management, data association and clutter filtering.

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