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Abstract
In this study, a set of consistent geometrical models is developed for fatigue assessment of partially overlapped circular hollow section K-joint. To accomplish the fatigue assessment, an accompanying set of automatic finite element mesh generation procedures is developed. These mesh generation procedures are able to discretize the geometrical models into different types of meshes for fatigue assessments of uncracked and cracked joints. The presentation of the work is divided into two parts. In Part 1, descriptions of the geometrical models and the mesh generation procedures are given. In Part 2, attention is focused on the validation of the numerical results.

Keywords: fatigue assessment, geometrical modelling, mesh generation, partially overlapped circular hollow section K-joint
1. Introduction

Despite that the first objective of structural tubular joint design is to align the centre lines of all members at a point to avoid inducing additional bending moments [1], in heavily loaded offshore and bridge frames, the use of partially overlapped circular hollow section (CHS) K-joints are often inevitable due to geometry limitation, member congestions and high strength requirement. In fact, previous studies [2-4] shown that a well designed partially overlapped CHS K-joint, despite its higher cost, could outperform its gapped joint counterpart in both ultimate strength capacity and cost effectiveness of construction. However, relatively less research efforts had been spent on the study of fatigue performance of partially overlapped CHS K-joints. The only numerical study was done by Efthymiou and Durkin [5]. Furthermore, their suggested stress concentration factor (SCF) equations were only verified in a small scale study conducted using four strain-gauged acrylic models [6]. Recently, full scale experimental tests conducted by Sopha et al. [7] found that the SCF formula suggested by Efthymiou and Durkin [5] are not always conservative. Due to the complexity of partially overlapped CHS K-joint, until now, no large scale parametric study result on the fatigue performance of this joint type is available [8]. The problem is further complicated by the fact that reliable hot spot stress and SCF values can only be obtained from 3D solid finite element model including weld profile details [1]. Together with the fact that detailed crack models are needed for fatigue life estimation of a cracked joint, geometrical model and FE meshes generations are two main hurdles that hinder large scale parametric study of partially overlapped CHS K-joints.

The main objective of this study is to overcome the above mentioned obstacles that prevent researchers from carrying out large scale parametric study for this joint type. In order to achieve this objective, two essential tools, namely, (i) a set of consistent and realistic geometrical models and (ii) an accompanying set of automatic mesh generation procedures
are developed. The whole study will be reported in two parts. In Part 1, focus will be given to the derivation of the geometrical models and the development of the automatic mesh generation procedures. In Part 2 [9], the validity of the geometrical models and the mesh generation procedures are verified by comparing the modelling results with those obtained from full scale fatigue tests.

In the next section, the geometrical models constructed are introduced. In Section 3, summaries of different mesh generation techniques developed are presented. In Section 4, the application range of the automatic mesh generators implemented and two mesh generation examples are given. Finally, conclusions for this part of the study are presented.

2. Geometrical modelling

In order to obtain realistic geometrical models of partial overlapped CHS K-joint, a hierarchical modelling approach which follows the fabrication sequence of real joints is developed. The geometrical models are constructed hierarchically in the following steps:

1. Modelling of the intersection curves and intersection points,
2. Modelling of weld profile, and
3. Modelling of the surface crack shape and crack front details.

2.1 Modelling of the intersection curves and intersection points

Fig. 1 shows the general configuration of a partially overlapped CHS K-joint and Table 1 lists the notations used in this study. From Fig. 1, it can be seen that the outer surface of the joint consists of three intersection curves which are formed by two successive intersections between the chord and the two (the through and the overlap) braces. Firstly, the through brace intersects with the chord to form a Y-joint and generates Curve 1. Secondly, the overlap brace intersects with the Y-joint so that Curve 2, Curve 3 and a unique junction point (on each side)
are formed. By treating the formation of the final joints as the results of two successive intersections, the definitions of these curves could be derived as simple intersections between two cylindrical surfaces at a time.

In this study, the double mapping method suggested by Cao et al. [10], which was validated by Lie et al. [11] is adopted as the solution for the intersection between two cylindrical surfaces. Fig. 2 shows the two coordinate systems $X-Y-Z$ and $x-y-z$ employed to determine the intersection curve and to define the chord and brace equations.

For chord:

$$X^2 + Y^2 = R^2$$  \hspace{1cm} (1a)

For brace

$$x^2 + y^2 = r^2$$  \hspace{1cm} (1b)

The origin of the $x-y-z$ system is at the intersection point between the brace’s axis and the surface of the chord. For the $X-Y-Z$ system, its origin is lying on the chord’s axis, directly beneath the origin of the $x-y-z$ system. The relationship between the two coordinate systems can be expressed as

$$
\begin{align*}
    x &= (X - R)\cos\theta - Z\sin\theta \\
    y &= Y \\
    z &= (x - R)\sin\theta + Z\cos\theta
\end{align*}
$$

so that the equation of the brace can be expressed as

$$[(X - R)\cos\theta - Z\sin\theta]^2 + Y^2 = r^2$$  \hspace{1cm} (3)

To obtain the intersection curve equations, the chord is unfolded onto a planar $Y'Z'$ plane [10] by the following transformation:

$$R \psi = Y'$$  \hspace{1cm} (4a)

and

$$X = R\cos\left(\frac{Y'}{R}\right), \quad Y = R\sin\left(\frac{Y'}{R}\right), \quad Z = Z'$$  \hspace{1cm} (4b)

In Eqn. 4 $\psi$ is the local polar angle defined on the chord cross section. Using Eqns. 3 and 4, the equation of the intersection curve can be expressed as
\[ R^2 \left( \sin \left( \frac{Y'}{R} \right) \right)^2 + \left[ Z' \sin \theta + R \left( 1 - \cos \left( \frac{Y'}{R} \right) \right) \cos \theta \right]^2 = r^2 \]  

(5)

By introducing a secondary mapping to a circle with radius equal to \( r \) and the driving angle \( \alpha \) such that

\[ u = rsin \alpha \quad \text{and} \quad v = rcos \alpha \]  

(6a)

the intersection curve equations on the \( Y'-Z' \) plane are given by

\[ Y' = R \sin^{-1} \left( \frac{u}{R} \right) \quad \text{and} \quad Z' = \left[ v - R \left( 1 - \cos \left( \frac{Y'}{R} \right) \right) \cos \theta \right] \frac{1}{\sin \theta} \]  

(6b)

and the angle \( \phi \) is defined as

\[ \phi = \arctan \left( \frac{Y'}{Z'} \right) \]  

(6c)

Eventually, by combining Eqns. 4 and 6, the parametric equations of the intersection curve in the \( X-Y-Z \) coordinate system can be expressed in terms of \( \alpha \) as

\[ \begin{align*}
X &= R \cos \left( \sin^{-1} \left( \frac{rsin \alpha}{R} \right) \right) \\
Y &= R \sin(\sin^{-1} u) = u = rsin \alpha \\
Z &= \left[ r \cos \alpha - R \left( 1 - \cos \left( \sin^{-1} \left( \frac{rsin \alpha}{R} \right) \right) \right) \cos \theta \right] \frac{1}{\sin \theta}
\end{align*} \]  

(7)

Eqn. 7 could be used to compute the coordinates of the intersection curve for any given value of \( \alpha \) (or \( \phi \)) during mesh generation.

Eqns. 1 to 7 could be directly employed for the definition of Curve 1 formed between the chord and the through brace. For the generation of Curve 2 and Curve 3, it is necessary to define additional coordinate systems and transform Eqns. 1 to 7 appropriately. Fig. 3 shows the four coordinate systems employed for the definitions of the three intersection curves. In Fig. 3, the coordinate systems \( X-Y-Z \) and \( x-y-z \) are renamed as \( X^{(1)}-Y^{(1)}-Z^{(1)} \) and \( X^{(4)}-Y^{(4)}-Z^{(4)} \), respectively. In addition, the following two coordinate systems are used.
(1) $X^{(2)}-Y^{(2)}-Z^{(2)}$: The coordinate system on the chord axis that defines Curve 2. Its origin is on the chord’s axis, directly beneath the intersection point between the overlap brace’s axis and the surface of the chord.

(2) $X^{(3)}-Y^{(3)}-Z^{(3)}$: The coordinate system that defines Curve 3. Its origin is at the intersection point of the through brace’s axis and the overlap brace’s axis.

To locate the junction point, it is equivalent to determinate the intersection between any two of the three intersection curves. In this study, an algorithm based on the bisection method is developed to locate the intersection point between Curve 1 and Curve 2 on the $Z'-Y'$ plane (Fig. 4). As shown in Fig. 4a, four starting points $a$, $b$, $c$ and $d$ on the $Z'-Y'$ plane are first identified. The positions of these points are then updated by bisecting the angles defining the points $a$ and $c$ so that the lines $ad$ or $cb$ are shortened in every iteration. The iteration is stopped when the lengths of $|ad|$ and $|cb|$ are both less than or equal to $10^{-4}R$.

All the above definitions of intersection curves and the junction point are derived based on pure surface to surface intersection. If the mid-surface equations of the chord and braces are substituted into the above equations, the geometrical model obtained could be employed to generate a shell FE model for the ultimate strength analysis. However, in case that a solid model is needed, the following three pairs of intersections between the inner and the outer surfaces of the CHS are needed.

(1) Intersections between the inner and outer surfaces of the through brace with the outer surface of the chord.

(2) Intersections between the inner and outer surfaces of the overlap brace with the outer surface of the chord.

(3) Intersections between the inner and outer surfaces of the overlap brace with the outer surface of the through brace.
The final outcomes of these three pairs of intersections are the formation of six intersection curves and two junction points (on each side of the joint) as shown in Fig. 5.

2.2 Modelling of weld profile

2.2.1 Minimum weld thickness

Welding is the most important step in the construction of tubular joints as its quality shall critically affects the ultimate strength of the joints. In terms of fatigue performance, it is found that the hot spot stress is sensitive to the weld thickness \([1,9]\) of the joint. For construction of tubular joints, in order to ensure complete load transfer among all members, the minimum weld thickness requirements are defined in all welding codes \([12,13]\). The minimum weld thickness along the intersection curve is specified as a function of the dihedral angle \(\gamma\). As shown in Fig. 6, \(\gamma\) at any point on the intersection curve is defined as the angle between the tangent planes of the two surfaces at that point. In general, the required weld thickness, \(T\), is specified as

\[ T = k(\gamma) t_b \]  

where \(t_b\) is the thickness of the brace and \(k(\gamma)\) is a thickness factor. In actual construction, it is compulsory that the thickness materialized, \(T_w\), should satisfy the following conditions:

If the American Welding Society code \([12]\) is used:

\[ T_w = k_w t_b \geq T_{AWS} = k_{AWS}(\gamma) t_b \quad \text{or} \quad k_w \geq k_{AWS}(\gamma) \]  

(9a)

If the American Petroleum Institute code \([13]\) is used:

\[ T_w = k_w t_b \geq T_{API} = k_{API}(\gamma) t_b \quad \text{or} \quad k_w \geq k_{API}(\gamma) \]  

(9b)

In Eqn. 9, \(k_w\) is the actual thickness factor achieved. Table 2 shows the minimum values of the factors \(k_{AWS}\) and \(k_{API}\) specified by the American Welding Society and the American Petroleum Institute codes, respectively. Note that the thickness factor \(k_{AWS}\) is continuous but not a smooth function of \(\gamma\). However, in practice, in order to avoid sudden change in weld thickness, the
actual weld thickness (and \( k_w \)) achieved in a real junction [14] is a smooth function greater than \( k_{AWS}(\gamma) \).

2.2.2 Weld thickness computation

As shown in Fig. 7, along a typical intersection curve between the brace and the chord, two weld paths, the *inner weld path* and the *outer weld path* are formed. Fig. 7 also shows the two theoretical intersection (dashed) curves which are obtained by considering the pure geometrical intersection between the chord and the brace. As shown in Fig. 7, for a given value of driving angle \( \phi \) (Eqn. 6c), the line \( OF \) intersects with the weld profile at four points \( A_i, W_i, A_o \) and \( W_o \). Figs. 8 and 9 show the cross sections corresponding to the two possible ranges of the *outer dihedral angle* \( \gamma_o \) at the outer intersection point \( A_o \). From Figs. 7 to 9, it can be seen that the thickness of the welding, \( T_w \), is equal to \( |W_iW_o| \) and can be expressed as

\[
T_w = T_1 + T_2 + T_3
\]

In Eqn. 10, \( T_1 = |A_oA_i| \) is the *contact thickness* which is generated based on the pure geometrical intersection while \( T_2 \) and \( T_3 \) are the additional *outer* and *inner thicknesses* generated by welding. Note that while it is required that \( T_2 > 0 \), when \( \gamma_o < 90^\circ \), it is possible that \( T_3 \leq 0 \) (Fig. 9). By expressing Eqn. 10a in a form similar to Eqns. 8 and 9, one has

\[
T_w = T_1 + T_2 + T_3 = k_1 t_b + k_2 t_b + k_3 t_b = (k_1 + k_2 + k_3) t_b = k_w t_b
\]

such that \( k_w = k_1 + k_2 + k_3 \) and \( k_i \), \( i = 1, 2, 3 \) are the weld factors for \( T_i \), respectively. In this study, the geometrical model adopted is a major extension from the works by Lie et al. [14]. In Lie’s model, it is assumed that the inner dihedral angle \( \gamma_i \) and the outer dihedral angle \( \gamma_o \) are identical so that the points \( A_i, W_i, A_o \) and \( W_o \) are lying on a straight line so that \( k_1 \) is given by

\[
k_1 \approx 1 / \sin \gamma_i
\]

While it was shown that Eqn. 11 could lead to realistic model in most cases, it is unrealistic to assume that \( \gamma_i \neq \gamma_o \) when \( r/R > 0.8 \). In here, a more accurate expression for \( k_1 \) is suggested [15].
for the correction of such assumption so that

\[ k_1 = \begin{cases} 
\frac{1}{\sin\gamma_i}, & \gamma_0 \leq \frac{3\pi}{4} \\
C_i\gamma_i + C_2, & \frac{3\pi}{4} \leq \gamma_0 \leq \pi
\end{cases} \quad (12a) \]

\[ C_i = \frac{\cos\gamma_i}{\sin^2\gamma_i} \bigg|_{\gamma_i = \frac{3\pi}{4}} - \frac{3}{5(\pi - \theta_{\min})}, \quad C_2 = \frac{3}{5} \left[ 1 - \left( \frac{0.75\pi - \theta_{\min}}{\pi - \theta_{\min}} \right)^2 \right] + \frac{1}{\sin\gamma_i} \bigg|_{\gamma_i = \frac{3\pi}{4}} \quad (12b) \]

where \( \theta_{\min} = \pi/6 \) is the minimum intersecting angle allowable between the two CHS.

In order to derive realistic expressions of \( k_2 \) and \( k_3 \), besides referring to the requirements specified by the welding codes, measurements were conducted on the actual weld profiles of many full scale and scaled joints [7,15-17]. Eventually, the following expressions of \( k_2 \) and \( k_3 \) are adopted:

\[ k_2 = \begin{cases} 
\frac{3}{5} \left[ 1 - \left( \frac{\gamma_1 - \theta_{\min}}{\pi - \theta_{\min}} \right)^2 \right], & \text{for } \gamma_o \leq \frac{3\pi}{4} \\
C_i\gamma_i + C_2 \cdot \frac{1}{\sin\gamma_i}, & \text{for } \frac{3\pi}{4} \leq \gamma_o \leq \pi
\end{cases} \quad (13a) \]

\[ k_3 = -\frac{1}{4} \left[ 1 - \left( \frac{\gamma_1 - \theta_{\min}}{\pi/2 - \theta_{\min}} \right)^{0.4} \right] \quad (13b) \]

From Eqn. 13, it can be seen that as

\[ \left( \frac{\gamma_1 - \theta_{\min}}{\pi - \theta_{\min}} \right) \quad \text{and} \quad \left( \frac{\gamma_1 - \theta_{\min}}{\pi/2 - \theta_{\min}} \right) < 1.0 \quad (14a) \]

one has

\[ \left( \frac{\gamma_1 - \theta_{\min}}{\pi - \theta_{\min}} \right)^2 \quad \text{and} \quad \left( \frac{\gamma_1 - \theta_{\min}}{\pi/2 - \theta_{\min}} \right)^{0.4} \quad (14b) \]

Hence, it can be concluded that

\[ |k_2| > |k_3| \quad \text{and} \quad |k_3| < 1 \quad (14c) \]

Eqn. 14c implies that the amount of fill in (cut out) along the inner weld path is smaller than the fill in along the outer weld path and the thickness of the brace. In fact, more detailed study
shows that $|k_2| >> |k_3|$ and $|k_3| << 1$. For example, assume that $\gamma_0 \leq 3\pi/4$, $\theta_{\text{min}} = \pi/6$ and $\gamma_i \approx 5\pi/12$ so that $\left(\frac{\gamma_i - \theta_{\text{min}}}{\pi - \theta_{\text{min}}}\right) = 0.5$. Simple calculations give $k_2 = 0.45$, $k_3 = -0.0272$ and $\left|\frac{k_2}{k_3}\right| = 16.54$. This result agrees with the characteristics of groove weld procedure used in tubular joints [12,13].

For partially overlapped CHS K-joints, results of experimental studies [7,9,16] show that the peak hot spot stress might occur at the brace side of the weld. Hence, the weld thickness on the brace side should be modelled. As shown in Figs. 8 and 9, the inner and outer weld paths on the brace side are determined by the points $B_i$ and $B_o$, respectively. For the outer weld path, by inspection of the actual outer weld thickness [7,15], it was found that the outer weld thickness on the chord and brace sides are almost similar. Hence, the distance $|A_oB_o|$ is taken as $T_2$. For the inner weld path, it was found that the length $|A_iB_i|$ could be taken as

$$|A_oB_o| = T_4 = k_4 t_b$$

so that

$$k_4 = \begin{cases} \frac{k_3}{\cos\gamma_i} & \gamma_i \leq \pi/4 \quad \text{or} \quad \gamma_i \geq \pi/4 \\ \frac{k_3}{\cos(3\pi/4)} & \text{otherwise} \end{cases}$$

After the thickness factors are defined, the coordinates of the points $W_i$ and $W_o$ which define the final weld path could be computed using the algorithm shown in Box 1.

(i) Calculate $k_1 (T_1)$ using Eqn. 12.

(ii) Determine the coordinates of $A_i = (Y^i_{A_i}, Z^i_{A_i})$ and $A_o = (Y^o_{A_o}, Z^o_{A_o})$ on the $Y'-Z'$ plane based on the geometry of the CHS, the intersection angles and $k_1$.

(ii) Determine the projection angles $\beta_i$ and $\beta_o$ corresponding to $\phi$ as shown in Fig. 10.

(iii) Calculate $k_2 (T_2)$ and $k_3 (T_3)$ using Eqn. 13.

(iv) Compute the coordinates of $W_i$ and $W_o$ using the following equations
\[
Y_{_{W}}' = Y_{_{A}}' + T_3 \sin \beta, \quad \text{and} \quad Z_{_{W}}' = Z_{_{A}}' + T_3 \cos \beta, \quad (16a)
\]
\[
X_{_{W}}' = R \cos \frac{Y_{_{W}}'}{R}, \quad Y_{_{W}}' = R \sin \frac{Y_{_{W}}'}{R}, \quad Z_{_{W}}' = Z_{_{W}}' \quad (16b)
\]
\[
Y_{_{W0}}' = Y_{_{A}}' + T_2 \sin \beta, \quad \text{and} \quad Z_{_{W0}}' = Z_{_{A}}' + T_2 \cos \beta, \quad (16c)
\]
\[
X_{_{W0}}' = R \cos \frac{Y_{_{W0}}'}{R}, \quad Y_{_{W0}}' = R \sin \frac{Y_{_{W0}}'}{R}, \quad Z_{_{W0}}' = Z_{_{W0}}' \quad (16d)
\]

Box 1: Computation of the coordinates of \(W_i\) and \(W_0\)

From the results obtained in experimental measurements, it was found that Eqns. 12 and 13 could give smooth weld thickness exceeds the minimum requirements set out in design codes [12,13] but lower than the thickness achieved in most constructed joints.

By including the three welding curves, the completed geometrical model with weld details near the joint junction is shown in Fig. 11. For each welding curve, due to the weld thickness, four weld paths are formed and they intersect at six junction points (three are visible from outside). For example, for Curve 1 between the through brace and the chord, the following four weld paths could be identified.

(i) The outer welding path on the chord side, Curve 1CO (corresponding to \(W_o\))

(ii) The Inner welding path on the chord side, Curve 1CI (corresponding to \(W_i\))

(iii) The outer welding path on the brace side, Curve 1TBO (corresponding to \(B_o\))

(iv) The inner welding path on the brace side, Curve 1TBI (corresponding to \(B_i\))

2.2.3 Modelling of surface crack shape and crack front details

Fatigue test results [7,15-17] shown that for tubular joints, peak hot spot stress develops along the outer weld paths (the weld toe) and surface crack is always initiated at the peak hot spot stress location. Furthermore, as the 3D curved crack front propagates along the weld path and penetrates into the cracked section, it traces out a 3D curved crack surface. To obtain a
concise mathematical description of the crack surface, a cross-section normal to the outer weld toe \((W_o)\) of the joint is considered (Fig. 12a) and a local coordinate system \(x'-y'-s'\) is set up with the origin at \(W_o\). The \(x'\) axis is parallel to the \(X\) axis while the \(y'\) and the \(s'\) axes are perpendicular and tangential to the outer weld path at \(W_o\), respectively. Furthermore, \(\phi\) is the angle between the \(s'\) axis and the \(Y\) axis. From the results of post-failure measurements done on cracked joints, it was found that the cross section of a surface crack (Fig. 12b) is a curve \(W_oD'\) so that the curved crack length, \(a'\), and its projection on the line \(OW_o\) (the projected crack length), \(a\), are equal to \(|W_oD'|\) and \(|W_oD|\), respectively. Furthermore, the crack surface angle \(\angle D'W_oD = \omega\) varies along the outer weld path and is a function of \(a\). However, repeated tests have shown that for CHS joint, \(\omega\) only varies in a small range both along the length and depth of the crack. Thus, in the current geometrical model, \(\omega\) is assumed to be a constant such that

\[
\omega = \omega_0 \quad \text{and} \quad a' = a / \cos \omega_0
\]

In Eqn. 17, \(\omega_0\) is the value of \(\omega\) when \(a=0\). From Eqn. 17 and Fig. 12, for a given value of \(a\) the relationship between the \(X\)-\(Y\)-\(Z\) and the \(x'\)-\(y'\)-\(s'\) coordinate systems can be expressed as

\[
\begin{bmatrix}
X' \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
x' \\
y' \\
s'
\end{bmatrix}
+ \begin{bmatrix}
\text{atan} \omega \\
-a \\
0
\end{bmatrix}
\]

In order to obtain a realistic model for the crack front shape, the alternating current potential drop technique was employed to track the advancement of surface crack during full scale fatigue tests [7,9,15-17]. In addition, the failed joints were opened up and the final crack surface and front were measured. The results obtained indicate that the crack front could be modeled as an unsymmetrical bi-elliptical curve on the unfolded \(s'\)-\(t'\) plane of the crack surface (Fig. 13). The definition of the crack front could be established by knowing the relative distances between a fixed reference point \(M\) and the two crack trips \((l_{cr1}\) and \(l_{cr2}\), and
the deepest point of the crack \( (l_{cr0}) \). Hence, for a given value of \( a' \), the position of the crack front is defined by the following parametric equations in the unfolded \( s'-t' \) plane

For \( l_{cr1} \leq s' \leq l_{cr0} \):

\[
\frac{(s'+l_{cr0})^2}{(l_{cr1} + l_{cr0})^2} + \frac{(t')^2}{(a')^2} = 1
\]  

(19a)

For \( l_{cr0} \leq s' \leq l_{cr2} \):

\[
\frac{(s'+l_{cr0})^2}{(l_{cr2} + l_{cr0})^2} + \frac{(t')^2}{(a')^2} = 1
\]  

(19b)

In practice, the values of \( l_{cr0}, l_{cr1}, l_{cr2} \) and \( a' \) could be obtained by in-situ measurements.

3. **Automatic mesh generation**

The existence of three intersection curves and intersection points makes the mesh generation of a partially overlapped CHS K-joint a nontrivial task. Similar to the derivation of the geometrical models, the mesh generation steps were carried out in a hierarchical manner. The starting point is the creation of a surface mesh, which is then converted into a solid mesh using an extrusion algorithm [18]. Eventually, additional details such as weld profile and surface crack are added to the mesh. This method of mesh generation is flexible in such a way that at different stages, meshes with different levels of details can be generated and employed for different applications. For example, the surface mesh (SURF_0W) could be employed for the quick assessment of the ultimate strength of the joint. In case that a quick assessment of SCF is needed, the hybrid model (HYBR_1W) which contains mainly surface elements and a small numbers of solid elements could be used. For a more detailed study on the distribution of hot spot stress, full solid meshes without (SOLID_0W) or with (SOLID_1W) welding details could be created by using the extrusion algorithm [18] and a weld profile meshing procedure. Eventually, when a full numerical study for a crack joint is needed, a solid mesh with weld and crack details (SOLID_CR) can be generated by employing a specially designed procedure that inserts the crack details at appreciate location along the weld toe.
3.1 Surface mesh generation

As the allowable overlap ratio could vary from 20% to 100% [1], unlike a gapped K-joint, the connection topology among the chord and the braces of a partially overlapped joint does not follow any predefined pattern. Hence, the use of a structured mesh generation procedure designed for a fixed connection topology is deemed to be infeasible. Therefore, a more complicated unstructured surface mesh generation algorithm [19,20] was selected as the basic surface mesh generation kernel. In order to facilitate the discretization procedure of the weld and crack details, the outer boundary surface is first considered and is discretized into surface mesh to form a reference to other surfaces. For the generation of the outer boundary mesh, the entire surface is divided in several zones (Fig. 14) and they are discretized one by one into a set of compatible surface meshes.

In the generation of the surface mesh, triangular elements are generated to cover all zones. After the outer surface mesh is generated, it is scaled down to fit the mid-surface of the model and forms the mesh SURF_0W (Fig. 14) without weld details. In case that the hybrid mesh with weld details (HYBR_1W) are needed, additional layers of quadrilateral shell elements will be generated along the three weld lines (Weld 1, 2 and 3) and the junction point (triangular shell elements only) where the weld profile will be imposed on the surface mesh (Fig. 15). The hybrid mesh is then created by further processing and converting those additional quadrilateral and triangular elements into solid prism elements and tetrahedral elements, respectively (Fig. 16).

3.2 Solid mesh generation

If solid meshes are required, the outer surface mesh will be converted into solid mesh by connecting the corresponding nodes on the respective boundary surface meshes using an extrusion algorithm specially designed for application to thin-walled structures [18]. As the surface meshes are divided into different zones and the surface mesh generator employed
allows users to specific graded element size within different zones, users could achieve an optimized element size distribution along the surface direction. A common practice is to specify a denser element distribution along all the intersection curves. In this study, the element size in the surface direction along the intersection curves is set equal to the weld thickness and increases linearly further from the intersection curves. Similar to surface mesh generation, the conversion procedure will generate the solid mesh first by dividing the whole mesh into four regions or *spaces* in order to facilitate the extrusion procedure. These four spaces are shown in Fig. 17 and a summary of them are given below.

- **Space 1**: This is the outer space for the joint and is defined by the outer surfaces of the three CHSs. The surface meshes of this space are identical to the outer surface meshes of the joint and are always generated first.

- **Spaces 2, 3 and 4**: These are the inner spaces corresponding to the chord section, the through brace and the overlap brace, respectively. The surface meshes for them are generated after the outer surface meshes are generated.

After the outer and inner surface meshes corresponding to the four spaces are generated, the mid-surface mesh will be formed which is then followed by the extrusion process. Two separate extrusions are used for the formation of solid elements: extrusion between the outer surface meshes and the mid surface meshes and then between the inner surface and the mid-surface meshes. Note that the extrusion procedure allows users to specify and refine the numbers of layers of elements at any part of the solid mesh. In this study, the chord and brace members are divided into four layers in the thickness direction within a radius of three times the weld thickness along all the intersection curves. Further always this distance, the sections are divided into two layers of elements. Fig. 17 shows the internal view for a typical extruded mesh without weld details (SOLID_0W).
3.3 Weld details mesh generation

In the weld detail mesh generation, the outer part for the four weld lines are discretized as strips of 3D prism elements, while the junction point is discretized by tetrahedral elements (Fig. 18). These strips of prism elements and the tetrahedral elements are merged with the solid mesh SOLID_0W to represent the fill in along the outer weld path (Fig. 19). For the inner weld path, as the fill in (or cut out) is much less than the outer weld path and the thickness of the sections, it is formed by slightly shifting the positions of those nodes lying the chord-brace intersection curves without adding of any new element (Fig. 20).

3.4 Surface crack mesh generation

From experimental studies [7], it is found that a surface crack can be formed at either the brace or the chord side of the weld path. The actual location is dependent upon the joint type and loading. Hence, the surface crack mesh generator was developed in such a way that it would able to generate meshes with surface cracks on either the brace or the chord side of the weld path. The surface crack mesh generator will create a local structured mesh surrounding the crack surface and crack front in the following steps:

(1) Identification of extracted zone

As mentioned in Section 2.2.3, once the location of a surface crack is known, the two crack tips could be determined by specifying their positions relative to the reference point M. Once the positions of the crack tips are defined, an extraction algorithm will search and determine the elements to be extracted to form an extracted zone (Figs. 21a). The extraction algorithm also adjusts the positions of the nodes for those elements containing the crack tips such that the best element aspect ratio is achieved in the subsequent generation steps. However, the boundary nodes of the extracted zone are kept unchanged in order to re-merge it back to the final mesh after the crack surface and the crack mouth blocks are generated.
(2) Generation of connection block

Since the solid mesh SOLID_1W is developed from an unstructured surface mesh but the crack surface and mouth are discretized into a structured mesh, a connection block (Fig. 21b) is generated for the smooth transition of element size and to maintain element compatibility.

(3) Generation of crack surface block

The crack surface block (Fig. 21c) is generated by considering the geometrical model of the surface crack. A tailor-made program is employed to determine the location of the crack surface. This program also determines the numbers of elements along the crack length direction. By default, at least 32 elements are generated along the crack. Moreover, based on the actual length of the crack front, this program is able to adaptively generate from 4 to 64 elements for very short or long cracks.

(4) Generation of crack mouth block

The mesh around the crack front is designed as a spider web consisting of three concentric rings of eight hexahedral elements (Figs. 21d and 21e) with collapsed quarter point elements [21] located in the innermost ring. Such mesh configuration was found to be efficient and reliable for the estimation of the stress intensity factors by either using the quarter-point displacement technique or the J-integral technique [7,9,15].

Finally, after all crack blocks are generated, they are combined together and inserted back to the extracted mesh to form the solid mesh with weld and crack details (SOLID_CR).

4. Application range of the mesh generation procedure and mesh generation examples

The automatic mesh generator implemented is applicable to a wide range of geometrical parameters commonly encountered in onshore and offshore structures. For the generation of the crack mouth block mesh, in order to ensure the quality of the elements there, at least four elements are generated to model the crack. Since the element size along the surface direction
is set equal to the weld thickness, this implies that the shortest crack length that could be modeled by the mesh generation program is $4T_w$. On the other hand, since at each crack tip a layer of transition elements with size approximately equal to $T_w$ is needed to connect the crack mouth block to the crack surface block, the longest crack length that could be modeled is $l_{int} - 2T_w$ where $l_{int}$ is the length of the weld. Along the thickness direction, in order to ensure that all the elements surrounding the crack mouth are well shaped, the allowable lower and the upper limits of the ratio $a'/t$ are 0.15 and 0.85, respectively. A summary of the geometrical application range of the mesh generator is given in Table 3.

One notable feature regarding the proposed mesh generation scheme is that it is applicable to the special case of identical chord and braces dimensions even when the joint has a high overlap ratio of $p=80\%$. Such geometrical configurations were not covered by the mesh generator created in the previous works by Cao et al. [10] and Lie et al. [11]. An example of the mesh with same chord and braces dimensions and $p=80\%$ is given in Fig. 22. Furthermore, as the current mesh generation scheme is also designed for different values of $\theta_1$ and $\theta_2$, it could be used to generate meshes for partially overlapped N-joints as shown in Fig. 23. Note that the meshes shown in Figs. 22 and 23 are the “raw” meshes that consist of linear elements with straight edges so that the ends of the CHS appear to be poorly represented. However, before they are used in the finite element analysis, they will be first converted to quadratic elements with curved edges. Mid-side nodes will be inserted into all the elements and their positions will be adjusted to fall on the surface of the CHS so that a much better representation will be achieved.

5. Conclusions

In this paper, a set of consistent and reliable geometrical models is developed to form the basis for a series of hierarchical mesh generation procedures for partial overlapped CHS K-
joints. Standard surface intersection procedure is employed to define the basic joint
cconfiguration. A new approach is adopted for the geometrical description of the welding
details. For the description of the crack surface and front, a general model for the inclined
crack surface and an unsymmetrical crack front are proposed. A set of hierarchical mesh
generation procedure based on the geometrical models also developed. The mesh of a
partially overlapped CHS K-joint is first created in the form of a surface mesh, which is then
converted into a solid mesh by an extrusion algorithm. Weld and crack details are then added
subsequently using a series of mapping procedures. The mesh generator is able to handle a
wide range of parameters including the case of identical chord and braces dimensions with a
large overlap ratio. It could able to generate a solid mesh with weld details and surface crack
of arbitrary length and locations on either the brace side or the chord side of the weld path. In
Part 2 [9], the accuracy and reliability of the proposed geometrical models and mesh
generation procedures are validated by comparing the numerical results generated by them
with the experimental results obtained by static and fatigue tests on two full scale partially
overlapped CHS K-joints.

References

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strength using numerical modelling), Journal of Offshore Mechanics and Artic


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Figure 22. A mesh of a partially overlapped CHS K-joint having $R=R_1=R_2$, $p=80\%$

Figure 23. A mesh of a partially overlapped CHS N-joint
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X-Y-Z) or (X^{(1)}-y^{(1)}-Z^{(1)})</td>
<td>Coordinate system defining the chord and through brace intersection curve (Curve 1)</td>
</tr>
<tr>
<td>(Y'-Z)</td>
<td>Unfold plane coordinate system of chord</td>
</tr>
<tr>
<td>(x-y-z) or (X^{(4)}-Y^{(4)}-Z^{(4)})</td>
<td>Coordinate system on the axis of the through brace</td>
</tr>
<tr>
<td>(X^{(2)}-y^{(2)}-Z^{(2)})</td>
<td>Coordinate system defining the chord and the overlap brace intersection curve (Curve 2)</td>
</tr>
<tr>
<td>(X^{(3)}-Y^{(3)}-Z^{(3)})</td>
<td>Coordinate system defining the through and the overlap braces intersection curve (Curve 3)</td>
</tr>
<tr>
<td>(x'-y'-s')</td>
<td>Local coordinate system for crack surface definition</td>
</tr>
<tr>
<td>(s'-t')</td>
<td>Unfold 2D coordinate system for crack front definition</td>
</tr>
<tr>
<td>(u,v)</td>
<td>Local planar coordinate system defining joint intersection</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Polar angle defined in the (u-v) coordinate system</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Polar angle on the chord cross section</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Angle corresponding to (\alpha) on the (Y'-Z) plane (Fig. 4)</td>
</tr>
<tr>
<td>(d)</td>
<td>Distance between the origins of the coordinate systems (OXYZ) and (O^{(2)}X^{(2)}Y^{(2)}Z^{(2)})</td>
</tr>
<tr>
<td>(R) or (R_0)</td>
<td>Radius of chord</td>
</tr>
<tr>
<td>(r) or (R_1)</td>
<td>Radius of through brace</td>
</tr>
<tr>
<td>(R_2)</td>
<td>Radius of overlap brace</td>
</tr>
<tr>
<td>(\theta) or (\theta_1)</td>
<td>Intersecting angle between through brace and chord</td>
</tr>
<tr>
<td>(\theta_2)</td>
<td>Intersecting angle between overlap brace and chord</td>
</tr>
<tr>
<td>(g)</td>
<td>Length of the overlap between overlap and through brace</td>
</tr>
<tr>
<td>(h)</td>
<td>Diameter of the overlap brace projected on chord axis</td>
</tr>
<tr>
<td>(p)</td>
<td>Overlap ratio, (p = g/2R_0)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Angle between the normal of the chord-brace intersection curve and the horizontal (Z)-axis in the (Y-Z) plane (Fig. 15)</td>
</tr>
<tr>
<td>(\gamma, \gamma_6, \gamma_9)</td>
<td>Dihedral angles (Figs. 9, 12 and 13)</td>
</tr>
<tr>
<td>(e)</td>
<td>Eccentricity between overlap and through brace to the chord axis</td>
</tr>
<tr>
<td>(T_{AWS}, T_{API})</td>
<td>Minimum weld thickness required by the AWS/API codes</td>
</tr>
<tr>
<td>(T_w)</td>
<td>Actual/modelled weld thickness</td>
</tr>
<tr>
<td>(T_1)</td>
<td>Original contact thickness</td>
</tr>
<tr>
<td>(T_{2}, T_{3})</td>
<td>Fill in/cut out on the outer and inner weld path, respectively.</td>
</tr>
<tr>
<td>(k_1)</td>
<td>Surface contact thickness factor</td>
</tr>
<tr>
<td>(k_2, k_3)</td>
<td>Outer and inner modification factors for weld model on the chord side</td>
</tr>
<tr>
<td>(k_{AWS}, k_{APS})</td>
<td>Actual/modelled thickness factors for the AWS/API codes</td>
</tr>
<tr>
<td>(k_{Tw})</td>
<td>Modification factor of modelled weld thickness</td>
</tr>
<tr>
<td>(\theta_{\text{min}})</td>
<td>Minimum intersecting angle = 30°</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Angle between the (s') and the (Y) axis for crack surface definition</td>
</tr>
<tr>
<td>(\omega)</td>
<td>Crack surface angle</td>
</tr>
</tbody>
</table>

Table 1. List of symbols used in this study
<table>
<thead>
<tr>
<th>Dihedral Angle $\gamma$</th>
<th>$k_{API}$</th>
<th>$k_{AWS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50°-135°</td>
<td>1.25</td>
<td>$1/sin\gamma$</td>
</tr>
<tr>
<td>35°-50°</td>
<td>1.5</td>
<td>$1/sin\gamma$</td>
</tr>
<tr>
<td>Below 35°</td>
<td>1.75</td>
<td>2.0 (for $\gamma &lt; 30^\circ$)</td>
</tr>
<tr>
<td>Over 135°</td>
<td>Build out to full thickness but need not exceed 1.75</td>
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</tr>
</tbody>
</table>

Table 2. Minimum weld thickness specified by the American Welding Society and the American Petroleum Institute codes

<table>
<thead>
<tr>
<th>Geometrical parameter</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intersecting angles</td>
<td>$30^\circ \leq \theta_1, \theta_2, \theta \leq 90^\circ$</td>
</tr>
<tr>
<td>Ratio of brace to chord radius</td>
<td>$0.1 \leq \beta_1, \beta_2 \leq 1.0$</td>
</tr>
<tr>
<td>Ratio of brace’s thickness to brace’s radius</td>
<td>$0.03 \leq 1/\gamma \leq 0.25$</td>
</tr>
<tr>
<td>Overlap ratio</td>
<td>$20% \leq p \leq 80%$</td>
</tr>
<tr>
<td>Position of crack</td>
<td>Anywhere along the welds at either the chord side or the brace side</td>
</tr>
<tr>
<td>Crack surface angle</td>
<td>$-20^\circ \leq \omega \leq 20^\circ$</td>
</tr>
<tr>
<td>Crack length</td>
<td>$4T_w \leq l_{ct} \leq l_{int} - 2T_w$</td>
</tr>
<tr>
<td>Crack depth</td>
<td>$0.15 \leq a'/t \leq 0.85$</td>
</tr>
</tbody>
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