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Adaptive Backstepping Stabilization of Nonlinear Uncertain Systems with Quantized Input Signal

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Abstract—In this paper, we study a general class of strict feedback nonlinear systems, where the input signal takes quantized values. We consider a stabilization problem for nonlinear uncertain systems via adaptive backstepping approach. The control design is achieved by introducing a hysteretic quantizer to avoid chattering and using backstepping technique. A guideline is derived to select the parameters of the quantizer. The designed controller together with the quantizer ensures the stability of the closed loop system in the sense of signal boundedness.

Keywords: Adaptive control, backstepping, quantized control, nonlinear systems, hysteretic quantizer

I. INTRODUCTION

In quantized control systems, the control signal to the plant is a piece-wise constant function of time. There has been a great deal of attention in the study of quantized control systems, in which a control system is interacted with information quantization, due to its theoretical and practical importance in the study of digital control, hybrid systems, networked control systems as discussed in [1], [2], [3], [4]. The main motivation for considering quantization in control systems comes from the observation that for many control systems, quantization is not only inevitable, but also useful. An important aspect is to use quantization schemes that have sufficient precision and require low communication rate. Much attention has been paid to quantized feedback control, in order to understand the required quantization density or information rate in stability analysis. Most of the work on quantized feedback control concentrates on understanding and mitigation of quantization effects [5], [6]. Recently, research on stabilization of linear and nonlinear systems with quantized control signals has received great attention, see for examples, [3], [4], [7], [8], [9], [10], [11]. The systems considered in above references are completely known.

In practice, it is often required to consider the case where the plant to be controlled is uncertain. Quantized control of systems with uncertainties has been studied by using robust approaches, see for examples, [12], [13], [14], [15], [16], [17]. Quantized robust control problem for uncertain strict-feedback systems has been addressed in [15], [16]. As well known, adaptive control is an useful and important approach to deal with system uncertainties due to its ability of providing on-line estimations of unknown system parameters with

measurements. However, results based on adaptive control approach are still very limited. It is noted that adaptive control schemes with quantized input have been reported only in [18], [19], [20]. In [18], [19], adaptive quantized control for linear systems was studied. In [20], adaptive quantized control of nonlinear systems was considered, where the idea of constructing hysteretic type of input quantization was originally introduced. However the stability condition in [18], [20] depends on the control signal, which is hard to be checked in advance as the control signal is only available after the controller is put in operation.

Since backstepping technique was proposed, it has been widely used to design adaptive controllers for uncertain systems [21]. This technique has a number of advantages over the conventional approaches such as providing a promising way to improve the transient performance of adaptive systems by tuning design parameters. Because of such advantages, research on adaptive control using backstepping technique has also received great attention, see for examples, [22], [23], [24], [25], [26], [27].

So far there is still no result available for backstepping based adaptive stabilization of strict-feedback nonlinear uncertain systems preceded by quantized input signal. In this paper, we provide a solution to this problem. A hysteretic type quantizer is studied. The quantization parameters will be chosen based on a derived inequality related to the given controller design parameters and certain system parameters. In this way, the stability in the sense of ultimate boundedness is achieved by choosing suitable quantization parameters and design parameters, which can be easily verified in advance. Thus our proposed scheme relaxes the stability condition in [18], [20]. It also ensures that the ultimate stabilization error is proportional to a design parameter and thus adjustable. Simulation results illustrate the effectiveness of our proposed scheme.

This note is organized as follows. Section II states the problem of this note and presents the hysteretic quantizer. Sections III presents the adaptive control design based on the backstepping technique and analyzes the stability and performance. Simulation results are presented in Section IV. Finally, Section V concludes this note.

II. PROBLEM STATEMENT

A. System Model

Consider a control system over the network with control input quantization. The control input is quantized and then coded in the coder to be sent over the network. We assume

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that the network is noiseless, so the quantized input signal is recovered in the decoder and applied to the plant. In this note, a class of nonlinear plants is considered in the following parametric strict-feedback form as in [21], [28].

$$\begin{aligned} \dot{x}_1 &= x_2 + \psi_1(x_1) \\ \dot{x}_2 &= x_3 + \psi_2(x_1, x_2) \\ &\vdots \\ \dot{x}_{n-1} &= x_n + \psi_{n-1}(x_1, \dots, x_{n-1}) \\ \dot{x}_n &= q(u(t)) + \phi^T(x)\theta + \psi_n(x) \end{aligned} \quad (1)$$

where $x(t) = [x_1(t), \dots, x_n(t)]^T \in \mathfrak{R}^n$ and $q(u(t)) \in \mathfrak{R}^1$ are the states and input of the system, respectively, $\bar{x}_i(t) = [x_1(t), \dots, x_i(t)]^T \in \mathfrak{R}^i$, the vector $\theta \in \mathfrak{R}^r$ is constant and unknown, $\phi \in \mathfrak{R}^r$, $\psi_i \in \mathfrak{R}^1$, $i = 1, \dots, n$ are known nonlinear functions and differentiable. The input $q(u(t))$ represents the quantizer and takes the quantized values, where $u(t) \in \mathfrak{R}^1$ is the control input signal to be quantized at the encoder side. For this class of nonlinear systems, we assume that the existence and uniqueness of solution are satisfied.

For the development of control laws, the following assumptions are also made.

Assumption 1: The nonlinear functions ϕ and ψ_i satisfy the global Lipschitz continuity condition such that

$$\|\phi(t, x) - \phi(t, y)\| \leq L_\phi \|x - y\| \quad (2)$$

$$\begin{aligned} \left\| \frac{\partial \phi}{\partial x}(t, x) \right\| &\leq L_\phi \\ \|\psi_i(t, \bar{x}_i) - \psi_i(t, \bar{y}_i)\| &\leq L_{\psi_i} \|\bar{x}_i - \bar{y}_i\| \quad (3) \\ \left\| \frac{\partial \psi_i}{\partial \bar{x}_i}(t, \bar{x}_i) \right\| &\leq L_{\psi_i} \end{aligned}$$

$\forall x, y \in \mathfrak{R}^n$, $\forall \bar{x}_i, \bar{y}_i \in \mathfrak{R}^i$, where L_ϕ and L_{ψ_i} are known constants.

Assumption 2: The unknown parameter vector θ is within a known compact convex set C such that $\|\theta_a - \theta_b\| \leq \theta_M$ for any $\theta_a, \theta_b \in C$ and a constant θ_M .

The control objective is to design a backstepping feedback control law for $u(t)$ to ensure that all closed-loop signals are bounded and the ultimate stabilization error is within an adjustable bound.

B. Hysteretic Quantizer

In this paper, we use hysteresis quantizer to avoid chattering. The hysteresis quantization was firstly introduced in [29]. In this paper, the quantizer $q(u(t))$ represents the hysteretic quantizer in the following form similar to those in [14], [29].

$$q(u(t)) = \begin{cases} u_i \text{sgn}(u), & \frac{u_i}{1+\delta} < |u| \leq u_i, \dot{u} < 0, \text{ or} \\ & u_i < |u| \leq \frac{u_i}{1-\delta}, \dot{u} > 0 \\ u_i(1+\delta) \text{sgn}(u), & u_i < |u| \leq \frac{u_i}{1-\delta}, \dot{u} < 0, \text{ or} \\ & \frac{u_i}{1-\delta} < |u| \leq \frac{u_i(1+\delta)}{(1-\delta)}, \dot{u} > 0 \\ 0, & 0 \leq |u| < \frac{u_{min}}{1+\delta}, \dot{u} < 0 \text{ or} \\ & \frac{u_{min}}{1+\delta} \leq u \leq u_{min}, \dot{u} > 0, \\ q(u(t^-)) & \dot{u} = 0 \end{cases} \quad (4)$$

where $u_i = \rho^{(1-i)} u_{min}$ with integer $i = 1, 2, \dots$ and parameters $u_{min} > 0$ and $0 < \rho < 1$, $\delta = \frac{1-\rho}{1+\rho}$. $q(u)$ is in the set $U = \{0, \pm u_i, \pm u_i(1+\delta)\}$. u_{min} determines the size of the dead-zone for $q(u)$. The map of the hysteretic quantizer $q(u(t))$ for $u > 0$ is shown in Figure 1. For this kind of quantizer, the following remarks are given. Similar discussions were also made in [14].

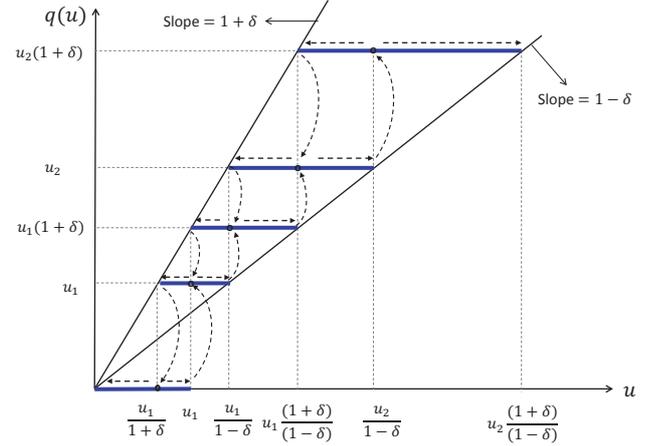


Fig. 1. The map of $q(u)$ for $u > 0$

Remark 1: The parameter ρ is considered as a measure of quantization density. The smaller the ρ is, the coarser the quantizer is. When ρ approaches to zero, δ approaches to 1, then $q(u)$ will have fewer quantization levels as u ranges over that interval.

Remark 2: The control action for the hysteretic quantizer (4) should be satisfied in terms of existence and uniqueness of solution of the closed-loop systems. Since the system (1) is uncertain so the parameter ρ of the hysteretic quantizer is not given a priori. Instead, it should be chosen based on a guideline that ensures the stability of the closed loop system.

Remark 3: Compared with the logarithmic quantizer in [12], [18], the quantizer in (4) has additional quantization levels, which are used to avoid chattering. Whenever $q(u(t))$ makes a transition from one value to another, some dwell time will elapse before a new transition can occur as shown in Figure 1. This can be seen as a way to add hysteresis to the quantizer system.

In order to propose a suitable control scheme, we decompose the hysteretic quantizer $q(u(t))$ into a linear part and a nonlinear part as follows.

$$q(u(t)) = u(t) + d(t) \quad (5)$$

where $d(t) = q(u(t)) - u(t) \in \mathfrak{R}^1$. Regarding the nonlinearity $d(t)$, we have the following lemma.

Lemma 1: The nonlinearity $d(t)$ satisfies the following inequality.

$$d^2(t) \leq \delta^2 u^2, \quad \forall |u| \geq u_{min}. \quad (6)$$

$$d^2(t) \leq u_{min}^2, \quad \forall |u| \leq u_{min}. \quad (7)$$

Proof: From Figure 2 and using sector bound property, we can get that for $u \geq u_{min}$

$$(1 - \delta)u \leq q(u) \leq (1 + \delta)u \quad (8)$$

$$|q(u) - u| \leq \delta u \quad (9)$$

Similarly for $u \leq -u_{min}$, it can be shown that $|q(u) - u| \leq -\delta u$. Thus, we have $|q(u) - u| \leq \delta|u|$ for $|u| \geq u_{min}$, which implies the property (6). For $u \leq u_{min}$, $q(u) = 0$ from the definition (4). So the property (7) is derived directly. \blacksquare

III. STATE FEEDBACK CONTROL

In this section, we will design adaptive backstepping feedback control laws for the nonlinear uncertain system (1) with hysteretic input quantization (4). The objective is to design an adaptive backstepping controller and a quantization rule (i.e. a guideline to choose the quantization parameters) for $u(t)$ to reduce the information to be sent over the communication channel in the presence of system uncertainties. We have no apriori information on how fine the quantizer should be to give a stable closed-loop system. To achieve the objective, a guideline of choosing the quantization parameters is derived. For nonlinear uncertain system (1), the number of design steps required is equal to n . At each step, an error variable z_i and a stabilizing function α_i is generated. Finally, the control u and a parameter estimate $\hat{\theta}$ are developed.

Introduce the change of coordinates

$$z_1 = x_1 \quad (10)$$

$$z_i = x_i - \alpha_{i-1}, \quad i = 2, 3, \dots, n \quad (11)$$

where α_i are virtual controllers. The design procedure is elaborated in the following steps.

Step i, ($i = 1, \dots, n - 1$). The design for the first $n - 1$ subsystems follows the backstepping design procedure in [21]. Design the stabilizing function α_i as

$$\alpha_i = -(c_i + 1)z_i - \psi_i + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (x_{j+1} + \psi_j) \quad (12)$$

where c_i is a positive constant.

Step n. In the last step n , the actual control input u appears and is at our disposal. The z_n dynamics is given as

$$\begin{aligned} \dot{z}_n &= q(u) + \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (x_{j+1} + \psi_j) + \theta^T \phi \\ &= u(t) + d(t) + \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (x_{j+1} + \psi_j) + \theta^T \phi \end{aligned} \quad (13)$$

We choose the following Lyapunov function

$$U_n = \sum_{i=1}^n \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad (14)$$

where Γ is a positive definite matrix, and $\tilde{\theta} = \theta - \hat{\theta}$. Then its derivative is given by

$$\begin{aligned} \dot{U}_n &= \sum_{i=1}^n \frac{1}{2} z_i \dot{z}_i + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &\leq - \sum_{i=1}^{n-1} c_i z_i^2 + \frac{1}{2} z_n^2 + z_n (u(t) + d(t) + \psi_n \\ &\quad - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (x_{j+1} + \psi_j) + \theta^T \phi) + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &\leq - \sum_{i=1}^{n-1} c_i z_i^2 + z_n d(t) + z_n (u(t) + \frac{1}{2} z_n + \psi_n \\ &\quad - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (x_{j+1} + \psi_j) + \hat{\theta}^T \phi) \\ &\quad + \tilde{\theta}^T \Gamma^{-1} (\Gamma \phi z_n - \dot{\hat{\theta}}) \end{aligned} \quad (15)$$

We are finally at the position to design a control law for u and update law for $\hat{\theta}$ as

$$\begin{aligned} u &= -(c_n + \frac{1}{2} + k_d) z_n - \psi_n + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (x_{j+1} + \psi_j) \\ &\quad - \hat{\theta}^T \phi \end{aligned} \quad (16)$$

$$\dot{\hat{\theta}} = Proj\{\Gamma \phi z_n\} \quad (17)$$

where c_n and k_d are positive constants. $Proj(\cdot)$ is the projection operator given in [21], which ensures that $\|\hat{\theta}\| \leq \theta_M$ for a given constant θ_M .

Using the property of the projection operator that $-\tilde{\theta}^T \Gamma^{-1} Proj(\tau) \leq -\tilde{\theta}^T \Gamma^{-1} \tau$ in [21] and from (15)-(17), the derivative of U_n satisfies

$$\begin{aligned} \dot{U}_n &\leq - \sum_{i=1}^n c_i z_i^2 + z_n d(t) - k_d z_n^2 \\ &\quad + \tilde{\theta}^T \Gamma^{-1} (\Gamma \phi z_n - Proj\{\Gamma \phi z_n\}) \\ &\leq - \sum_{i=1}^n c_i z_i^2 + z_n d(t) - k_d z_n^2 \end{aligned} \quad (18)$$

Now we will find an appropriate quantization parameter δ to ensure the boundedness of U_n . To do this, we need the following lemma.

Lemma 2: The virtual control law α_i , $i = 1, \dots, n - 1$ and the final control $u(t)$ satisfy the Lipschitz condition, namely

$$|\alpha_i| \leq k_{\alpha_i} \|\bar{z}_i(t)\| \quad (19)$$

$$|u(t)| \leq k_u \|z(t)\| \quad (20)$$

for positive constants k_{α_i} and k_u which depend on the given design parameters and certain system parameters, $\bar{z}_i(t) = [z_1, z_2, \dots, z_i]^T$, $z(t) = [z_1, z_2, \dots, z_n]^T$.

Proof: From Assumption 1, the functions ϕ and ψ_i satisfy the Lipschitz continuity condition. Since $\alpha_1 = -(c_1 + \frac{1}{2})z_1 -$

ψ_1 , it can be shown that

$$|\alpha_1| \leq (c_1 + \frac{1}{2})|z_1| + L_{\psi_1}|z_1| = k_{\alpha_1}|z_1|, \quad (21)$$

where $k_{\alpha_1} = c_1 + \frac{1}{2} + L_{\psi_1}$

$$\begin{aligned} |\frac{\partial \alpha_1}{\partial x_1}| &\leq (c_1 + \frac{1}{2}) + |\frac{\partial \psi_1}{\partial x_1}| \\ &\leq (c_1 + \frac{1}{2}) + L_{\psi_1} = k_{\alpha_1} \end{aligned} \quad (22)$$

$$\begin{aligned} |x_2| &\leq |z_2 + \alpha_1| \\ &\leq |z_2| + k_{\alpha_1}|z_1| \leq (2k_{\alpha_1}^2 z_1^2 + 2z_2^2)^{1/2} \\ &\leq \max(\sqrt{2k_{\alpha_1}}, \sqrt{2}) \|\bar{z}_2\| = k_{x_2} \|\bar{z}_2\|, \end{aligned} \quad (23)$$

where $k_{x_2} = \max(\sqrt{2k_{\alpha_1}}, \sqrt{2})$.

Following the similar procedure based on $\alpha_i = -(c_i + 1)z_i - \psi_i + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(x_{j+1} + \psi_j)$ in (12), we have

$$\begin{aligned} |\alpha_i| &\leq (c_i + 1)|z_i| + |\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(x_{j+1} + \psi_j)| \\ &\quad + |\psi_i| \\ &\leq (c_i + 1)|z_i| + L_{\psi_i} \|\bar{x}_i\| \\ &\quad + |\sum_{j=1}^{i-1} k_{\alpha_{i-1}}(|x_{j+1}| + L_{\psi_j} \|\bar{x}_j\|)| \\ &\leq k_{\alpha_i} \|\bar{z}_i\| \end{aligned} \quad (24)$$

$$|\frac{\partial \alpha_i}{\partial x_i}| \leq k_{\alpha_i} \quad (25)$$

$$|x_i| \leq |z_i + \alpha_{i-1}| \leq |z_i| + k_{\alpha_{i-1}} \|\bar{z}_i\| \leq k_{x_i} \|\bar{z}_i\| \quad (26)$$

$$\begin{aligned} \|\bar{x}_i\| &= (\sum_{j=1}^i x_j^2)^{1/2} \\ &\leq (\sum_{j=1}^i k_{x_j} \|\bar{z}_j\|^2)^{1/2} \\ &\leq (\sum_{j=1}^i k_{x_j})^{1/2} \|\bar{z}_i\| \leq k_{\bar{x}_i} \|\bar{z}_i\| \end{aligned} \quad (27)$$

From $\|\hat{\theta}\| \leq \theta_M$ and according to Assumption 1 and (24)-(27), the final control u in (16) satisfies

$$|u(t)| \leq k_u \|z(t)\|. \quad (28)$$

We are now at the position to state our main results in the following theorem. ■

Theorem 1: Consider the closed-loop adaptive system consisting of plant (1), the adaptive backstepping controller (16) with virtual control laws (12), parameter estimator with updating law (17), and the hysteretic quantizer (4). The boundedness of all the signals in the system is ensured if the quantized parameter δ satisfies

$$\beta - \delta k_u \geq \epsilon > 0. \quad (29)$$

where $\beta = \min\{c_1, c_2, \dots, c_n\}$ and ϵ is a positive constant. Furthermore, the stabilization error is ultimately bounded as

follows:

$$\|z(t)\| \leq B, \text{ where } B = \sqrt{\frac{u_{min}^2}{4k_d\epsilon}}. \quad (30)$$

Proof: We consider two cases to get a bound for \dot{U}_n .

Case 1. $|u(t)| \leq u_{min}$. Using $|d(t)| \leq u_{min}$ in (7), we have from (18) that

$$\begin{aligned} \dot{U}_n &\leq -\sum_{i=1}^n c_i z_i^2 + |z_n|u_{min} - k_d z_n^2 \\ &\leq -\sum_{i=1}^n c_i z_i^2 + \frac{1}{4k_d} u_{min}^2 \\ &\leq -\beta \|z(t)\|^2 + \frac{1}{4k_d} u_{min}^2, \quad \forall |u(t)| \leq u_{min}. \end{aligned} \quad (31)$$

Case 2. $|u(t)| > u_{min}$. Using the property that $d^2(t) \leq \delta^2 u^2$ in (6), (18) can be written as

$$\dot{U}_n \leq -\sum_{i=1}^n c_i z_i^2 + \delta |u(t)| \|z\| \quad (32)$$

From (19)-(20) in Lemma 2 we have

$$\begin{aligned} &\sum_{i=1}^n c_i z_i^2 - \delta |u| \|z\| \\ &\geq \beta \|z(t)\|^2 - \delta k_u \|z(t)\|^2 \\ &= (\beta - \delta k_u) \|z(t)\|^2, \quad \forall |u(t)| > u_{min} \end{aligned} \quad (33)$$

If the quantization parameter δ is chosen to satisfy (29), the derivative of U_n in (32) satisfies

$$\begin{aligned} \dot{U}_n &\leq -\sum_{i=1}^n c_i z_i^2 + \delta |u(t)| \|z\| \\ &\leq -(\beta - \delta k_u) \|z(t)\|^2 \\ &\leq -\epsilon \|z(t)\|^2 < 0, \quad \forall |u(t)| > u_{min}. \end{aligned} \quad (34)$$

Combining the two cases from (31) and (34), we obtain, for all $t \geq 0$, that

$$\dot{U}_n \leq -\epsilon \|z(t)\|^2 + \frac{1}{4k_d} u_{min}^2. \quad (35)$$

It follows that $\dot{U}_n < 0, \forall \|z(t)\| > \sqrt{\frac{u_{min}^2}{4k_d\epsilon}}$. Then the ultimate bound of $z(t)$ satisfies (30). From (35) as well as the projection operation (17), z_1, z_2, \dots, z_n and $\hat{\theta}$ are bounded under condition (29). This further implies that x_1 is bounded. The boundedness of x_2 follows from the boundedness of α_1 and the fact that $x_2 = z_2 + \alpha_1$. Similarly, the boundedness of x_i ($i = 3, \dots, n$) can be ensured from the boundedness of α_i in (12) and the fact that $x_i = z_i + \alpha_{i-1}$. Combining this with (16) or (20) we conclude that the control $u(t)$ is also bounded. Thus all the signals are bounded. ■

Remark 4: The ultimate stabilization error is proportional to u_{min} . As we can choose u_{min} , so such a stabilization error is adjustable and can be made arbitrarily small.

Remark 5: The controller designed in this section achieves the goals of stabilization with quantized input signal. The proof of these properties is a direct consequence of the recursive procedure, because a Lyapunov function is constructed for the entire system including the parameter estimates.

Remark 6: Note that the choice of δ is arbitrary so long as (29) holds for the given design parameters c_i . So (29) can be considered as a guideline to choose this quantization parameter. Based on Theorem 1, the required number of quantization levels for δ is finite since the control signal (16) is bounded and δ is bounded.

Remark 7: As stated in Theorem 1, the quantized parameter δ is chosen to satisfy (29). To do this, k_u should be known. It is noted that k_u depends on L_ϕ , L_{ψ_i} and other system parameters in Lemma 2 to guarantee that $u(t)$ is bounded by $k_u \|z(t)\|$. It follows that L_ϕ and L_{ψ_i} should be known which are assumed in Assumption 1.

IV. SIMULATION RESULTS

In this section we consider a nonlinear system with a hysteretic quantized input as follows.

$$\ddot{x} + \theta \sin(\dot{x}) + \tanh(x) = q(u) \quad (36)$$

where $q(u)$ represents hysteretic quantizer as in (4), parameters θ is unknown, $u_{min} = 0.02$. The objective is to design a quantized control input for u to stabilize the system. The actual parameter value is chosen as $\theta = 1$ for simulations. In the simulations, we choose $c_1 = c_2 = 1, \gamma = 1$. The quantization parameter is chosen as $\delta = 0.2$ which satisfies the stabilizing condition (29), where $k_u = 5$. The initial states and parameter are chosen as $x(0) = \dot{x}(0) = 0.5$, and $\hat{\theta}(0) = 0.82$. Figures 2- 4 show the trajectories of states x and \dot{x} , the estimated parameter $\hat{\theta}$, the actual input $u(t)$ and the quantized input $q(u)$, respectively. Figure 2 shows that the state x_1 tends to zero, which is within the bound given in (30). Clearly, the simulation results verify our theoretical findings and show the effectiveness of our proposed control scheme.

V. CONCLUSION

In this paper, we develop an adaptive backstepping feedback stabilization scheme for a general class of strict feedback nonlinear systems preceded by quantized input signal. A hysteretic type quantizer is studied. The quantization parameter is chosen based on an established inequality depending on the given controller design parameters and certain system parameters, which ensures system the stability and stabilization error within an adjustable bound. Simulation results illustrate the effectiveness of our proposed scheme.

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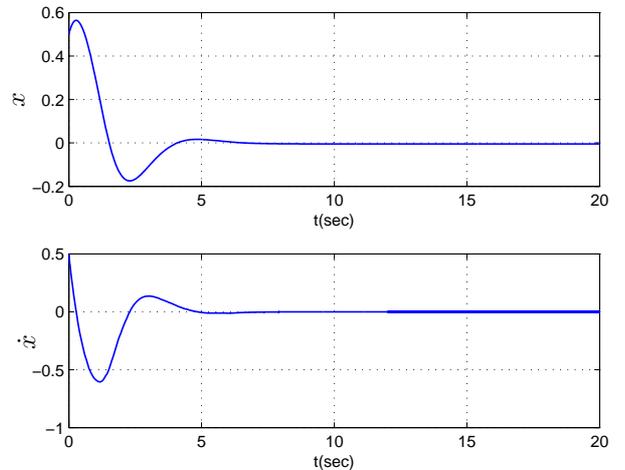


Fig. 2. Stabilization of x_1 and x_2

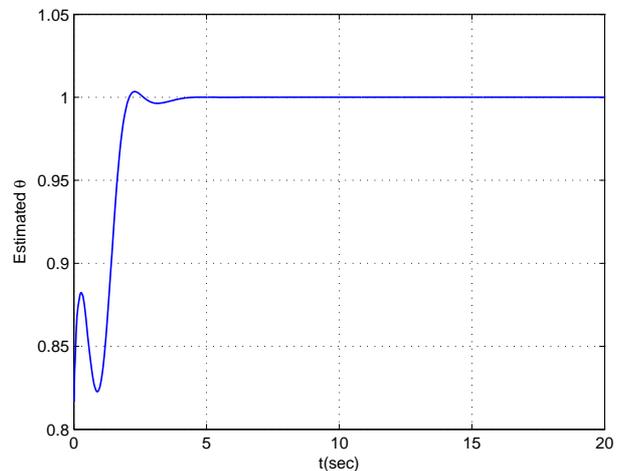
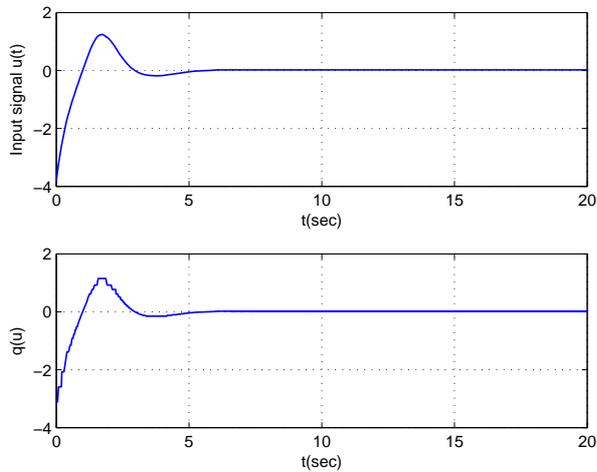


Fig. 3. Estimation of parameter θ

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Fig. 4. Input $u(t)$ and quantized input $q(u)$

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