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Tunable optical bistability at the graphene-covered nonlinear interface
Yuanjiang Xiang, Xiaoyu Dai, Jun Guo, Shuangchun Wen, and Dingyuan Tang

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Tunable optical bistability at the graphene-covered nonlinear interface

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We investigate theoretically the optical bistability of reflection at the interface between graphene and Kerr-type nonlinear substrates. We derive a simple procedure to calculate the nonlinear reflectivity with graphene, and discuss the influence of the graphene sheets on the hysteretic response of the TM-polarization reflected light. It is found that the bistable behavior of the reflected light can be electrically controlled via suitably varying the applied voltage on the graphene. In THz, the bistable thresholds can be lowered markedly by increasing the Fermi energy. However, in near-infrared frequency, it requires multiple graphene layers to exhibit significant influence on the bistable thresholds. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4863927]

Due to ultra-high bandwidth and the elimination of optical-to-electrical conversion, all-optical signal processing systems are likely to provide a promising alternative to their electronic counterparts.1 Nonlinear optical effects can be used for high-speed processing of optical signals. A feasible approach to all-optical switching is based on optical bistability.2,3 Optical bistability offers many intriguing applications, such as optical transistor,4 all-optical switching,5 and optical memory.6 Fabry-Perot cavity filled by the nonlinear medium is the classical configuration of optical bistability. Another particular interesting case of producing optical bistability is the light reflected at the interface of a nonlinear medium. Kaplan showed that the light reflected at the boundary of a nonlinear medium can exhibit hysteresis.7 Wysin et al. suggested that the critical power of optical bistability can be reduced due to the resonant enhancement of the evanescent fields in the nonlinear medium associated with the surface plasmons in a metal film.8 However, few related work has been carried out to explore tunable optical bistability properties.9,10 It is known that the tunability of the optical bistability is intrinsically important for the applications in the all optical devices. In this paper, we will extend the investigation of optical bistability to the graphene, and discuss the controllable optical bistability of light reflected from graphene-on-nonlinear dielectric surface.

Graphene has attracted intensive scientific interest owing to its incredible physical properties showing great potential applications in nano-electronic devices and opto-electric devices with ultrahigh electron mobility, ultrafast relaxation time for photo-excited carriers, and gate-variable optical conductivity.11,12 Graphene also has the outstanding optical properties,13,14 such as, strong light-graphene interaction, broadband and high-speed operation, etc. It seems to be a good candidate for designing tunable optical device that operates in both THz and optical frequency ranges due to the tunability of the charge carrier density and conductivity by the bias voltage of graphene. Therefore, electrical tunability of conductivity of graphene could potentially open a new possibility of tunable optical sensor,15 tunable metamaterials,16,17 tunable terahertz absorber,18 and Goos-Hänchen effect,19 etc. It also provides a scheme for controlling the optical bistability via suitably varying the applied voltage on the graphene.

We consider an interface between a linear dielectric and a Kerr nonlinear medium with self-focusing properties, and the graphene is coated on the nonlinear interface. The geometrical setup is shown in Fig. 1(a), x direction is along the interface and z direction is perpendicular to the interface. A plane wave is incident from the linear dielectric with the dielectric constant \(\varepsilon_1\) at an angle \(\theta\) on the nonlinear medium with the dielectric constant \(\varepsilon_2\). The Kerr medium is characterized by \(\varepsilon_2 = \varepsilon_0^2 + \varepsilon_2(\varepsilon_0^2 + \varepsilon_0^2)\), where \(\varepsilon_0\) is the dielectric constant of the nonlinear medium at zero intensity; \(\varepsilon\) represents the nonlinear coefficient, which is connected to the optical Kerr constant \(n_2\) through the relation \(\varepsilon = \varepsilon_0 n_2\). \(E_2\) is the local electric-field amplitude in the Kerr medium. The graphene sheet is coated on the nonlinear substrate with the surface conductivity \(\sigma\). Within the random-phase approximation, the graphene surface conductivity \(\sigma\) is the sum of the intraband \(\sigma_{\text{intra}}\) and the interband term \(\sigma_{\text{inter}}\), where

\[
\sigma_{\text{intra}} = \frac{ie^2 k_B T}{\pi \hbar^2 (\omega + i/\tau)} \left( \frac{E_F}{k_B T} + 2 \ln \left( \frac{e^{\frac{-E_F}{k_B T}} + 1 \right) \right),
\]

\[
\sigma_{\text{inter}} = \frac{ie^2}{4\pi \hbar} \ln \left| \frac{2E_F - (\omega + i\tau)^{-1}}{2E_F + (\omega + i\tau)^{-1}} \right|.
\]

where \(\omega\) is the frequency of the incident light, \(e\) and \(h\) are universal constants related to the electron charge and reduced Planck’s constants, respectively, \(E_F\) and \(\tau\) are the Fermi energy (or chemical potential) and electron-phonon relaxation time, respectively. \(k_B\) is the Boltzmann constant, and \(T\) is a temperature in K. Equation (2) is deduced on the condition for \(k_B T \ll E_F, \hbar \omega\). The Fermi energy \(E_F\) can be straightforwardly obtained from the carrier density \((n_2D)\) in a graphene sheet, \(E_F = \hbar \nu_F (\pi n_2D)^{1/2}\), \(\nu_F = 10^6 \text{m/s}\) is the Fermi velocity of electrons. Here, the carrier density \(n_2D\) can be electrically controlled by an applied gate voltage, thereby leading to a voltage-controlled Fermi energy \(E_F\) and hence the voltage-controlled surface conductivity \(\sigma\), this could

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provide an effective route to achieving electrically controlled optical bistability in graphene-on-Kerr nonlinear surface.

The electric field and magnetic field of the plane wave component of the incident beam is denoted by $E_{0}(z) = E_{1} \exp(i k_{i} \cdot r)$ and $H_{0}(z) = H_{1} \exp(i k_{1} \cdot r)$, where $k_{i} = k_{\perp} \hat{e}_{z}$, $k_{1} = k_{1} \sin \theta$, $k_{1z} = k_{1} \cos \theta$, $k_{1} = \omega_{0} \sqrt{\epsilon_{1}}$, and $k_{0} = \omega/c$. The field is assumed to be uniform in the $y$ direction. All of the electric field and magnetic field in the two media can be written out in terms of their $x, y, z$ components, respectively. Especially, for the TM polarization, the incident magnetic and electric fields are

$$E_{1x}, E_{1z} = \left[ 1, k_{1z}/(\omega_{0} \epsilon_{1}) - k_{z}/(\omega_{0} \epsilon_{1}) \right] H_{1} \exp[i(k_{x}x + k_{1z}z)].$$

The reflected fields are

$$E_{1x}, E_{1z} = \left[ 1, -k_{1z}/(\omega_{0} \epsilon_{1}), -k_{z}/(\omega_{0} \epsilon_{1}) \right] r H_{1} \exp[i(k_{x}x - k_{1z}z)],$$

where $r$ is the reflection coefficient. The wave equation for the magnetic field $H_{2y}$ in Kerr medium can be written as

$$\frac{\partial^{2} H_{2y}}{\partial z^{2}} + \alpha^{2} \left( \epsilon_{2} + \alpha^{2} |H_{2y}|^{2} - \epsilon_{1} \sin^{2} \theta_{1} \right) H_{2y} = 0,$$

where $\alpha = \kappa/(\omega_{0} \epsilon_{2} \sin^{2} \theta_{1})$. For the self-focusing Kerr nonlinear medium ($\alpha > 0$), the solutions to the wave equation (Eq. (3)) is well known, $H_{2y}(z) = \sqrt{2\alpha q_{z}^{2}} \text{sech}(k_{0} q_{z} z + \kappa) \exp[i(k_{0} x)]$, where $q_{z}^{2} = \epsilon_{1} \sin^{2} \theta_{1} - \alpha^{2} \kappa$, $\kappa = \cos^{-1}(\sqrt{2\alpha q_{z}^{2}/|H_{2y}(0)|})$ with $|H_{2y}(0)|$ being the amplitude of the magnetic field at the interface $z = 0$. For a given value of the magnetic field $H_{2y}(0)$, the effective dielectric constant $\tilde{\epsilon}_{2}$, which is determined by the average boundary value of the field intensity in Kerr medium, is obtained and can be written as $\tilde{\epsilon}_{2} = \epsilon_{0}^{2} + \alpha |H_{2y}(0)|^{2}/2$.

The electric field in the Kerr medium can be expressed as $E_{2x} = \frac{ik_{0} q_{z}}{\omega_{0} \tilde{\epsilon}_{2}} \text{tanh}(k_{0} q_{z} + \kappa) H_{2y}$, and $E_{2z} = -\frac{k_{0} q_{z}}{\omega_{0} \tilde{\epsilon}_{2}} H_{2y}$. Considering the graphene sheet, and from the boundary condition of $E_{z}$ and $H_{y}$ at $z = 0$, we have the following relations, $H_{1x}(z = 0) + H_{1x}(z = 0) = H_{2y}(z = 0)$, and $E_{1x}(z = 0) + E_{1x}(z = 0) = E_{2z}(z = 0)$. Finally, we can obtain the reflection coefficient and transmission coefficient for the plane wave reflected from the nonlinear interface covered by the graphene

$$r = \frac{k_{1z}}{\tilde{\epsilon}_{1}} \left( 1 + \frac{ik_{0} q_{z}}{\omega_{0} \tilde{\epsilon}_{2}} \text{tanh}(\kappa) \right) + \frac{k_{0} q_{z}}{\tilde{\epsilon}_{2}} \text{tanh}(\kappa);$$

$$t = \frac{k_{1z}}{\tilde{\epsilon}_{1}} \left( 1 + \frac{ik_{0} q_{z}}{\omega_{0} \tilde{\epsilon}_{2}} \text{tanh}(\kappa) \right) + \frac{k_{0} q_{z}}{\tilde{\epsilon}_{2}} \text{tanh}(\kappa).$$

If the graphene sheet is absent, by neglecting the conductivity $\sigma$, we can return to the reflection coefficient and transmission coefficient for the plane wave reflected from the nonlinear interface without the graphene. The reflection coefficient and transmission coefficient for TE polarization can also be derived by the similar method $r = \frac{k_{1z} - ik_{0} q_{z} \text{tanh}(\kappa) - \mu_{0} \sigma}{k_{1z} + ik_{0} q_{z} \text{tanh}(\kappa) + \mu_{0} \sigma}$, $t = \frac{k_{0} q_{z}}{\tilde{\epsilon}_{2}} \text{tanh}(\kappa) + \mu_{0} \sigma$.

Furthermore, once the magnetic field $H_{2y}(0)$ is known, the value of the input intensity $I_{in}$ can be written as

$$I_{in} = \frac{1}{2\tilde{\epsilon}_{1} \omega_{0} c} |H_{2y}(z = 0)|^{2}/|q_{z}|^{2}.$$

Next, we discuss the role of the graphene sheets in the optical bistability according to Eqs. (4)–(6). In the present paper, we only discuss the optical bistability of TM polarization, and TE polarization can be discussed similarly. It is clear from Eqs. (4) and (5) that the optical properties of the interface, including the transmission and reflection, will be influenced greatly by introducing the graphene sheets, which result in the dependence of the hysteretic response on the optical properties of the graphene sheets. Both the real part and imaginary part of graphene conductivity effect on the hysteretic response. However, besides the step-like behaviour of the real part of the conductivity $\sigma_{r}$ in the near-infrared frequency (NIR), $\sigma_{r}$ is not strong dependent on the Fermi energy $E_{F}$ compared to the imaginary part of graphene conductivity $\sigma_{i}$, as shown in Figs. 1(b) and 1(c). Hence, we can neglect the influence of $\sigma_{r}$ on the dependence of the hysteretic response on the Fermi energy in theory. From Eq. (5), we can know that the negative $\sigma_{i}$ will decrease the amplitude of the transmission coefficient, leading to the increase of the bistable thresholds according to Eq. (6). On the contrary, the positive $\sigma_{i}$ will increase the amplitude of the transmission coefficient, leading to the decrease of the bistable thresholds. The detailed analyses are outlined below.

The first considered case is the optical bistability without the graphene sheet depicted in Fig. 2. The parameters are the following: $\epsilon_{2} = 2.6$, $\epsilon_{0}^{2} = 2.25 + 0.02i$, the incident angles are $\theta = 81^{\circ}, 82^{\circ}$, and $83^{\circ}$, respectively. To avoid damaging the graphene sheets at a high power level, the
nonlinear medium with a higher Kerr constant is adopted here, \( n_2 = 2 \times 10^{-12} \text{m}^2/\text{W} \), which can be obtained in the Nano-Ag:polymeric composite material\(^{25}\) or Au:SiO\(_2\) composite films.\(^{26}\) Moreover, as we know, the other nonlinear effects, such as charge or thermal dispersion, also could change the refractive index of substrate in similar order of magnitude and realize optical bistable effect as proposed in the papers.\(^{27,28}\) However, in the present Letter, we only consider the electronic nonlinearity. In the absence of the nonlinearity, it is clear that the critical angle for total internal reflection (TIR) is \( \theta_c \approx 68.5^\circ \). However, in the presence of the nonlinearity, the critical angle \( \theta_i \) will depend on the intensity of the incident light wave, and consequently it appears possible to optically switch from TIR to transmission mode, or in reverse, simply by changing the incident light intensity, as shown in Fig. 2(b). Here, the hysteresis effects are observed when the incident angle \( \theta_i \) is larger than the critical angle \( \theta_c \). As the input intensity \( I_{in} \) increases, the reflected intensity \( I_{out} \) increases linearly as shown in Fig. 2(a), and the phase of the reflection coefficient increases accordingly as shown in Fig. 2(c); however the reflectance \( R \) keeps the TIR mode. When \( I_{in} \) reaches the high switch threshold intensity \( I_{on} \), the mode of the system will switch from TIR mode to the transmission mode, and the reflected intensity \( I_{out} \) becomes very small suddenly, however the reflectance phase switches to high value. In contrast, as \( I_{in} \) decreases from \( I_{on} \) to the low switch threshold intensity \( I_{off} \), the system will maintain the transmission mode, and finally switches to the TIR mode when \( I_{in} < I_{off} \), however, the phase \( \phi \) jumps from the high value to the low value. Moreover, if we decrease the incident angle \( \theta_i \), both the high threshold \( I_{on} \) and low threshold \( I_{off} \) of the bistability move to the lower light intensity, which is induced by the decreased \( \Delta = |\sin^2 \theta_i - \varepsilon_2/\varepsilon_1| \) as the incident angle decreased, implying that the requirement of intensity-dependent nonlinear index change for switching the system from TIR mode to transmission mode or in reverse should be decreased.

Upon insertion of graphene, the bistable behavior will be influenced by the graphene, but the impact depends on the graphene surface conductivity. At NIR frequency, for low Fermi energy \( 2E_F < \hbar \omega_0 \), the interband transitions in graphene are allowed. Here we choose the wavelength \( \lambda = 1.55 \mu \text{m} \), the interband transitions are allowed for \( E_F < 0.4 \text{eV} \), as shown in Fig. 1(b). For \( E_F < 0.4 \text{eV} \), the real part of the graphene conductivity \( \sigma_r \) tends toward the universal optical conductivity of graphene \( \sigma_0 = e^2/4\hbar \), however the imaginary part of the graphene conductivity \( \sigma_i \) is the sum of the interband transitions and intraband transitions. \( E_F < 0.4 \text{eV} \) is negative for and positive for \( E_F > 0.4 \text{eV} \). The properties of the graphene conductivity \( \sigma \) have great influence on the bistable behavior, as shown in Fig. 3. Fig. 3(a) shows the effect of the tunable \( E_F \) by bias voltage applied on graphene on the hysteresis effects. It is clear that \( I_{on} \) and \( I_{off} \) can move to low or high input intensity depending on the value of \( E_F \). For \( E_F < 0.4 \text{eV} \) and in the TIR mode, \( \sigma_i \) is negative, then the real part of the denominator of the transmission coefficient \( t \) (Eq. (5)) increases, \( |t| \) decreases and hence it needs more input intensity to maintain the optical bistability of the TIR mode, which leads to the higher input intensity shifts of \( I_{on} \) and \( I_{off} \) as shown in Fig. 3(A1) and (A2). However, for \( E_F \gg 0.4 \text{eV} \), \( \sigma_r \) becomes positive and \( \sigma_i \approx 0 \), then the real part of the denominator of the transmission coefficient \( t \) decreases, \( |t| \) increases and hence it needs less input intensity to maintain the optical bistability of the TIR mode, which leads to the lower input intensity shifts of \( I_{on} \) and \( I_{off} \). At NIR frequency, the influence of the graphene on the reflectance bistability is not very significant due to the finite graphene surface conductivity. Hence, it requires multiple graphene layers to exhibit significant influence on the bistable thresholds. In Fig. 3(b), we have given the influence of the layer number of graphene sheets \( N \) on the bistable behavior. Here we only consider the case for \( E_F = 0.3 \text{eV} \). As the layer number \( N \) is increased, both \( I_{on} \) and \( I_{off} \) move to higher input intensity.

For frequencies \( 2E_F > \hbar \omega_0 \), the interband transitions in graphene are forbidden by the Pauli exclusion principle, hence \( \sigma \approx \sigma_{\text{min}} \). Here we choose the wavelength \( \lambda = 30 \mu \text{m} \) (\( f = 10 \text{THz} \)), the dependence of \( \sigma \) on the Fermi energy has been shown in Fig. 1(c). It is seen that \( \sigma_i \) is positive and becomes more and more large, and \( \sigma_r \) is small hence can be neglected compared to \( \sigma_i \). Obviously, the large value of \( \sigma_i \) will make great influence on the hysteresis effects of the nonlinear TIR mode, as shown in Fig. 4. In THz, the Kerr-nonlinear effect has been observed in liquids (CS\(_2\), CCl\(_4\), and CH\(_2\)I\(_2\), etc.),\(^{29,30}\) here in order to compare with the bistable behavior in NIR frequency we still choose the same parameters as those in Fig. 3. Fig. 4(a) gives the effect of \( E_F \) on the bistable behavior. By increasing the Fermi energy, \( \sigma_r \) is increased, which will lead to the increase of absorption and make the reflectance decrease. In the meanwhile, \( \sigma_i \) is increased significantly, which will lead to the decrease of \( |t| \) obviously and hence need less input intensity to maintain the optical bistability of the TIR mode, leading to the lower input intensity shifts of \( I_{on} \) and \( I_{off} \), as shown in Fig. 4(a).
Here, $I_{on}$ moves from 7.37 MW/cm$^2$ (without graphene) to 7.07 MW/cm$^2$ for $E_F = 1.2$ eV, at the same time $I_{off}$ moves from 5.53 MW/cm$^2$ to 5.31 MW/cm$^2$. The high and low bistable thresholds have been lowered about 0.30 MW/cm$^2$ and 0.22 MW/cm$^2$, respectively.

Further, we discuss the role of the layer number of graphene sheets $N$ in the bistable behavior for the THz wavelength, as shown in Fig. 4(b). Here, we also consider the case for $E_F = 0.3$ eV. As layer number $N$ is increased, both $I_{on}$ and $I_{off}$ move to lower input intensity markedly, which is induced by the great increased graphene conductivity. The high and low bistable thresholds have been lowered about 0.43 MW/cm$^2$ and 0.32 MW/cm$^2$, respectively. Clearly, the hysteresis effects at the nonlinear interface can be flexibly adjusted by varying the bias voltage or the layer number of the graphene sheets. Finally, we want to explain the influence of the losses in graphene on the tunability, and we have plotted the dependence of the reflection on the light intensity as shown in Fig. 4(c). It is clear that the bistable thresholds are enhanced as the relaxation time $\tau$ is decreased (losses are increased), implying that the input intensity must be higher in order to reach the same hysteretic state. Correspondingly, the intensity required for switching the system should be increased.

In summary, we have investigated theoretically the optical bistability of the plane wave reflected from graphene-on-nonlinear dielectric surface. It is found that the hysteretic response occurs near the critical angle. It is very important that the hysteretic effects can be electrically controlled through electrical or chemical modification of the charge carrier density of the graphene. Specially, the threshold intensity in near-infrared frequency or THz frequency of optical bistability can be lowered by adjusting the Fermi energy.
or by adopting the multiple graphene sheets. In THz, the bistable thresholds can be lowered markedly by increasing the Fermi energy. The electrical tunability of optical bistability with graphene could potentially open a new possibility of optical transistor, all-optical switching, and optical memory, etc.

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