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<td><strong>Author(s)</strong></td>
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Impact of Operational Systems on Supplier’s Response Under Performance-based Contracts

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Abstract
Performance-based contract has developed into a new service strategy in after-market operations that aims to compensate for performance outcomes. One of the key components of a performance-based contract is performance measure that is affected by factors such as base stock, reliability and response time. To achieve a desired performance level, the supplier has to improve the component reliability and the repair time. The component reliability in a repairable part inventory system is a function of the number of operational systems and the service capacity. In this research, we consider a single echelon repairable part inventory system and conduct a parametric analysis to evaluate the impact of the total number of systems on the supplier’s decisions under a performance contract. The results show that the numbers of systems plays a key role on the supplier’s response when the customer uses the average number of backorders as the negative incentive.

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1. Introduction

The profitability of support services, especially for systems with long operational life cycles, encourages companies in private and public sectors to pay attention to supportability that is a core performance measure of the logistics system. Accordingly, performance based supportability has become an established sustainment strategy that aims for the best industry practices implementation in a cost-effective manner [1]. By outsourcing the support services, the sustainment contract is being reshaped into a novel approach called performance-based contract (PBC) that organizes an agreement between two parties (customer and supplier).

Investigation of PBC implementation in the defense sector was launched in 1998 when a team of 60, including the office of the Secretary of Defense and Logistics Agency, evaluated shifting process from traditional contracts to PBC. They set a target of 50% migration to performance-based contracts by the end of 2005 in Army, Air and Navy

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U.S. force [2]. One of the earliest PBC implementation dates back to 1998 when Lockheed Martin offered a system to DoD for supporting F-117 Nighthawk, which tied its compensation to the fighter’s performance outcome.

Solving intercity traffic problem in Norwegian rail transport is an instance of PBC’s successful implementation in industry which was launched in 2003 between the Ministry of Transport and Communication (MOT) and the Norwegian State Railways (NBS). They designed an incentivized mechanism based on demand and production to reach the social goals of the Ministry of Transport while maximizing the Norwegian State Rail’s profit. Payment was made in accordance with train-kilometre and seat-kilometre output leading to increased service frequency and reduced crowding. To capture the risk of ignoring punctuality, NBS was also penalized for train cancellations and delays [3].

To achieve a desired performance outcome, an appropriate incentive should be in place to motivate the supplier. In previous studies [4, 5], average number of backorders, average downtime and cumulative downtime have been used as three efficient incentives under performance contracts. However, a contract based on these three performance measures may cause different responses from the supplier(s) depending on the system features. Also, anticipating the supplier’s reaction to each incentive mechanism is very important in performance contract as the customer may not directly specify the supplier’s investment decision.

In [5] the authors show that when the supplier does not have any control on the system failure rate, using average downtime rather than cumulative downtime leads to higher performance. They also found that when the supplier has the ability to reduce the system failure rate, cumulative downtime is superior to average downtime. They focused on systems with infrequent failure where there are many examples of systems with a high failure rate. To simplify the analysis, they also assume a constant failure rate and ignore the dependency of the failure rate to the number of operational systems and the service rate.

In [6] the authors found that to achieve a desired performance level in a repairable parts inventory system, the supplier has to improve the component reliability and the repair time, rather than invest in building up a stock of spares. The most effective incentive is defined based on the sensitivity of the parameters that can enhance performance. The best incentive must have the highest sensitivity to the failure rate and repair rate since it can motivate the supplier to improve these parameters.

The failure rate in a repairable parts inventory system is a function of the number of operational systems and the service rate. The number of operational systems, in turn, is directly dependent on the total number of systems which should be supported by the supplier. Therefore, the number of systems is one of the main parameters that may affect the supplier’s response under performance contract. Against this backdrop, the following question becomes important in the successful implementation of a performance-based contract: How does the number of systems influence the supplier’s decision under a performance-based contract?

In this research, we consider a single echelon repairable part inventory system consisting of a repair facility and a single warehouse. We conduct a parametric analysis to evaluate the impact of the total number of systems on the supplier’s decision under performance contract. The results show that the numbers of systems plays a key role on the supplier’s response when the customer applies the average number of backorders as the negative incentive.

To the best of our knowledge, this research is the first attempt to evaluate the role of the total number of systems on the effectiveness of incentives under performance contract. The rest of the paper is organized as follows: Section 2 discusses the relevant literature on performance based contract. In section 3 we present the structure of the inventory system. The incentive mechanism is discussed in Section 4. The performance contracting is modelled in Section 5 and Section 6 rounds off the paper with conclusions and future research directions.

2. Literature Review

After-sales services are, often, contracted between two parties (customer and the supplier) and sometimes contractual problems do come up during the contract period. These issues become even more important in performance contracts where the customer requires a minimum level of performance and the supplier is rewarded or penalized in accordance with the delivered outcome. The literature offers some guidelines on understanding the concept of PBC and its fundamental issues by studying real-life implementation of PBC. In [7] the author conducts a study to find the most frequent barriers and enablers to effective PBC implementation. His findings indicate that enhancing the performance, metric and incentive can lead to successful implementation of a PBC.

The performance of an inventory system with repairable parts can be defined in different ways. Modelling the performance in such systems has been studied extensively in the literature [8, 9]. In [10] the author compares several
system performance measures, such as fill rate, ready rate, operational rate and backorder. He concludes that backorders are the most reasonable performance measure.

The literature of performance and metrics in repairable systems strongly suggests using availability as the key performance measure for systems under performance-based contracts. There is a common approach to measuring the availability of the system, namely, the average number of backorders. In [4] the authors study a repairable inventory model under PBC in which the availability of the system is formulated based on the average number of backorders. In this research the failure rate and repair rate have been assumed constant as relaxing these assumptions may change the results.

Keeping the availability of complex technical systems (such as computer networks, defence and medical systems) at a certain level is vital since user operations may suffer when the systems fail. In such systems, the failure of systems leads to significant losses for the users; if the duration of the backorders is long, the losses may grow exponentially. In [11] the authors believe that the availability of the system is specified by its reliability and the speed of its repair facility. The speed of system repair activities is improved by the repair-by-replacement approach in which a failed part is replaced by a ready-for-use one.

In [12] the authors evaluate performance-based logistics (PBL) as a way to integrate acquisition and sustainment of systems, and leverage commercial best practices to reduce costs, improve performance, and ensure operational readiness. Their research also highlights the tracking of performance data to ensure PBL is achieving the expected results of reduced costs, improved performance and reliability. In [13] the authors look at outcome-based contracts as an excellent evidence of organizations shifting from goods-dominated logic to service-dominated logic. They analyse two contracts between defence contractors and the UK Ministry of Defence (MoD) that provide maintenance, repair and overhaul (MRO) services to the fighter jet and the missile system for their entire operational life.

Performance-based contracts have been used widely in both commercial and military sector, and quite a few articles [4, 5, 6, 11, 14, 15] study design and implementation of PBC. The literature review reveals a lack of understanding of how such contracts should be designed and implemented taking into account the number of operational systems. Evaluating the impact of the number of systems on the supplier’s decision under performance contract is the focus of this research.

3. Inventory System

We consider a single echelon repairable part inventory system consisting of a repair facility and a single warehouse. In this system, the supplier can improve the system performance by managing base stock level at the warehouse, component reliability, and efficiency of the repair facility. The system operates under the following assumptions:

- Each component’s failure follows an independent Poisson distribution with constant rate ($\lambda$).
- The demand for each component occurs according to a Poisson process with the variable rate $\lambda(z)$, where $z$ is the number of operational systems.
- The repair time at each server in the repair facility, which consists of the repair time and transportation delay, is an independent exponential random variable with mean $(\mu)^{-1}$.
- A one-for-one base stock ($S$) replenishment policy is used where a failed component is immediately replaced by a ready-to-use component (new or refurbished) from the warehouse and the failed one is sent to the repair facility.

In [6] the authors apply Markovian approach for such an inventory system to develop the steady-state probability of inventory level ‘$x$’ at the warehouse ($\pi_x$). The flow balance equations are:

$$\pi_x(z \lambda + \mu) = (D) \times \pi_{x-} + \pi_+ \times (F) \times \pi_{x+} + \pi_{[\text{Min}(z+1, N)])}$ for $-N \leq x \leq S$

$$D = \begin{cases} 1 & \text{if } -N + 1 \leq x \leq S \\ 0 & \text{otherwise} \end{cases} \quad F = \begin{cases} 1 & \text{if } -N \leq x \leq S - 1 \\ 0 & \text{otherwise} \end{cases}$$ (1)
By solving the balance equations and the normalizing constraint, we can specify \( \pi_x \) as follows:

\[
\pi_x = \frac{\left( \frac{\mu}{\lambda} \right)^x}{A} N^x \left( 1 - \left( \frac{\mu}{N \lambda} \right)^{x+k} \right) e^{\frac{\mu}{\lambda}} \Gamma(N, \frac{\mu}{\lambda})
\]

\[
A = \begin{cases} 
(N + x)! & x < 0 \\
N! & x \geq 0 
\end{cases}
\]

(2)

4. Performance Measure

Given the steady-state probabilities, the average number of backorders can be computed as follows:

\[
E(B|\lambda, \mu, S) = \sum_{x} x \pi_x = \left[ \frac{\left( \frac{\lambda}{\mu} \right)^N e^{\frac{\mu}{\lambda}} \Gamma(N, \frac{\mu}{\lambda}) (N \lambda - \mu) + \lambda}{\left( \frac{\lambda}{\mu} \right)^N e^{\frac{\mu}{\lambda}} \Gamma(N+1, \frac{\mu}{\lambda}) (N \lambda - \mu) - \left( \frac{\mu}{N \lambda} \right)^S \mu + \mu} \right] \lambda
\]

(3)

As can be seen, the average number of backorders is a function of the ratio of the failure rate to the service rate. Let \( \phi = \lambda / \mu \), so we can calculate the average number of backorders based on \( S \) and \( \phi \):

\[
B(S, \phi) = \frac{\phi^N e^{\phi} \Gamma(N, \frac{1}{\phi}) \left( N - \frac{1}{\phi} \right) + 1}{\phi^N e^{\phi} \Gamma(N+1, \frac{1}{\phi}) \left( N - \frac{1}{\phi} \right) + 1 - \frac{1}{\phi} N} = \left( \frac{1}{\phi^N} \right) \phi^{N+1} \Gamma(N+1, \frac{1}{\phi}) \left( N - \frac{1}{\phi} \right) + 1 - \frac{1}{\phi} N
\]

(4)

To evaluate the impact of the base stock on the average number of backorders, we define the sensitivity function of the number of backorders to the base stock as:

\[
\Delta B(S, \phi) = B(0, \phi) - B(S, \phi) \quad 0 \leq \phi \leq 1
\]

(5)

Figure 1 illustrates the behaviour of \( \Delta B \) by varying \( \phi \) for different levels of the base stock.
As can be seen from Figure 1, adding the base stock decreases the number of backorders only around a special value of $\Phi (\phi_1 < \Phi < \phi_2)$, especially when the number of systems is high. Also, we find that this value of $\phi$ is directly related to the number of systems ($\phi = I/N$). This means that keeping the base stock can be helpful for the supplier only when he chooses a service rate close to the failure rate ($\mu = \lambda I$). If the supplier sets a service rate much smaller than the failure rate ($\mu << \lambda I$) the repair queue increases rapidly and the supplier cannot control the number of backorders, even by adding base stock. On the other hand, when the service rate is much larger than the failure rate ($\mu >> \lambda I$) there is no queue at the repair centre and the supplier does not need to keep any base stock. We can simplify the average backorders formula as:

Fig 1. The sensitivity of the number of backorders to the base stock
\[
B = \left( N - \frac{1}{\varphi} \right) \times \left( \frac{a}{a-b} \right)
\]

\[
a = \varphi^{-\frac{1}{\varphi}} \left( N + 1, \frac{1}{\varphi} \right) \left( N \varphi - 1 \right) + 1
\]

\[
b = \left( \frac{1}{N \varphi} \right)^{5}
\]

We notice that for each value of the number of systems \((N)\) there is a range of \(\varphi (\varphi_2 \leq \varphi \leq 1)\) in which the value of ‘\(a\)’ is much larger than the value of ‘\(b\)’, regardless of the base stock level. Therefore, the second term in \(B\) is one:

\[
\frac{a}{a-b} \approx 1 \quad \text{for} \quad \varphi_2 \leq \varphi \leq 1
\]

We also observe that there is a special range of \(\varphi (0 \leq \varphi \leq \varphi_1)\) for each value of the number of systems such that the value of \(a/(a-b)\) is almost zero. In that range the repair centre is fast enough to replace the failed part in a short time so there is no backorder.

\[
\frac{a}{a-b} \approx 0 \quad \text{for} \quad 0 \leq \varphi \leq \varphi_1
\]

We conclude that the average number of backorders can be approximated by the following formula:

\[
B = \begin{cases} 
    \left( N - \frac{1}{\varphi} \right) \times \left( \frac{a}{a-b} \right) & \text{for} \quad \varphi_2 \leq \varphi \leq 1 \\
    \left( N - \frac{1}{\varphi} \right) \left( \frac{a}{a-b} \right) & \varphi_1 < \varphi < \varphi_2 \\
    0 & 0 \leq \varphi \leq \varphi_1 
\end{cases}
\]

5. Performance Contract

As a first step in performance-based contract, the customer offers a compensation model which determines the payment to the supplier over the lifetime of the contract. The supplier, as the other interested party in the contract, sets the parameters of the system (such as inventory level, reliability of components and repair rate) to deliver the performance level required. The supplier’s total cost consists of subsystem design cost, production cost and repair facility cost. We assume that all cost functions are linear in \(S\) and \(\varphi\) with unit cost \(C_S\), \(C_P\) respectively.

\[
TC(S, \varphi) = S \times C_S + \frac{1}{\varphi} C_P
\]

For providing insurance to the supplier, a fixed payment \((w)\) is included in the model. To incentivize the supplier to improve the system (increase his performance), both positive and negative incentives are also included in the model. As per this policy, the customer pays its share \((0 \leq \alpha \leq 1)\) of supplier’s total cost \((TC)\) and penalizes the supplier based on the average number of backorders.

\[
P = w + \alpha(TC) - \beta E(B)
\]
where $\alpha$ represents a percentage of supplier’s total cost that is compensated for by the customer and parameter $\beta$ denotes the penalty rate for each backorder. Given the contract terms $(w, \alpha, \beta)$, the supplier then sets $(\varphi, S)$ to maximize his profit:

$$\begin{align*}
\text{Max} & \quad \Pi = w - (1 - \alpha) (SC_s + \frac{1}{\varphi} C_p) - \beta (N - \frac{1}{\varphi}) \\
\text{S.t} & \quad \varphi_2 \leq \varphi \leq 1
\end{align*}$$

(12)

In this range of $\varphi$, the supplier’s decision for $S$ and $\varphi$ is as follows:

$$S = 0 \quad , \quad \varphi = \begin{cases} 
\varphi_2 & \beta > (1 - \alpha) C_p \\
1 & \text{Otherwise}
\end{cases}$$

(13)

When $\varphi_2 \leq \varphi \leq 1$ the base stock does not affect the number of backorders. Therefore, the supplier keeps no base stock due to its associated costs. Also, the supplier has motivation to improve the performance only when the penalty rate ($\beta$) is larger than the cost of such improvement ($(1-\alpha) C_p$). Offering appropriate penalty ($\beta$) and cost sharing ($\alpha$) rates by the customer incentivizes the supplier to choose the lowest $\varphi$ in this range ($\varphi_2 \leq \varphi \leq 1$).

As a next step, the supplier evaluates the benefit of choosing a lower $\varphi$ ($\varphi < \varphi_2$):

$$\begin{align*}
\text{Max} & \quad \Pi = w - (1 - \alpha) (SC_s + \frac{1}{\varphi} C_p) - \beta (N - \frac{1}{\varphi}) (\varphi - (\frac{1}{N})^s) \\
\text{S.t} & \quad \varphi_1 < \varphi < \varphi_2
\end{align*}$$

(14)

From the experiments, we again observe that the supplier’s benefit will be maximum around a special value of $\varphi$ ($\varphi = 1/N$) regardless of the value of base stock ($S$). To verify this, we show in Table 1 how $\varphi_1$ and $\varphi_2$ converge to this value ($1/N$) as the number of systems is increased.

<table>
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<tr>
<th>N</th>
<th>$\varphi_1$</th>
<th>$1/N$</th>
<th>$\varphi_2$</th>
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<tr>
<td>50</td>
<td>0.015</td>
<td>0.02</td>
<td>0.025</td>
</tr>
<tr>
<td>100</td>
<td>0.007</td>
<td>0.01</td>
<td>0.012</td>
</tr>
<tr>
<td>500</td>
<td>0.0018</td>
<td>0.002</td>
<td>0.0022</td>
</tr>
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</table>

Now, we can find the optimal base stock level by setting the optimal $\varphi$ ($1/N$). The supplier’s objective can be simplified as follows:

$$\begin{align*}
\text{Max} & \quad \Pi = w - (1 - \alpha) (SC_s + NC_p) - \beta (\frac{N^2}{(\frac{1}{N})^s} Ne^{N \Gamma(N+1,N)+SN})
\end{align*}$$

(15)
Proposition 1. The optimal base stock ($S^\star$) level set by the supplier is

$$S^\star = \frac{N\beta}{\sqrt{(1-\alpha)C_s}} - \frac{1}{N}\gamma \Gamma(N+1,N)$$  \hspace{1cm} (16)

Proof: The supplier’s profit is a concave function of $S$. Observe that:

$$\frac{\partial \Pi}{\partial S} = -(1-\alpha)C_s + \frac{N^2\beta}{\left(\frac{1}{N}\gamma \Gamma(N+1,N)+SN\right)^2}$$  \hspace{1cm} (17)

Given that $\frac{\partial n}{\partial S} = 0$ at optimality (the supplier’s profit is a concave function with respect to $S$), we obtain

$$S^\star = \frac{N\beta}{\sqrt{(1-\alpha)C_s}} - \frac{1}{N}\gamma \Gamma(N+1,N)$$  \hspace{1cm} (18)

As the result, if the customer offers the contract parameters ($\alpha, \beta$) such that ($\beta > (1-\alpha)C_s$), the supplier chooses ($\varphi, S$) as follows

$$\varphi^\star = \frac{1}{N} \hspace{0.5cm}, \hspace{0.5cm} S^\star = \frac{N\beta}{\sqrt{(1-\alpha)C_s}} - \frac{1}{N}\gamma \Gamma(N+1,N)$$  \hspace{1cm} (19)

From the above formulas for $\varphi^\star$, $S^\star$, we find the following results which offer insights into the impact of the contract terms on supplier’s optimal decision.

$$\begin{align*}
(1)\mu^\star &= N\lambda \\
(2)\frac{\partial S^\star}{\partial \beta} &> 0 \\
(3)\frac{\partial S^\star}{\partial \alpha} &> 0
\end{align*}$$  \hspace{1cm} (20)

From (1) we find that when the supplier cannot change the failure rate ($\lambda$), the optimal service rate is $N\lambda$, which is directly proportional to the number of systems. Parts (2) and (3) show the supplier’s reaction to the contract parameters ($\alpha, \beta$). A larger penalty rate ($\beta$) creates a stronger incentive for the supplier to decrease the number of backorders by increasing the base stock. Also, increasing the cost sharing rate ($\alpha$) by the customer motivates the supplier to increase the base stock as the customer reimburses a larger portion of the associated costs.

6. Conclusions

This study draws upon the dependency of the supplier’s reaction to the number of operational units in performance-based contracts. Under performance contracts, the customer cannot directly impact the supplier’s investment decision. Therefore, customers need to design the contracts in a way that positively motivates the suppliers to improve and maintain the system reliability. Also, the most effective incentive mechanism is introduced based on the sensitivity of the parameters that can improve the performance such as failure and service rates. As the
failure rate is a function of the number of operational systems and the service rate, the number of systems is a prominent parameter that affects the supplier’s response under performance contract. We conduct a parametric analysis to evaluate the role of total number of systems on the effectiveness of average number of backorders as an incentive under performance contract. The results show that the supplier’s decision is directly influenced by the number of systems when the customer applies the average number of backorders as a negative incentive. Our findings also highlight the impact of the penalty and cost sharing rates as contract terms (from the customer) on the decision of the base stock level by the supplier. Finally, finding the most effective incentive mechanism that is affected by system parameters such as the number of operational systems is a direction for future research.

References