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<td><strong>Author(s)</strong></td>
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Minimax Design of Nonnegative Finite Impulse Response Filters

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Abstract—Nonnegative impulse response (NNIR) filters have found many applications in signal processing and information fusion areas. Evidence filtering is one of the examples among others. An evidence filter is required to satisfy a nonnegativity condition and a normalization condition on its impulse response coefficients, and thus is basically an NNIR filter. This paper considers the design of nonnegative finite impulse response (FIR) filters based on frequency response approximation and proposes a constrained minimax design formulation using the fundamental limitations on the NNIR filter’s frequency responses recently developed in the literature. The formulation is converted into a linearly constrained positive-definite quadratic programming and then solved with the Goldfarb-Idnani algorithm. The proposed method is applicable to nonnegative FIR lowpass as well as other types of filters. Design examples demonstrate the effectiveness of the proposed method.

Keywords—nonnegative impulse response filter; finite impulse response filter; minimax design; quadratic programming

I. INTRODUCTION

Distributed sensor networks consisting of a large number of low-cost and low-power sensors have found many applications in military surveillance, homeland security, traffic surveillance, environment monitoring, etc., for collecting observations and processing information. The use of distributed nodes with multiple sensing modalities can significantly enhance the robustness and the accuracy of the decision making process in such environments [1]. Data collected by nodes are fused at various levels and in different ways in order to make useful inferences at the decision-making center. The inferences made based on the fused information range from distinguishing among different event (or threat) types, estimating important parameters such as location or velocity of an object, target tracking, as well as detecting the presence or absence of events with certain “frequency” or “spectral” characteristics [2].

Dempster-Shafer belief theory [3] has been extensively used in surveillance and security applications and is excellent in modeling imperfect sensor data by mapping into Dempster-Shafer theoretic beliefs (evidences) which are nonnegative in nature [4]. By integrating the Dempster-Shafer theory with discrete time filtering techniques, an evidence filtering method is established in [2] to fuse temporally ordered information from multiple sensor modalities and directly infer on the frequency characteristics of events.

Filtering of temporally ordered evidences can be described by a finite impulse response (FIR) or infinite impulse response (IIR) filter [2]. If an FIR filter is used, the updated (filtered) evidence is a weighted sum of the incoming evidences. If an IIR filter is used, the filtered evidence is a weighted sum of the past and incoming evidences. In order to ensure the validity of the updated evidence, the impulse response coefficients (or, the weights) of an evidence filter should be nonnegative (the nonnegativity condition), and all impulse response coefficients should sum to 1.0 (the normalization condition). These are the inherent constraints should be imposed on the evidence filters when one considers their designs.

Because of the nonnegativity of the impulse response, an evidence filter does not have arbitrarily specified frequency responses. Instead, the frequency response of an evidence filter has some fundamental limitations [5] [6]. For example, the frequency response of a nonnegative impulse response (NNIR) filter inherently has a low-frequency passband, and the maximum roll-off of its frequency response that can be achieved in the transition-band is determined by the magnitude response at the passband edge frequency [5] [6]. Realizing these fundamental frequency domain limitations, a procedure is provided...
in [6] to check whether a set of specifications are feasible for an NNIR lowpass filter and to tune the specifications when they are infeasible for the filter.

Although much research has been conducted on the time-domain and frequency-domain characteristics of NNIR filters [5]-[11], very little work has been done on the design of NNIR filters. Since nonnegative FIR filters have some time-domain and frequency-domain limitations, the design of such filters involves time-domain and frequency-domain constraints. The existing eigenfilter approaches [12] [13] and constrained least-squares methods [14] [15] can incorporate with time-domain and frequency-domain constraints, and thus are prospective methods for nonnegative FIR filter design after exploiting the filters’ fundamental frequency response limitations.

In [2], a heuristic procedure was proposed to design the nonnegative FIR filters for evidence filtering applications. This method first designs a conventional filter using the Hamming window method, and then offset the impulse response of the conventional filter to satisfy the nonnegativity and normalization conditions and iteratively smooth the offset filter to improve the stopband performance until the stopband degradation becomes satisfactory. This offset-smoothing method is not an optimization method, the designed filter is not optimal in any sense.

In this paper, we present a constrained minimax formulation for nonnegative FIR filters. At this point, we restrict the designed filters to have linear phases. Then, the constrained minimax problem is a linear programming problem. In light of Lemma 1 and Theorem 2 of [14], we convert it into a linearly constrained positive-definite quadratic programming problem, and then use the Goldfarb-Idnani algorithm to solve the quadratic programming problem. Design examples demonstrate the effectiveness of the proposed method.

II. LIMITATIONS OF NONNEGATIVE FIR FILTERS

An M-th order nonnegative FIR filter can be described by a transfer function

\[ H(z) = h_0 + h_1 z^{-1} + \cdots + h_M z^{-M}, \]  

(1)

with an impulse response \( h = [h_0, h_1, \ldots, h_M]^T \) of length \((M+1)\) satisfying the nonnegativity condition

\[ h_k \geq 0 \text{ for } k = 0, 1, \ldots, M. \]  

(2)

For an evidence filter, the impulse response of the nonnegative FIR filter should also satisfy the normalization condition

\[ h_0 + h_1 + \cdots + h_M = 1. \]  

(3)

The frequency response of the filter \( H(z) \) is described by

\[ H(e^{j\omega}) = h_0 + h_1 e^{-j\omega} + \cdots + h_M e^{-jM\omega}, \]  

(4)

and the magnitude frequency response of the filter is denoted by \( |H(e^{j\omega})| \). From Lemma 6 of [6], if the square magnitude frequency response of \( H(z) \) at \( m\omega_0 \), \( |H(e^{im\omega_0})|^2 \), is bounded by:

\[ |H(1)|^2 - \delta \leq |H(e^{im\omega_0})|^2 \leq |H(1)|^2 \]  

(5)

where \( \delta > 0, \omega_0 \in [0, \pi/(m+k)], m > 0 \) and \( k > 0 \) are integer, then the decrease in the squared magnitude response from \( m\omega_0 \) to \((m+k)\omega_0\) is bounded by

\[ |H(e^{im\omega_0})|^2 - |H(e^{i(m+k)\omega_0})|^2 \leq m^2 (k^2 + 2mk)\delta \]  

(6)

It follows that the maximum roll-off of the magnitude response that can be achieved in the transition band from the passband edge frequency \( m\omega_0 \) to the stopband edge frequency \((m+k)\omega_0\) is determined by the magnitude response at the passband edge frequency \( m\omega_0 \), which implies that the passband ripple and stopband ripple cannot be independently specified.

Considering the above limitation of the frequency response and the requirement of minimum signal-to-noise ratio in practical applications, Ref. [6] presents a set of frequency domain specifications for NNIR lowpass filters as follows:

\[ S_{LP} = \{ \omega_p, \omega_s, K, \delta_p, \delta_s \}, \]  

(7)

where \( \omega_p, \omega_s, \delta_p, \) and \( \delta_s \) respectively represent the passband and stopband edge frequencies, and the passband and stopband magnitude responses, and \( K \) is the minimal transition band attenuation must be maintained. The minimal transition band attenuation is considered as a hard requirement for the NNIR filter.

III. FORMULATION OF THE MINIMAX DESIGN PROBLEM

In this paper, we will only consider the design of nonnegative FIR filters with exact linear phases. Then an additional constraint exploiting the symmetry of the impulse response coefficients should be imposed on the filters as follows:

\[ h_k = h_{M-k} \text{ for } k = 0, 1, \ldots, M. \]  

(8)

With the above symmetry constraint, the frequency response of the filter can be described by

\[ H(e^{j\omega}) = P(\omega)e^{-j\phi(\omega)}, \]  

(9)

where \( \phi(\omega) = -M/2\omega \) is the linear phase response of the filter, and \( P(\omega) \) is the magnitude frequency response (up to a negative sign) of the filter described by

\[ P(\omega) = 2h_0 \cos(M\omega) + \cdots + 2h_{M/2-1} \cos(\omega) + h_{M/2} \]  

(10a)

for even \( M \), and
\[ P(\omega) = 2h_0 \cos(M\omega/2) + \cdots + 2h_{(M-1)/2} \cos(\omega2) \] (10b)

for odd \( M \).

By introducing
\[ x = [h_0, h_1, \ldots, h_{M/2}]^T, \] (11)
where \(|M/2|\) represents the integer part of \( M/2 \), the magnitude response \( P(\omega) \) can be rewritten as
\[ P(\omega) = c(\omega)^T x, \] (12)
where
\[ c(\omega) = \left[ 2 \cos \frac{M\omega}{2}, \ldots, 2 \cos \omega, 1 \right]^T \] (13a)
for even \( M \), and
\[ c(\omega) = \left[ 2 \cos \frac{M\omega}{2}, \ldots, 2 \cos \frac{3\omega}{2}, 2 \cos \frac{\omega}{2} \right]^T \] (13b)
for odd \( M \).

Since the maximum transition band roll-off of the magnitude response is determined by the magnitude response at the passband edge frequency, as stated in the previous section, the magnitude response \( P(\omega) \) can be determined by the magnitude response at the passband and stopband edges. For odd \( M \), the passband and stopband ripples \( \delta_p \) and \( \delta_s \) cannot be independently specified. If we require a design to maintain a minimal transition band attenuation \( K \) (in dB), then the passband and stopband ripples \( \delta_p \) and \( \delta_s \) should satisfy
\[ 20 \log_{10}(1-\delta_p) - 20 \log_{10}(1-\delta_s) \geq K, \] (14a)
or
\[ \delta_s \leq 10^{-K/20}(1-\delta_p). \] (14b)

When \( K \) is given, we choose the maximum passband magnitude response error (magnitude ripple) \( \delta_p \) to minimize and let the stopband magnitude response ripple \( \delta_s \) satisfy (14), leading to the following constrained minimax design problem:
\[ \begin{align*}
\text{minimize} & \quad \delta_p, \\
\text{subject to} & \quad 1 - \delta_p \leq |H(e^{j\omega})| \leq 1, \text{ for } \omega \in \Omega_p, \\
& \quad |H(e^{j\omega})| \leq \delta_s \leq 10^{-K/20}(1-\delta_p), \text{ for } \omega \in \Omega_s, \\
& \quad h_k \geq 0 \text{ for } k = 0, 1, \ldots, M. \\
& \quad h_0 + h_1 + \cdots + h_M = 1.
\end{align*} \] (15)

where \( \Omega_p \) and \( \Omega_s \) denote the passband and stopband of the filter. By using Eqs. (9), (11), (12), and (13), we may rewrite the above minimax problem as:
\[ \begin{align*}
\text{minimize} & \quad \delta_p, \\
\text{subject to} & \quad c(\omega)^T x \leq 1, \text{ for } \omega \in \Omega_p, \\
& \quad -c(\omega)^T x - \delta_p \leq -1, \text{ for } \omega \in \Omega_p, \\
& \quad c(\omega)^T x + 10^{-K/20} \delta_p \leq 10^{-K/20}, \text{ for } \omega \in \Omega_s, \\
& \quad -c(\omega)^T x + 10^{-K/20} \delta_p \leq 10^{-K/20}, \text{ for } \omega \in \Omega_s, \\
& \quad -x \leq 0, \\
& \quad a^T x = 1,
\end{align*} \] (16)
where \( a = [2, \ldots, 2, 1] \) for even \( M \) or \( a = [2, \ldots, 2, 2] \) for odd \( M \).

If we replace the passband \( \Omega_p \) and stopband \( \Omega_s \) by sufficiently dense discrete frequency sets, say \( \Omega_p \rightarrow \Omega_p \cap \Omega \) and \( \Omega_s \rightarrow \Omega \cap \Omega \) where \( \Omega = \{k\pi/L, k = 0, 1, \ldots, L\} \) and \( L \) is a sufficiently large positive integer (e.g. \( L = 10M \)), the above problem becomes a finite linear programming problem, the minimum cost value of which is unique.

**IV. SOLVING THE MINIMAX DESIGN PROBLEM**

In the previous section, the minimax design problem of a nonnegative FIR filter has been formulated by a finite linear programming. There are several existing algorithms for finite linear programming problems. The “linprog” program in the Matlab optimization toolbox is one among others. However, when the numbers of the optimization variables and constraint equations are large, the “linprog” program may fail to obtain the optimal solution. To this end, we convert the finite linear programming problem into an equivalent finite positive-definite quadratic programming and then solve the quadratic programming using the Goldfarb-Idnani algorithm [16].

Noting \( \delta_p \geq 0 \), the linear programming problem (16) is equivalent to the following quadratic programming problem:
\[ \begin{align*}
\text{minimize} & \quad \delta_p^2, \\
\text{subject to linear constraints} & \quad (16b) \text{ to } (16g).
\end{align*} \] (17a)

Then, by Lemma 1 and Theorem 2 of [14], we may further convert it into another quadratic programming problem as follows:
\[ \begin{align*}
\text{minimize} & \quad \delta_p^2 + \lambda x^T x, \\
\text{subject to linear constraints} & \quad (16b) \text{ to } (16g),
\end{align*} \] (18a)
where $\lambda > 0$ is a sufficiently small number. According to Lemma 1 and Theorem 2 of [14], there must exist a sufficiently small positive number $\lambda^* > 0$, such that for any $\lambda$ satisfying $0 < \lambda < \lambda^*$, the unique solution to problem (18) is also a solution to problem (17). In addition, it has been shown in [14] that for FIR filter design, $\lambda = 10^{-6}$ can be considered as sufficiently small for the equivalence between problems (17) and (18). We will choose this value for $\lambda$ in our designs.

Now the cost function in (18a) is positive definite. Then the Goldfarb-Idnani algorithm [16] can be directly used to solve the linearly constrained quadratic programming problem (18).

V. DESIGN OF LOWPASS FILTERS

For the minimax design of nonnegative FIR lowpass filters by the proposed method, the frequency domain specifications are the filter order $M$, the passband and stopband edge frequencies $\omega_p$ and $\omega_s$, and the minimum transition band magnitude response attenuation $K$ in dB, i.e.,

$$S_{LP} = \{M, \omega_p, \omega_s, K\}.$$ \hspace{1cm} (19)

The passband and stopband are defined by

$$\Omega_p = [0, \omega_p] \text{ and } \Omega_s = [\omega_s, \pi].$$ \hspace{1cm} (20)

**Example 1.** Design of 80-th order nonnegative FIR lowpass filters with a passband edge frequency $\omega_p = 0.1\pi$, a stopband edge frequency $\omega_s = 0.175\pi$, and different transition band attenuations $K$ ranging from 35 to 65 dB.

**TABLE I. DESIGN RESULTS OF THE NONNEGATIVE FIR LOWPASS FILTERS IN EXAMPLE 1**

<table>
<thead>
<tr>
<th>Method</th>
<th>Transition band attenuation $K$</th>
<th>Passband magnitude ripple $\delta_p$</th>
<th>Stopband magnitude ripple $\delta_s$</th>
</tr>
</thead>
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<tr>
<td>Proposed</td>
<td>35 dB</td>
<td>-5.54 dB</td>
<td>-40.54 dB</td>
</tr>
<tr>
<td></td>
<td>40 dB</td>
<td>-5.80 dB</td>
<td>-45.80 dB</td>
</tr>
<tr>
<td></td>
<td>45 dB</td>
<td>-5.98 dB</td>
<td>-50.98 dB</td>
</tr>
<tr>
<td></td>
<td>50 dB</td>
<td>-6.16 dB</td>
<td>-56.16 dB</td>
</tr>
<tr>
<td></td>
<td>55 dB</td>
<td>-6.34 dB</td>
<td>-61.34 dB</td>
</tr>
<tr>
<td></td>
<td>60 dB</td>
<td>-6.46 dB</td>
<td>-66.46 dB</td>
</tr>
<tr>
<td></td>
<td>65 dB</td>
<td>-6.54 dB</td>
<td>-71.54 dB</td>
</tr>
<tr>
<td>Offset-smoothing</td>
<td>33.42 dB</td>
<td>-6.58 dB</td>
<td>-40.0 dB</td>
</tr>
</tbody>
</table>

Table I lists the passband and stopband magnitude response ripples of the resultant filters with $K = 35, 40, 45, 50, 55, 60,$ and $65$ dB. For a comparison, corresponding results of the filter obtained in [6] using the offset-smoothing method in [2] are also listed in the table. It is seen that the proposed method has obtained much better filters than the offset-smoothing method. It can also be seen that the stopband magnitude ripple is much easier to reduce than the passband magnitude ripple.

The magnitude frequency responses of some of the designed nonnegative FIR filters by the proposed method are shown in Fig. 1(a) and the impulse response of the filter with a transition band attenuation of $K = 35$ dB is drawn in Fig. 1(b).

**Example 2.** Design of different order nonnegative FIR lowpass filters with a passband edge frequency $\omega_p = 0.5\pi$, a stopband edge frequency $\omega_s = 0.6\pi$, and a minimum transition band attenuation 90 dB.

**TABLE II. DESIGN RESULTS OF THE NONNEGATIVE FIR LOWPASS FILTERS IN EXAMPLE 2**

<table>
<thead>
<tr>
<th>Filter order $M$</th>
<th>Transition band attenuation $K$</th>
<th>Passband magnitude ripple $\delta_p$</th>
<th>Stopband magnitude ripple $\delta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>90 dB</td>
<td>-8.95 dB</td>
<td>-98.95 dB</td>
</tr>
<tr>
<td>70</td>
<td>90 dB</td>
<td>-8.83 dB</td>
<td>-98.83 dB</td>
</tr>
<tr>
<td>80</td>
<td>90 dB</td>
<td>-8.72 dB</td>
<td>-98.72 dB</td>
</tr>
<tr>
<td>90</td>
<td>90 dB</td>
<td>-8.64 dB</td>
<td>-98.64 dB</td>
</tr>
<tr>
<td>100</td>
<td>90 dB</td>
<td>-8.46 dB</td>
<td>-98.46 dB</td>
</tr>
</tbody>
</table>

Table II lists the passband and stopband magnitude response ripples of the resultant filters with orders $M = 60, 70, 80, 90,$ and $100$. It is seen that the magnitude response ripple is difficult to reduce by increasing the filter order if the transition
band magnitude attenuation keeps invariant. The magnitude frequency response and impulse response of the 60-th order filter are shown in Fig. 2.

![Figure 2](image_url)

Figure 2. (a) Magnitude response and (b) impulse response of the 60-th order nonnegative FIR lowpass filter in Example 2

VI. DESIGN OF HIGHPASS AND BANDPASS FILTERS

The minimax design method presented in Sections III and IV is not only able to design nonnegative FIR lowpass filters, but also able to design other types of nonnegative FIR filters, such as bandpass filters and highpass filters. However, besides the desired passband, there will be a spurious passband $[0, \omega_{sp}]$ at the low frequency region for these two types of filters due to the fundamental frequency domain limitations of the nonnegative FIR filter.

The frequency domain specifications for nonnegative FIR bandpass filters are the filter order $M$, the spurious passband edge frequency $\omega_{sp}$, the passband edge frequencies $\omega_p$, and the minimum transition band magnitude response attenuation $K$ in dB, i.e.,

$$S_{bp} = \{ M, \omega_{sp}, \omega_p, K \}. \quad (21)$$

The passband and stopband should be

$$\Omega_p = [ \omega_p, \pi ] \text{ and } \Omega_s = [ \omega_{sp}, \omega_p ] \text{.} \quad (24)$$

Example 3. Design of a 399-th order nonnegative FIR bandpass filter with a low stopband edge frequency $\omega_{s1} = 0.75\pi$, a passband left edge frequency $\omega_{p1} = 0.77\pi$, a passband right edge frequency $\omega_{p2} = 0.83\pi$, and a transition band attenuation $K = 85$ dB. The spurious passband edge frequency is $\omega_{sp} = 0.05\pi$.

The resultant filter has a passband ripple of -11.2 dB and a stopband attenuation of 96.2 dB. The magnitude frequency response and impulse response of the filter are drawn in Fig. 3.

Example 4. Design of a 100-th order nonnegative FIR high-
pass filter with a stopband edge frequency \( \omega_s = 0.75\pi \), a passband edge frequency \( \omega_p = 0.85\pi \), and a transition band attenuation \( K = 40 \text{ dB} \). The spurious passband edge frequency is \( \omega_{sp} = 0.15\pi \).

The resultant filter has a passband ripple of \(-8.78 \text{ dB}\) and a stopband attenuation of \(48.78 \text{ dB}\). The magnitude frequency response and impulse response of the filter are drawn in Fig. 4.

**Figure 4.** (a) Magnitude response and (b) impulse response of the 100-th order highpass nonnegative FIR filter in Example 4

VII. CONCLUSIONS

The design of nonnegative FIR filters has been investigated in this paper, and a constrained minimax formulation has been presented. The constrained minimax formulation has taken into account the nonnegativity and normalization constraints on the NNIR filter’s impulse response as well as fundamental limitations on the NNIR filter’s frequency response. By converting the constrained minimax formulation into a linearly constrained quadratic programming, the design problem can be efficiently solved by the Goldfarb-Idnani algorithm. The proposed method has been effectively applied to the design of nonnegative FIR lowpass, highpass and bandpass filters. Design results show that the proposed method has obtained much better filters than the filter obtained in [6] by the offset-smoothing method in [2].

**REFERENCES**


