Highly sensitive refractive index sensor based on two cascaded microfiber knots with vernier effect

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ABSTRACT

A highly sensitive refractive index (RI) sensor based on two cascaded microfiber knots with vernier effect is proposed and demonstrated by theoretical arithmetic. Deriving from high proportional evanescent field of microfiber and sharp spectrum fringes induced by vernier effect, a slight change of ambient RI will cause large variation of effective RI and significant wavelength shift of resonant peaks, indicating high sensitivity and resolution of the proposed compound resonator. Numerical analysis demonstrates a high sensitivity of 10000nm/RIU and a resolution of $5.57 \times 10^{-5}$ RIU at the ambient RI around 1.33 for the fiber diameter of 1 $\mu$m and cavity radii of $R_1 = 500\mu$m, $R_2 = 547.62\mu$m.

Keywords: microfiber, knot resonator, cascaded knots, vernier effect, refractive index sensor

1. INTRODUCTION

Compared with standard optical fiber, micro/nano fiber (MNF) has many unique properties such as tight optical confinement, high fraction of evanescent fields, low optical loss through sharp bending[1], making it a reliable candidate to measure ambient refractive index (RI). Until now, abundant microfiber-based structures and schemes have been proposed for RI sensing such as microfiber-based Mach-Zender interferometer [2], micro/nano fiber Bragg grating [3] and microfiber loop resonator [4], etc.

Due to its ability to induce transmission spectrum with high Q factor and sensitivity to cavity length, vernier effect yielded by cascading double fiber rings has already been applied to high performance sensors such as strain sensor [5] and micro-displacement sensor [6]. However, microfiber-based vernier effect has not yet been reported to the best of our knowledge.

In this letter, we design a RI sensor based on a compound resonator consisting of two microfiber knot resonators (MKRs) cascaded through a bus microfiber. The slightly different cavity length of two MKRs would generate vernier effect. Because of the vernier effect, the compound resonator is capable of better performance in RI sensing than monolithic MKR, by offering wider free spectrum range (FSR), better fringe fineness and higher Q value. A theoretical analysis is carried out to investigate the vernier effect and the enhancement mechanism of RI sensing. After that the simulation result is presented to verify the ability of proposed resonator to measure RI with high sensitivity and large measuring range. The influences of structure parameters on RI sensing characteristics are also discussed in detail.

2. DESIGN AND OPERATION PRINCIPLE

2.1 Vernier effect

The proposed compound resonator is a special parallel arrangement of two MKRs with four twist regions connecting them together, as shown in Fig.1. By assembling two rings of slightly unequal length it is possible to induce vernier effect and thus to extend FSR. The working principle of vernier effect can be explained as follows: because of the high fractional evanescent fields, the parallel placed two MNF segments could function as a coupler, leading to transfer of light from one segment to another. Assume that light is launched into the first MKR (MKR1) through coupling area I and...
subsequently oscillates in counter-clockwise direction. At coupling area II, part of the power can be coupled to the bus microfiber and then transmit to the second MKR (MKR2) through coupling area III. Similarly, the light oscillates unidirectionally in MKR2 and is partly coupled to the output port at coupling area IV. Being restricted by resonant principle, only signal light whose frequency satisfies the phase resonant conditions of both MKRs could exist solidly, while others will be suppressed and thus can’t be output. Therefore, an interference pattern with broadened FSR could be expected by measuring the transmission spectrum of the device. The phase resonant condition of compound resonance can be described as:

\[
\begin{align*}
\beta_1 S_1 &= (m+1/2) \pi, m = 0, 1, 2, \ldots \ldots \\
\beta_2 S_2 &= (n+1/2) \pi, n = 0, 1, 2, \ldots \ldots \\
n &= m + N, N = 1, 2, 3, \ldots \ldots
\end{align*}
\]

where \(\beta_1, \beta_2\) are the propagation constants in MKR1 and MKR2, \(S_1 = 2\pi R_1, S_2 = 2\pi R_2\), \(R_1\) and \(R_2\) are radii of MKR1 and MKR2, respectively. FSR of MKR1 (FSR\(_1\)) and MKR2 (FSR\(_2\)) are respectively given by

\[
FSR_1 = \frac{c}{2\pi n_{\text{eff}1} R_1}, \quad FSR_2 = \frac{c}{2\pi n_{\text{eff}2} R_2}
\]

where \(c\) denotes the velocity of light in vacuum, \(n_{\text{eff}1}, n_{\text{eff}2}\) are effective RIs of two MKRs.

To present vernier effect more clearly, transmission spectra of MKR1, MKR2 and cascaded MKRs are shown in Fig. 2 (a), (b) and (c) respectively. In our design, \(R_1 < R_2\) results in FSR\(_1\) > FSR\(_2\). The maximal peaks of the vernier transmission spectrum are located at the mutual resonant wavelength of MKR1 and MKR2, while other resonant peaks of individual MKR1 or MKR2 are suppressed to some extent. Note that in spite of the existence of subordinate peaks in Fig. 2(c) resulting from nonzero linewidth of single MKR, we still define FSR of the compound resonator as the wavelength difference of two adjacent maximal peaks. Specially, if \(R_1/R_2 = (m + 0.5)/(m+1.5)\) (where \(m=1, 2, 3 \ldots \ldots\) ) is chosen, every \(m\)-th resonant peak of MKR1 will overlap with every \((m+1)\)-th resonant peak of MKR2, implying that the period of cascaded MKRs is \(m\) times of MKR1 or \((m+1)\) times of MKR2. Therefore, FSR of the compound resonator (FSR\(_{\text{total}}\)) can be given by

\[
FSR_{\text{total}} = m \times FSR_1 = (m+1) \times FSR_2
\]

2.2 Refractive index sensing

If we change the effective RI of MKR2 (\(n_{\text{eff}2}\)) by slightly varying its ambient RI (\(n_a\)), the resonance wavelength of MKR2 (\(\lambda_2\)) will shift as \(\Delta \lambda_2 = \lambda_2 \left( \Delta n_a / n_{\text{eff}2} \right) \left( \Delta n_{\text{eff}2} / \Delta n_a \right)\), where \(\Delta n_{\text{eff}2}, \Delta n_a\) are the variation of effective RI and ambient RI of MKR2. According to Eq. (3), the corresponding change in resonant wavelength of compound resonator (\(\Delta \lambda\)) can be described as:

\[
\Delta \lambda = \Delta \lambda_2 \left[ \frac{FSR_1}{FSR_1 - FSR_2} \right]
\]

Then the sensitivity can be deduced as:

\[
S = \Delta \lambda / \Delta n_a = \left( \lambda_2 / n_{\text{eff}2} \right) \left[ \Delta n_{\text{eff}2} / \Delta n_a \right] \left[ \frac{FSR_1}{FSR_1 - FSR_2} \right]
\]
From Eq. (5), it is clear that when compared with single MKR2, the sensitivity of the compound resonator is magnified by $FSR_r/(FSR_1 - FSR_2)$. With Eq. (2) being taken into consideration, to get a higher amplification, the radius of MKR1 should be designed as small as possible.

The measuring range could also be inferred from vernier effect. Since $FSR_r$ is fixed, the minimum and maximum change of resonant wavelength in compound resonator are $\Delta \lambda_{\text{min}} = FSR_r$ and $\Delta \lambda_{\text{max}} = FSR_{\text{total}}$ respectively. Therefore, the lower and upper limit of ambient RI variations could respectively be denoted as:

$$\Delta n_{\text{min}} = n_{\text{eff},2} \left( FSR_2 - FSR_r \right) / \left( \lambda_2 \cdot \Delta n_{\text{eff},2} / \Delta n_a \right)$$  \hspace{1cm} (6)$$

and

$$\Delta n_{\text{max}} = n_{\text{eff},2} FSR_r / \left( \lambda_2 \cdot \Delta n_{\text{eff},2} / \Delta n_a \right)$$  \hspace{1cm} (7)$$

3. NUMERICAL SIMULATION AND DISCUSSIONS

A theoretical model is set up to simulate the response of compound resonator to ambient RI variation. The dependence of sensitivity on the structure parameters is discussed to optimize these parameters for better sensing performance. The coupling parameters are set as $k_{s1} = 1 - k_{s2} = 1, k_{s3} = k_{s4} = 0.99$, $r_{s1} = r_{s2} = r_{s3} = r_{s4} = 0.1$, where $k_{s1}, k_{s2}, k_{s3}, k_{s4}$ and $r_{s1}, r_{s2}, r_{s3}, r_{s4}$ are coupling efficiencies and coupling losses of four coupling areas respectively.

![Fig. 3. (a) Transmission spectra of a compound resonator with cavity radius $R_1$ of 500μm and fiber diameter of 1500nm evolving as the variation of ambient RI; (b) Comparison of spectra after RI varying one measuring period.](image)

Fig. 3 presents the transmission spectra of a compound resonator evolving as the variation of ambient RI. The compound resonator has cavity radii of $R_1 = 500\mu m$, $R_2 = 547.62\mu m$ and fiber diameter of 1.5μm. We carry out the simulation around the index value of 1.33 with an increase at an interval of $\Delta n_{\text{min}} = 1.0733 \times 10^{-4}$. The resonant wavelength experiences a red shift as the ambient RI increasing (Fig. 3(a)), and will almost coincide with the adjacent major peak of initial spectrum (Fig. 3(b)). A further analysis of the dependence of the resonant wavelength on ambient RI shows that the sensitivity as high as 5300nm/RIU can be achieved, as depicted with purple triangles in Fig. 4(a).

The role of compound resonator on enhancing RI sensitivity is confirmed by comparing its sensitivity with that of individual MKR with the same parameters, as shown in Fig. 4(a). The sensitivity 5300nm/RIU of compound resonator is 10.5 times larger than the sensitivity 460nm/RIU of single MKR, implying an overwhelmingly good performance of compound resonator. Furthermore, compound resonator has a narrower linewidth than individual MKR, which enables a higher RI measuring resolution and thus a more precise sensing ability.

To discuss the effect of fiber diameter on RI sensitivity, the cavity radii and the initial ambient RI are fixed to $R_1 = 500\mu m$, $R_2 = 547.62\mu m$ and 1.33 respectively, while different fiber diameters of 1μm, 1.139μm and 1.5μm are set to compare their sensing characteristics, as shown in Fig. 4(b). It is clear that the sensitivity of the sensor increases with the decrease of fiber diameter. The highest sensitivity of 10000nm/RIU for a 1μm diameter microfiber compound resonator is almost two times of the value of 5300nm/RIU for a 1.5μm diameter microfiber compound resonator. Besides, the resolution of the 1μm diameter microfiber compound resonator could be down to 5.57×10^{-5} as deduced from Eq. (6). The reason lies in that the decreasing microfiber diameter will allow more fundamental mode to leak out and make the microfiber more vulnerable to ambient RI variation.

Besides, the influence of cavity radius is also taken into consideration. Fig. 4(c) presents the dependence of resonant wavelength on the ambient RI variation for three compound resonators with the same fiber diameter of 1.5μm but different cavity radii of $R_1 = 250\mu m$, 500μm and 1000μm. Three parallel fitting lines indicate their identical RI...
sensitivities. However, the measuring resolution and the maximum measurable value for the 250μm radius cavity are $2.1467 \times 10^{-4}$ and $2.5231 \times 10^{-3}$, while that of 1000μm radius cavity are $5.367 \times 10^{-5}$ and $5.6308 \times 10^{-4}$. Therefore, in practical application, a compromise should be made between resolution and measuring range.

Fig. 4. The dependence of resonant wavelength on ambient RI variation at different sensing structure (a), different microfiber diameters (b) and different cavity radii (c).

### 4. CONCLUSION

A highly sensitive RI sensor based on an all-microfiber compound resonator with veriniier effect is proposed and demonstrated by numerically simulating. Compared with single MKR, the compound resonator has a RI sensing enhancement in sensitivity. For microfiber knots with 500μm cavity radius and 1.5μm fiber diameter, the sensitivity of $5300\text{nm/RIU}$ for the compound resonator is 10.5 times larger than the value of $460\text{nm/RIU}$ for monolithic MKR. The analysis of the effects of fiber diameter and cavity radius on RI sensing characteristics indicates that RI sensitivity increases as fiber diameter decreases, but keeps unchanged with the cavity radius variation. For ambient RI around 1.33, high sensitivity of $10000\text{nm/RIU}$ and resolution of $55.57 \times 10^{-3}$ could be achieved by the compound resonator with fiber diameter of 1μm and cavity radii of $R_1 = 500\mu m$ and $R_2 = 547.62\mu m$. The sensing performance of the compound resonator could be further enhanced by reducing the fiber diameter or working in the RI region from 1.35 to 1.4 [2].

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