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<td>Wang, Ran; Wang, Ping; Xiao, Gaoxi; Gong, Shimin</td>
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Power Demand and Supply Management in Microgrids with Uncertainties of Renewable Energies

Ran Wang, Ping Wang, Gaoxi Xiao and Shimin Gong

Nanyang Technological University, Singapore

Abstract

In the operation of microgrids, an important task is to match the power generation and consumption at a minimum cost. Since the highly fluctuant renewable energies constitute a significant portion of the power resources in microgrids, the microgrid system central controller (MGCC) faces the challenge of effectively utilizing the renewable energies while fulfilling the requirements of customers. In this paper, a power demand and supply management framework is proposed to tackle the problem stated above. A novel uncertainty model is first developed to capture the randomness of renewable energy generation. Specifically, we introduce a reference distribution according to the past observations and empirical knowledge, and then define a distribution uncertainty set to confine the uncertainty of renewable energies. The new model allows the renewable energy to fluctuate around the reference distribution. An optimization problem is then formulated to determine the optimal power consumption and generation scheduling for minimizing the fuel cost. A two-stage optimization approach is presented to first transform and then solve the prime problem. Numerical results indicate the properties of our problem formulation and provide some illuminations on the policy making for the MGCC. It is also shown that the proposed power demand and supply management mechanism can effectively reduce the energy cost.

Keywords: microgrids, demand and supply management, renewable energy uncertainties, reference distribution, robust optimization

Email address: {wang0686, wangping, egxxiao, gong0012}@ntu.edu.sg
(Ran Wang, Ping Wang, Gaoxi Xiao and Shimin Gong)
1. Introduction

The smart grid is the innovative future electric power system that will improve the conventional electrical grid network to be more clean, reliable, secure, cooperative, and efficient. The growth and evolution of the smart grid is expected to come with the plug-and-play integration of the basic structures called microgrids. Specifically, microgrids are small-scale low voltage power supply networks designed to supply electrical load for a small community such as a university campus, a commercial area and a trading estate, etc. Microgrids can autonomously coordinate local generations and demands in a dynamic manner. It can operate in either grid-connected mode or islanded mode \cite{1}. There have been world wide deployments of pilot microgrids, such as those in US, Germany, Greece and Japan \cite{2}.

Microgrids are expected to be more robust and cost-effective than the traditional approach of centralized grids. However, to achieve a stable and economical operation, a number of technical and regulatory issues have to be resolved before the microgrid can become a commonplace. One problem that would require due attention is the effective management of power supply and demand loads, which amounts to matching the power generation and consumption profiles \cite{3} \cite{4}. Specifically, the power generators or microsources employed in microgrids are usually renewable or non-conventional distributed energy resources. While the incorporation of such renewable resources shall certainly bring great environmental benefits, it imposes new challenges as well: different from that in the traditional power systems with conventional controllable electric generators, generation scheduling in microgrids with fluctuant, climate-dependent renewable energy sources has to cope with the nontrivial uncertainties.

The microgrids may adopt hierarchical or decentralized demand control schemes \cite{5} \cite{6}. The decentralized control schemes facilitate distributed control and management of large complex systems. However, such control requires significant experiments before implementation and sophisticated coordination. Also it may introduce new security challenges. Hierarchical control is performed by a master controller which is responsible for matching the generation and load. When the demand resources are controlled upon the occurrence of disturbance, the strategy is often known as direct load control \cite{7} \cite{8}. In direct control program, based on an agreement between the central controller and customers, the controller can remotely control the operations of certain appliances in a household. This capability can be especially effective where there are electric devices allowing flexible usage time and/or energy storage, such as electric water heater (EWH) equipped
with hot water storage tank and plug-in hybrid electric vehicles (PHEVs), etc. The Kyotango microgrid project in Japan is an example of hierarchically controlled microgrid [2].

This paper tackles the basic problem faced by the microgrid system central controller (MGCC), namely to achieve a good match between power demand and supply subject to uncertainties of renewable energies. On the power demand side, we envision a scenario with real-time communication between the controller and energy consumer premises. Specifically, in each time period, the operator controller receives consumer power demands with different power level requirements, durations and time elasticity levels. The MGCC needs to minimize the electricity generation cost by optimally scheduling the operation of each appliance subject to the requirements set by the users. Here the generation fuel cost is modeled as a convex function of instantaneous total power consumption.

On the power supply side, MGCC has to focus on how to effectively manage power generation so as to match the user load and maintain system reliability. We propose a novel uncertainty model to capture the fluctuant nature of renewable energies. Compared with previous robust optimization based approaches which confine the renewable energy within a lower and upper bounds, the proposed model provides more statistical details in describing the underlying uncertainty (The previous robust approaches will be discussed in Section 2.). Specifically, an empirical distribution is extracted as a useful reference, which allows the actual distribution of renewable energies to vary around it. To the best of our knowledge, this is the first time that the distribution uncertainty model is adopted to depict the indeterminacy property of renewable energy generation. The load balance constraint is aptly approximated using the chance constraint representation, which allows convenient tuning of the conservation level of the solution using a single parameter. A tractable robust optimization approach is developed for transforming the chance constraints into linear constraints, and some of the desired properties of the problem formulation are investigated, providing illuminations for the policy making of MGCC. It is shown that the proposed power demand and supply management scheme can greatly reduce the energy cost for the microgrid system.

The remainder of this paper is organized as follows. Section 2 provides a brief survey of the related work. In Section 3, we show the mathematical depiction of the power demand and supply management problem and the uncertainty model of the renewable energies. Section 4 presents the robust approach for handling the load balance constraint, and the problem decomposition process. We provide the simulation results and discussions in Section 5 and conclude the paper in Section 6.
2. Related Work

The problem we are tackling can be viewed as containing two different parts. On the power demand side, we try to build a hierarchical demand control scheme so as to achieve the economic consumption scheduling and fulfill the requirements set by energy users; on the power supply side, we need to properly model the randomness of renewable energy generation, which may account for a significant portion of power supply in microgrids. Note that load balance constraints act as the connection between power consumption and generation.

Demand control techniques can be categorized into either price based load control techniques, referred to as demand response methods, or direct load control, referred to as demand side management. Under price based load control scheme, users are encouraged to make energy consumption decisions individually according to the price information. Demand side management strategies, however, are usually applied directly by a central controller and require consumer subscription to an economic incentive program. Several representative works have studied demand control techniques in residential microgrids. A recent paper [9] developed a real-time pricing scheme that aims at reducing the peak to average load ratio (PAR) through demand response management in smart grid systems. A two-stage optimization problem was then proposed and solved. Fathi et al. developed a stochastic model of scheduling in a local area network with the objective of cost minimization and PAR minimization [10]. The work [11] presented a linear programming formulation for minimizing the energy cost through direct load control. In [12] an optimization model was presented to adjust the hourly load level of a given consumer in response to hourly electricity prices. The electricity price uncertainty was modeled through robust optimization techniques. The uncertainties of renewable energies, however, are not considered in these studies. As such, the control schemes may not be readily optimal and applicable to the microgrid scenario where renewable energies constitute a significant portion of power resources.

There also exist some studies considering renewable energy uncertainties when scheduling the energy generation. Such work can be categorized into two groups: the stochastic based approaches and the robust optimization based approaches. For instance, Wang et al. defined stochastic upper and lower supply curves to capture a broad range of fluctuations in the power system, where energy generated by each power source was modeled as stochastic arrivals in the queuing model [13]. In [14], scenario-based stochastic operation management methods were developed to tackle the fluctuant demands and renewable energies using the probability
distribution function (PDF) of each uncertain variable. Hidden Markov models have also been adopted to characterize renewable energy generation [15] [16] [17]. Stimulated by observations that in practical scenarios, obtaining an accurate distribution function could be computationally costly and renewable energy may not follow Markov process or any simple distributions, robust optimization has recently received growing attention as a modeling framework for optimization under uncertainty. Instead of assuming explicit probability distribution, robust optimization confines the renewable generation in a pre-defined uncertainty set containing the worst-case scenario. For example, Zhang et al. considered a distributed economic dispatch problem for microgrid with high penetration of renewable energies [18]. The intrinsically stochastic properties of renewable energy sources are captured by a polyhedral uncertainty set with deterministic lower and upper bounds. Similar methods for modeling renewable energies can also be found in other recent work [19] [20]. Different from the existing work, our approach jointly considers power demand and supply management. Rather than assuming there is available knowledge of the specific distribution of renewable energy generation, the proposed approach describes the underlying uncertainty in a more detailed yet flexible manner. It allows more information of renewable energy generation to be effectively incorporated into the uncertainty model when such information is available.

3. Formulation of the Microgrid Demand and Supply Management Problem

In this section, a mathematical representation of the energy consumption and generation scheduling problem in an islanded microgrid with renewable energies is provided. An MGCC is responsible for scheduling the operations of the microgrid as well as performing optimization for minimizing the electricity generation cost for the microgrid system. The operations of the system and its mathematical depictions are introduced from the energy user side and energy generation side, respectively. The uncertainty model for describing the randomness of renewable energies is then demonstrated.

3.1. Energy Demand Side

Consider a group of energy consumers participating in this energy consumption scheduling program. It is assumed that there are two-way communication infrastructures (e.g., a local area network (LAN)) between MGCC and energy consumers. Let \( A \) denote the set of appliances belonging to these consumers, which may include PHEVs, dishwashers, cloth dryers, air conditioners, etc. Time
is divided into discrete time slots with equal length. For each appliance $a$ that is switched on, the active power consumed during one unit of time slot is $x_a$. An energy consumption scheduling vector $y_a$ is also defined for each appliance $a$ as follows:

$$y_a = [y_a^1, ..., y_a^H]$$

(1)

where $H \geq 1$ is the scheduling horizon indicating the number of time slots ahead that are taken into account for decision making in the energy consumption scheduling. For each coming time slot $h \in \mathcal{H} = [1, 2, ..., H]$, a binary variable $y_a^h = 0/1$ denotes the state of appliance $a$ (on/off). Under such case, the actual energy consumption for appliance $a$ at time slot $h$ can be expressed as $x_a \cdot y_a^h$.

There is usually an upper limit on the total energy consumption in the microgrid in each time slot. Denoting this limit as $E^{max}$, we have:

$$\sum_{a \in A} x_a \cdot y_a^h \leq E^{max}, \quad \forall h \in \mathcal{H}. \tag{2}$$

Next, assume that for each appliance $a \in A$, the user indicates $\alpha_a, \beta_a \in \mathcal{H}$ as the beginning and end of a time interval in which the appliance $a$ can be scheduled, respectively. Obviously, $\alpha_a < \beta_a$. For instance, the user may select $\alpha_a = 8$ PM and $\beta_a = 6$ AM (the next day) for his PHEV so that he could plug it in at night and get it fully charged before going to work the next day. Denote the minimum number of time slots needed for appliance $a$ to finish its preset work as $T_a$. Given the predetermined parameters $\alpha_a, \beta_a, T_a$, the appliance scheduling is subject to the following constraints:

$$\sum_{h=\alpha_a}^{\beta_a} y_a^h \geq T_a, \quad \forall a \in A, \tag{3}$$

and

$$y_a^h = 0, \quad \forall a \in A, \quad \forall h \in \mathcal{H} \setminus [\alpha_a, \beta_a]. \tag{4}$$

Constraint (3) shows that the time length $\beta_a - \alpha_a$ needs to be large enough to allow finishing the normal operation of appliance $a$. In addition, the energy user can choose proper $\alpha_a, \beta_a$ and $T_a$ to indicate whether the operation of appliance $a$ needs to be started immediately ($\beta_a - \alpha_a = T_a$) or can be deferred ($\beta_a - \alpha_a > T_a$).
To reveal the ramping down and ramping up limits on load levels of each time slot, we have:

\[
\sum_{a \in A} x_a \cdot y_a^h - \sum_{a \in A} x_a \cdot y_a^{h+1} \leq r^D, \quad h \in [1, 2, \ldots, H - 1],
\]

(5)

\[
\sum_{a \in A} x_a \cdot y_a^{h+1} - \sum_{a \in A} x_a \cdot y_a^h \leq r^U, \quad h \in [1, 2, \ldots, H - 1].
\]

(6)

In this regard, it is assumed that each household participating in this energy consumption scheduling program is equipped with a smart meter, which is capable of detecting the electric power level of each appliance. The energy consumer announces to the MGCC his needs by selecting parameters \(\alpha_a, \beta_a\) and \(T_a\) for each appliance \(a \in A\).

The above constraints (2) to (6) describe common characteristics of household appliances. However, there exist some appliances of which the operation cannot be interrupted. Such kind of loads are called as uninterruptible loads. Discussions on how such loads may be handled are presented below.

**Operation of Uninterruptible Loads:** Some loads are interruptible, such as PHEV, which means that it is possible to charge the battery for some time, stop charging for some time and then switch on the charging process again. Some other loads, however, are not interruptible, e.g., microwave oven. Appliances generating such loads, once started, have to be finished in one go. For each uninterruptible appliance \(a \in A'\), where \(A'\) represents the set of uninterruptible appliances, and each time slot \(h\), let \(z_a^h\) denote an auxiliary binary variable such that \(z_a^h = 1\) if appliance \(a\) starts operation at time slot \(h\) and \(z_a^h = 0\) otherwise. We have

\[
\sum_{h=\alpha_a}^{\beta_a-T_a+1} z_a^h = 1
\]

(7)

and

\[
z_a^h = 0, \quad \forall h \in \mathcal{H}\setminus[\alpha_a, \beta_a - T_a + 1].
\]

(8)

Then we relate start time vector \(z_a^h\) with decision variable vector \(y_a^h\) as follows:

\[
y_a^h \geq z_a^h, y_a^{h+1} \geq z_a^{h+1}, \ldots, y_a^{h+T_a-1} \geq z_a^{h+T_a-1} = z_a^h.
\]

(9)

From (9), if \(z_a^h = 1\), then \(y_a^h = y_a^{h+1} = \ldots = y_a^{h+T_a-1} = 1\).
3.2. Energy Supply Side

We now turn to the energy supply side to consider the load balance constraint in the microgrid. The microgrid may be considered as a graph consisting of three nodes as illustrated in Fig. 1. The first node represents the renewable energy generation sources such as wind turbines, solar panels and fuel cells. At time slot $h$, denote the total energy generated in this node as $\xi_h$, where $\xi_h$ is a random variable of which the probability density function may not be known. Node 2 in Fig. 1 represents the load connected through the transmission line to node 1 and node 3. The load at time $h$, denoted as $l^h$, is dependent on the energy consumption from the user side which, from the above analysis, can be expressed as:

$$l^h = \sum_{a \in A} x_a \cdot y^h_a.$$  \hfill (10)

Finally the third node includes a group of controllable electricity generators, which has a total amount of generation $P_{cg}^h$ as commanded by MGCC. Controllable generators in microgrid typically include gas turbines, micro-turbines, reciprocating internal combustion engines with generators, etc. These generators are powered by fossil fuels and can be controlled to compensate the mismatch between load and renewable power supply. A key requirement to the MGCC is to set the generation source power such that the supply could meet the demand. This statement can be mathematically described as

$$\xi_h + P_{cg}^h \geq l^h.$$  \hfill (11)

3.3. Problem Formulation

The objective function of MGCC can be defined in terms of minimizing the energy cost of the whole microgrid system. Hence the optimal energy consump-
The generation scheduling problem is formulated as follows:

$$\min \sum_{h=1}^{H} C_h(P_{cg}^h)$$  \hspace{1cm} (12)

subject to (2) to (11)

where $C_h(\cdot)$ is the cost function of electricity plant in the microgrid, which is assumed to be an increasing convex function. The convex property reflects the fact that each additional unit of power needed to serve the demands is provided at a higher cost. Example cases include the quadratic cost function $[21][22]$ and the piecewise linear cost function $[23][3]$, etc. Without loss of generality, we consider quadratic cost function $C_h(P_{cg}^h) = a_h P_{cg}^{h2} + b_h P_{cg}^h + c_h$ throughout this paper, where $a_h \geq 0$, $b_h \geq 0$ and $c_h \geq 0$ are known parameters for each time slot $h$. In practice, the coefficient of the quadratic term is usually small. Therefore, the quadratic cost function can be reduced to a linear cost function. As to the renewable energy cost, for typical renewable energies (e.g. solar and wind energy), capital cost dominates. The operation and maintenance costs are typically very low or even negligible $[24][25]$. In this paper, we consider the scenario where renewable energy generators such as solar panels and wind turbines have already been installed; hence the marginal cost of renewable energy is neglected, leading to its omission in the objective function $[26]$. The main difficulty in solving problem (12) is the indeterminacy of renewable energy generation $\xi_h$ that exists in constraint (11). Note that to optimize over the space defined by (11) amounts to solving an optimization problem with potentially large or even infinite number of constraints. Obviously, this realization of uncertainty is intractable. Next, a practical and flexible model will be developed to capture the uncertainty of $\xi_h$.

3.4. Probability Distribution Measure of Renewable Energies

It is generally difficult to characterize the renewable energy generation. In previous optimization approaches, operations on the random variable $\xi_h$ is cumbersome and computationally intractable. Moreover, in practice, we may not know the precise distribution of $\xi_h$. Solutions based on assumed distributions hence may not be justified. The variability of a random variable is usually measured using its variance or second moments which, however, may not provide sufficient details in describing the random variable. In this paper, a reference distribution, rather than moment statistics is extracted from historical data that will capture the distribution properties. Since renewable energy generation distribution is fluctuating over time and hard to be described in a closed-form expression, empirical distribution may
adopted as a useful reference and allow the actual distribution to fluctuate around it. For example, it may be assumed that the renewable energy generation distribution $f_0(\xi^h)$ is shifting around a known Gaussian distribution (or other distribution) $g_h(\xi^h)$, which can be obtained based on long-term field measurement.

The discrepancy between $f_0(\xi^h)$ and its reference $g_h(\xi^h)$ can be described by a probabilistic distance measure, for example the Kullback-Leibler (KL) divergence [27], which is a non-symmetric measure of the difference between two probability distributions. Name these two distributions as $f(\xi^h)$ and $g(\xi^h)$, respectively. Generally, one of the distributions, say, $f(\xi^h)$, represents the real distribution through precise modeling, while the reference $g(\xi^h)$ is a closed-form approximation based on the theoretic assumptions and simplifications. The definition of the KL divergence between two continuous distributions is given as follows:

$$D_{KL}(f(\xi^h), g(\xi^h)) = \int_{\xi^h \in S} [\ln f(\xi^h) - \ln g(\xi^h)] f(\xi^h) d\xi^h,$$

where $S$ is the integral domain. When distributions $f(\xi^h)$ and $g(\xi^h)$ are close to each other, the distance measure is close to zero. Adopting the KL divergence, the distribution uncertainty set is defined as follows:

$$U_r(g(\xi^h), D_0) = \{f(\xi^h) \mid \mathbb{E}_f[\ln f(\xi^h) - \ln g(\xi^h)] \leq D_0\},$$

where $D_0 \geq 0$ represents a distance limit and is obtained from empirical data or real-time measurement. It indicates energy generation’s variation level. If the energy generation is very volatile, we have less confidence on the reference distribution and thus may set a larger distance limit.

Considering the renewable energy generation distribution $f_0(\xi^h)$ with reference distribution $g_h(\xi^h)$ and distance limit $D_h$, we have the following constraints for renewable energy generation distribution $f_0(\xi^h)$:

$$\mathbb{E}_{f_0}[\ln f_0(\xi^h) - \ln g_0(\xi^h)] \leq D_h,$$

$$\mathbb{E}_{f_0}[1] = 1.$$  

Given (15) and (16), now it is ready to transform the load balance constraint (11) to allow efficient solution of problem (12).

**Remark:** Finding a proper reference distribution and obtaining an appropriate distance limit sometimes may be difficult, especially when still in lack of historical data. However, there are good reasons to expect that in most cases, situation could be improved quickly with continuous accumulation of historical records.
4. Optimization Algorithms

In this section, the optimization algorithms for solving the prime problem \((12)\) is developed. First a robust approach for handling the load balance constraint is presented, and then the prime problem is decomposed into a subproblem and a main problem to allow easier solution. Finally, the possible extensions of the proposed algorithm are briefly discussed.

4.1. Robust Approach for the Load Balance Constraint

As shown in \((11)\), the load balance constraint is \(\xi^h + P_{eq}^h \geq l^h\). In practice, a decision criterion is to set \(P_{eq}^h\) and \(l^h\) in such a way that we are confident that the load balance constraint is achieved. To achieve that, we may introduce a small value \(\epsilon\) to control the degree of conservatism and change the above expression into a chance constraint:

\[
P(\xi^h \leq l^h - P_{eq}^h) \leq \epsilon
\]  

(17)

where \(\epsilon\) is the fault tolerance limit of the power grid, representing the acceptable probability that the desirable power supply is not attained. Then its robust expression can be obtained:

\[
\max_{f_0(\xi^h) \in U_r(\xi^h, D^h)} P(\xi^h \leq l^h - P_{eq}^h) \leq \epsilon
\]

(18)

which is equivalent to:

\[
\max_{f_0(\xi^h) \in U_r(\xi^h, D^h)} \int_{0}^{l^h - P_{eq}^h} f_0(\xi^h) d\xi^h \leq \epsilon.
\]

(19)

Defining \(\delta^h = l^h - P_{eq}^h\) as the robust renewable energy usage (REU) decision, which equals the amount of energy dispatched to renewable energy plants at time slot \(h\). In addition, an auxiliary function can be introduced as follows:

\[
h(\xi^h, \delta^h) = \begin{cases} 
1, & \xi^h \leq \delta^h; \\
0, & \xi^h > \delta^h.
\end{cases}
\]

(20)

The left part of inequality \((19)\) then can be formulated into an optimization problem:

\[
\max_{f_0(\xi^h)} \int_{0}^{+\infty} h(\xi^h, \delta^h) \cdot f_0(\xi^h) d\xi^h
\]

(21)

s.t. \(\mathbb{E}_{f_0}[\ln f_0(\xi^h) - \ln g_h(\xi^h)] \leq D_h\)

\(\mathbb{E}_{f_0}[1] = 1\)
Define $Q^h_f(\delta^h) = \max_{f_0(\xi^h) \in U_r(g_h, D_h)} \int_0^{+\infty} h(\xi^h, \delta^h) \cdot f_0(\xi^h) d\xi^h$ as the worst-case fault probability. We can then obtain a worst-case mapping $\mathcal{M}^h_{ws}$ which maps robust REU decision $\delta^h$ to $Q^h_f(\delta^h)$:

$$\mathcal{M}^h_{wc} : \delta^h \rightarrow Q^h_f(\delta^h).$$  \hfill (22)

### 4.2. Sub-Problem: Determine the Robust REU Decision Threshold

Since there exists a random variable $\xi^h$ in the constraints, we cannot solve energy generation and consumption scheduling problem (12) directly. As aforementioned, problem (12) is decomposed into a subproblem and a main problem. The goal of our sub-problem is to determine the robust REU decision threshold $\delta^h*$ so that the load balance constraint can be transformed into a solvable form.

**Proposition 1:** Problem (21) is a convex optimization problem.

**Proof:** Rewrite (21) as follows:

$$\begin{align*}
\max_{f_0(\xi^h)} \quad & \int_0^{+\infty} h(\xi^h, \delta^h) \cdot f_0(\xi^h) d\xi^h \\
\text{s.t.} \quad & \int_0^{+\infty} \left[ \ln f_0(\xi^h) - \ln g_h(\xi^h) \right] f_0(\xi^h) d\xi^h \leq D_h \\
\quad & \int_0^{+\infty} f_0(\xi^h) d\xi^h = 1.
\end{align*}$$  \hfill (23-25)

It can be seen that the objective function (23) and equality constraint function (25) are affine with respect to $f_0(\xi^h)$. Next it is shown that the inequality constraint function (24) is convex.

**Lemma 1:** If $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is convex, then the perspective of $f$, which is denoted as a function $g : \mathbb{R}^{n+1} \rightarrow \mathbb{R}$ that

$$g(x, t) = tf(x/t),$$  \hfill (26)

with domain

$$\text{dom} g = \{(x, t)| x/t \in \text{dom} f, \ t > 0\}$$  \hfill (27)

preserves convexity.

That is to say, if $f$ is a convex function, so is its perspective function $g$. Similarly, if $f$ is concave, so is $g$. This can be proved in several ways, e.g., by direct verification of the defining inequality or using epigraphs and the perspective mapping on $\mathbb{R}^{n+1}$. Readers can refer to [28] for more detailed discussions.
We consider the convex function \( f(x) = \ln x \) on \( \mathbb{R}^+ \). Its perspective is
\[
g(x, t) = -t \ln(x/t) = t \ln(t/x) = t(\ln t - \ln x)
\]
(28)
and it is convex on \( \mathbb{R}^2^+ \). The function \( g \) is called the relative entropy of \( t \) and \( x \). Then we have that the KL divergence
\[
\int_{x \in S} [\ln f(x) - \ln g(x)] f(x) dx
\]
between distribution \( f(x) \) and \( g(x) \) is convex in \( f(x) \) (and \( g(x) \) as well). In this case, it is claimed that the inequality constraint (24) is convex in distribution \( f_0(\xi^h) \).

Through Slater’s condition, strong duality holds for problem (23)-(25). Adopting the Lagrangian method, the worst-case fault probability \( Q_f^h(\delta^h) \) can be obtained as follows:
\[
Q_f^h(\delta^h) = \min_{\tau, \eta} \max_{f_0(\xi^h)} \mathbb{E}_{f_0} \left[ h(\xi^h, \delta^h) - \eta - \tau \frac{f_0(\xi^h)}{g_0(\xi^h)} \right] + \tau D_h + \eta
\]
where \( \tau \geq 0 \) and \( \eta \) are Lagrangian multipliers associated with constraints (24) and (25), respectively. Let
\[
P(\delta^h, f_0, \tau, \eta) = \mathbb{E}_{f_0} \left[ h(\xi^h, \delta^h) - \eta - \tau \frac{f_0(\xi^h)}{g_0(\xi^h)} \right],
\]
(29)
the derivative of \( P(\delta^h, f_0, \tau, \eta) \) with respect to \( f_0 \) can be derived as
\[
\frac{\partial P}{\partial f_0} = \lim_{t \to 0} \frac{1}{t} \left[ P(f_0(\xi^h) + t \cdot g_0(\xi^h)) - P(f_0(\xi^h)) \right]
\]
(30)
\[
= \int_{0}^{+\infty} \left( h(\xi^h, \delta^h) - \eta - \tau \frac{f_0(\xi^h)}{g_0(\xi^h)} \right) g_0(\xi^h) d\xi^h.
\]
Using the Karush-Kuhn-Tucker (KKT) optimality conditions, we thus have
\[
h(\xi^h, \delta^h) - \tau \frac{f_0(\xi^h)}{g_0(\xi^h)} - \eta - \tau = 0
\]
(31)
\[
\int_{0}^{+\infty} f_0(\xi^h) d\xi^h = 1
\]
(32)
\[
\mathbb{E} \left[ \ln \frac{f_0(\xi^h)}{g_0(\xi^h)} \right] - D_h \leq 0
\]
(33)
\[
\tau \left( D_h - \mathbb{E} \left[ \ln \frac{f_0(\xi^h)}{g_0(\xi^h)} \right] \right) = 0
\]
(34)
\[
\tau \geq 0
\]
(35)
From (31), the optimal distribution function can be expressed as follows:

\[ f^*_0(\xi^h) = g_h(\xi^h) \exp \left(\frac{h(\xi^h, \delta^h) - \eta}{\tau} - 1\right). \]  

(36)

The dual variables \((\tau, \eta)\) in (36) should be chosen properly such that conditions (32)-(35) are satisfied. Specifically, the following results can be obtained:

**Proposition 2:** The choice of \((\tau, \eta)\) is a solution of the following nonlinear equations.

\[ H_1(\tau, \eta) = R(\delta^h)e^{-\eta/\tau} + S(\delta^h)e^{(1-\eta)/\tau} - 1 = 0 \]  

(37)

\[ H_2(\tau, \eta) = S(\delta^h)e^{(1-\eta)/\tau} - \eta - \tau(1 + D_h) = 0, \]  

(38)

where \(S(\delta^h) = (1 - G_h(\delta^h))\exp(-1), R(\delta^h) = G_h(\delta^h)\exp(-1), \) and \(G_h(\delta^h) = \int_{\xi^h \geq \delta^h} g_h(\xi^h)d\xi^h\) denotes the complementary cumulative distribution function of reference distribution \(g_h(\xi^h)\).

**Proof:** By substituting the optimal distribution \(f^*_0(\xi^h)\) back into (32) and \(f^*_0(\xi^h), (31)\) into (34), we have

\[ \int_0^{+\infty} g_h(\xi^h) \exp \left(\frac{h(\xi^h, \delta^h) - \eta}{\tau} - 1\right) d\xi^h = 1 \]  

(39)

\[ \int_0^{+\infty} (h(\xi^h, \delta^h) - \eta - \tau) g_h(\xi^h) \cdot \exp \left(\frac{h(\xi^h, \delta^h) - \eta}{\tau} - 1\right) d\xi^h - D_h \cdot \tau = 0, \]  

(40)

which are equivalent to:

\[ \exp \left(-1 - \frac{\eta}{\tau}\right) \cdot \int_{\delta^h}^{+\infty} g(\xi^h)d\xi^h \]  

(41)

\[ + \exp \left(-1 + \frac{1-\eta}{\tau}\right) \cdot \int_0^{\delta^h} g(\xi^h)d\xi^h - 1 = 0 \]

(42)

\[ (1 - \eta - \tau) \exp \left(-1 + \frac{1-\eta}{\tau}\right) \cdot \int_0^{\delta^h} g(\xi^h)d\xi^h \]

\[ + (\eta - \tau) \exp \left(-1 - \frac{\eta}{\tau}\right) \int_{\xi^h}^{\infty} g(\xi^h)d\xi^h - \tau D_h = 0. \]
Equation (37) can be easily obtained from (41) by introducing $S(\delta^h)$ and $R(\delta^h)$. Through (41), (42) can be transformed into:

$$(1 - \eta - \tau) \exp \left( -1 + \frac{1 - \eta}{\tau} \right) \cdot \int_0^{\delta^h} g(\xi^h) d\xi^h +$$

$$(-\eta - \tau) \left[ 1 - \exp \left( -1 + \frac{1 - \eta}{\tau} \right) \cdot \int_0^{\delta^h} g(\xi^h) d\xi^h \right] - \tau D_h = 0.$$  

Then we have

$$\exp \left( -1 + \frac{1 - \eta}{\tau} \right) \cdot \int_0^{\delta^h} g(\xi^h) d\xi^h - \eta - \tau - \tau D_h = 0,$$  

which is equivalent to (38). Hence, Proposition 2 is proved.

It is, however, still rather difficult to obtain an explicit solution from (37) and (38). Hence we propose the Newton iterations as detailed in Algorithm 1.

Once the solutions for (37) and (38) in Proposition 2 is determined, through (31) and (34), the worst-case fault probability can be obtained as follows:

$$Q^h_f(\delta^h) = E_{\xi^0} [h(\xi^h, \delta^h)] = (1 + D_h) \tau + \eta$$  

(44)

Our next step is then to find the robust REU decision threshold $\delta^{h*}$ such that $Q^h_f(\delta^{h*}) = \epsilon$, which involves the calculation of inverse function of $Q^h_f(\delta^h)$ and it is not directly possible from (44). The following property of function $Q^h_f(\delta^h)$, however, may help us design such a search method.

**Proposition 3:** The worst-case fault probability $Q^h_f(\delta^h)$ is non-decreasing with respect to the REU decision $\delta^h$.

The conclusion in Proposition 3 is straightforward since we have $dQ^h_f(\delta^h)/d\delta^h = dE_{\xi^0} [h(\xi^h, \delta^h)]/d\delta^h = f^*_0(\delta^h) \geq 0$. Though direct solution is not available, the monotonicity of $Q^h_f(\delta)$ enlightens us a bisection method to search for the solution for $Q^h_f(\delta^h) = \epsilon$. The main idea is to perform the search within an interval of $[0, \rho]$, where $\rho$ is an empirical constant such that $Q^h_f(\rho) > \epsilon$.

Details of the algorithm for searching the robust REU decision threshold are presented in Algorithm 1. Note that, from the 3rd to the 11th lines of the algorithm, Newton iteration is adopted to solve the equation in Proposition 2 and obtain the worst-case probability with fixed robust REU decision. Then the worst-case probability at $\delta^h_-$ and $\delta^h_-$ is compared with the fault tolerant limit $\epsilon$, respectively. The comparison results help shrink the search region as shown in lines 12-14.
Algorithm 1 Search for robust REU decision threshold $\delta^{h*}$

**Input:** Reference distribution $g_h(\xi^h)$; 
- Distance limit $D_h$; 
- Search radius $\rho$; 
- Load balance fault tolerant limit $\epsilon$; 
- Tolerance $\varepsilon$.

**Output:** Robust REU decision threshold such that $Q^h_f(\delta^{h*}) = \epsilon$;

1: **Begin**
2: initialize $\delta^h_0 = 0$, $\delta^h_+ = \rho$, and set $H(\tau, \eta) = [H_1(\tau, \eta), H_2(\tau, \eta)]^T$
3: while $|\delta^h_- - \delta^h_+| > \varepsilon$
4: set $\tilde{\delta}^h = \frac{\delta^h_- + \delta^h_+}{2}$ and initiate the time iteration $k = 1$
5: while $H(\tau, \eta) > \varepsilon$
6: evaluate $H(\tau, \eta)$ and Jacobian matrix $J(\tau, \eta)$
7: solve $J(\tau, \eta)\Delta x_k = -H(\tau, \eta)$
8: update $\tau_{k+1} = [\tau_k + \Delta \tau_k]^+$, $\eta_{k+1} = \eta_k + \Delta \eta_k$
9: update $Q^h_f(\delta^h) = (1 + D_h)\tau_{k+1} + \eta_{k+1}$
10: set $k = k + 1$
11: end while
12: if $(Q^h_f(\tilde{\delta}^h) - \epsilon)(Q^h_f(\delta^{h*}) - \epsilon) < 0$
13: then set $\delta^h_- = \tilde{\delta}^h$ else set $\delta^h_+ = \delta^h$ end if
14: if $|Q^h_f(\tilde{\delta}^h) - \epsilon| < \varepsilon$ break end if
15: end while
16: set $\delta^{h*} = \tilde{\delta}^h$
17: **End**

4.3. Main-Problem: Determine the Optimal Energy Consumption and Generation Scheduling

Once the robust REU decision threshold $\delta^{h*}$ for the robust load balance constraint (19) is obtained, the energy generation and consumption management problem can be reformulated. Specifically, the following optimization problem can be
tackled rather than the original Eq. (12)

\[
\min \sum_{h=1}^{H} C_h(P_{cg}^h) \tag{45}
\]

s.t. \[\sum_{a \in A} x_a \cdot y_a^h - P_{cg}^h = \delta^h, \quad \forall h \in \mathcal{H},\]

and (2) to (10),

where the optimization variables include the controllable energy generation variable \( P_{cg}^h \) for all time slots \( h \in \mathcal{H} \), and the energy consumption scheduling vector \( y_a \) for all appliances \( a \in A \). The objective function aims at minimizing the overall energy cost in microgrid over the whole time horizon.

It can be seen that all the constraints of (45) are linear and the objective function is quadratic. This problem is a mixed integer quadratic programming problem. Algorithms that can be adopted to tackle this kind of problem include the cutting plane method and the branch and bound method. This problem can also be effectively solved by some commercial optimization softwares including CPLEX, Mosek, FortMP and Gurobi, etc.

4.4. Extensions of the Proposed Algorithm: A Brief Discussion

With trivial or, sometimes, non-trivial extensions, the proposed algorithm may be applied to solve some other power demand and supply problems in microgrids. A few possible extensions are briefly discussed as follows.

- The scenario that has been considered in this paper assumes that the end-users control their power consumption in accordance with the guideline that MGCC suggests. Under such case, the uncertainties from the end-user side are expected to be limited and can be handled by the system, since the control sequences obtained by the proposed algorithm are of reasonably good robustness. For the cases where uncertainties from the user-side exceed the range of tolerance (e.g., many end-users do not follow the guideline for whatever reasons), the proposed uncertainty model can be extended to include the uncertainty from the end-user side by properly integrating user-side and supply-side uncertainties. Once the integrated reference distribution of the combined uncertainties is obtained, the proposed algorithm can be used with virtually no changes. Detailed discussions on such cases, however, are out of the scope of this paper. For the even worse case where unexpected real-time changes go beyond what have been modeled, a few
classic approaches may be adopted, e.g., by increasing the power output of energy generators, turning on stand-by fast response generators, importing electricity from the power grid, or shedding load if necessary, etc.

- The power demand and supply management framework discussed so far is an offline approach suitable for planning the energy consumption and generation ahead of time. When real-time adjustment is of a big concern yet response time limit is not too rigid, the proposed algorithm can be easily extended to handle such cases. One option is to adopt the model predictive control (MPC) approach (also known as ”receding horizon approach”) \cite{29} \cite{30}, of which the basic idea is to calculate the optimal control sequences yet implement only the first step of them. In other words, the power demand and supply management problem is solved at time \( h = \tau (\tau \in \mathcal{H} \) denotes the current time index.) for the remaining horizon \( [\tau, \tau + 1, ..., H] \), yet only the solution for the current time slot \( \tau \) is implemented. In the next time slot, MGCC shall update the system information (e.g., the requirements of end-users and robust REU decision thresholds) and re-do the calculations. For the cases where response time has to be very short, however, different algorithms with lower complexities and faster speed probably have to be developed, in order to support real-time operation and small-step scheduling more efficiently. For such cases, enhancing the system fault tolerance against the noises of real-time data shall also be considered.

- The problem formulation can also be easily extended to handle the case where microgrids import electricity from outside power grid. Specifically, assume that the microgrid imports \( P^h_E \) units of electricity from the power grid, the problem formulation (45) can be modified as:

\[
\begin{align*}
\min & \quad \sum_{h=1}^{H} a_h \cdot P_{cg}^h \cdot 2 + b_h \cdot P_{cg}^h + c_h + d_h \cdot P^h_E \\
\text{s.t.} & \quad \sum_{a \in A} x_a \cdot y_a^h - P_{cg}^h = \delta^h + P^h_E, \quad \forall h \in \mathcal{H}, \\
& \quad \text{and (2) to (10),}
\end{align*}
\]

where \( d_h \) is the electricity price of power grid at time slot \( h \). The problem is essentially still a mixed integer quadratic programming problem which can be solved by using the same algorithm. Whereas if electricity price from the power grid \( d_h \) is time varying with non-trivial uncertainty, the problem
will become more complicated. A feasible option is to develop a worst-case robust optimization approach for the problem. Readers may refer to [31] [32] [12] for more exhaustive descriptions on dealing with bounded uncertainties in the coefficients of objective function. Detailed discussions are not too difficult yet rather lengthy, and therefore have to be left to a separate report.

5. Simulation Results and Discussions

In this section, simulation results are presented for assessing the performance of the proposed power demand and supply management scheme and evaluating the effects of different system parameters. Here, an assumption is made on top of paper [33] [34], where Gaussian random process has been adopted to describe the renewable energy generation. Specifically, it is assumed that the reference distribution is a Gaussian distribution $g_h(\xi^h)$ with mean $\bar{m}_h$ and standard deviation $\sigma_h$. In addition, the parameters of the cost function in (45) for each time slot are set as $a_h > 0$, $b_h = 0$, and $c_h = 0$.

5.1. The Impacts of Distribution Uncertainty Set

We first set the fault tolerant limit $\epsilon = 10^{-3}$ and try to investigate the relations between robust REU decision threshold $\delta^{h*}$ and distance limit $D_h$ for different values of $\bar{m}_h$ and $\sigma_h$. The results are plotted in Fig. 2 and Fig. 3. It is shown that the robust REU decision threshold decreases with the increase of the distance limit. This observation is intuitive since a larger distance limit defines a larger distribution set which allows the renewable energy output to fluctuate more intensively. Given the required fault tolerant limit, REU decision threshold has to be set at a lower value so as to rely less on the more uncertain renewable energy and guarantee the system reliability. Note that when $D_h = 0$, the renewable energy follows exactly the reference distribution $g_h(\xi^h)$. In this special case, renewable energy generation is a random variable with determinate distribution $g_h(\xi^h)$. While $D_h > 0$, our reference model considers a more general case which allows discrepancy between actual distribution and its reference. The discrepancy however is limited and confined by a probabilistic distance measure. Simply put, the reference model allows the actual renewable energy generation to follow a different distribution function from the reference distribution, but not be too disparate based on historical data or empirical knowledge.

From Fig. 2 and Fig. 3, the following statement can also be proposed: when the reference distribution is Gaussian, the robust REU decision threshold $\delta^{h*}$ lin-
Figure 2: Robust REU Decision Threshold $\delta^h_*$ with Distance Limit $D_h$ for different $\overline{m}_h$

Figure 3: Robust REU Decision Threshold $\delta^h_*$ with Distance Limit $D_h$ for different $\sigma^h$
Fault Tolerant limit $\epsilon$

Robust REU Decision Threshold $\delta h^*$

$D_n = 0$

$D_n = 0.005$

$D_n = 0.01$

$D_n = 0.015$

$D_n = 0.02$

Figure 4: Robust REU Decision Threshold $\delta h^*$ with Fault Tolerant Limit $\epsilon$ for different $D_h$

early increases with the mean of reference distribution $m_h$ and linearly decreases with the standard deviation of reference distribution $\sigma_h$. This statement can be explained analytically as follows. $G_h(\delta^h)$ in (37) and (38) is first transformed into:

$$G_h(\delta^h) = \int_{\xi^h \geq \delta^h} g_h(\xi^h) d\xi^h = \int_{\delta^h - \overline{m}_h}^{+\infty} n(x) dx,$$

(47)

where $n(x)$ is the probability density function of the standard Gaussian distribution. Since fault tolerant limit $\epsilon$ is of a relatively small value, we have that $\delta^h$ is less than $\overline{m}_h$. As $\overline{m}_h$ and $\sigma_h$ vary, in order to preserve the same worst-case probability $Q_f(\delta^h)$, the solutions $(\eta, \tau)$ of the equations (37) and (38) need to remain unchanged, indicating that $S(\delta^h)$, $R(\delta^h)$ and $G(\delta^h)$ also need to be constants. In this regard:

$$\frac{\delta^h - \overline{m}_h}{\sigma_h} = C \quad \Rightarrow \quad \delta^h = C\sigma_h + \overline{m}_h,$$

where $C$ is a negative constant. Thus, $\delta^h$ linearly increases with $\overline{m}_h$ and linearly decreases with $\sigma_h$.

5.2. Effects of Fault Tolerant Limit $\epsilon$

We set $\overline{m}_h = 36$ and $\sigma_h = 2$ and investigate how the robust REU decision threshold varies when fault tolerance limit increases. Fig. 4 plots the mapping
from fault tolerant limit $\epsilon$ to robust REU decision threshold $\delta^{h*}$ under different values of the distance limit $D_h$. The figure indicates that a larger fault tolerant limit permits a higher reliance on renewable energy (a larger robust REU decision threshold), which is straightforward to understand. Also note that the worst-case fault probability is an increasing function of the REU decision threshold. Thus it is justified to adopt the bisection method as presented in Algorithm 1 to search for the REU decision threshold which satisfies the fault tolerant limit requirement. Note that in this figure, the red triangle line is the special case where renewable energy follows reference distribution exactly. It is also observed that robust REU threshold $\delta^{h*}$ is more sensitive to $\epsilon$ when $D_h$ increases.

5.3. The Impacts of Uninterruptible Loads

For the experiment studied in this part, we set the power consumption scheduling horizon $|\mathcal{H}| = 12$ h. That is, the MGCC solves optimization problem (45) to decide on the operations of each appliance for the next 12 hours. In this paper, 30 household appliances including electric cookers (EC), air conditioners (AC), electric water heaters (EWH), cloth dryers (CD), dish washers (DW) and plug-in hybrid electric vehicles (PHEVs) are considered to study the optimal power consumption scheduling with a mixed integer quadratic programming approach. The detailed operation data used in this paper is modified based on the information from [35] [36] [37] [38] [39], and is shown in Table 1. Note that a user elasticity index is also introduced as follows

$$\gamma_a = \frac{T_a}{\beta_a - \alpha_a + 1}$$


to describe the scheduling flexibility of appliance $a$. Obviously, $\gamma_a \in (0, 1]$, and a larger $\gamma_a$ implies a more inflexible arrangement property. The operation window $[\alpha_a, \beta_a]$ of each appliance is chosen according to the preferences of different users. In this paper, the values of $\alpha_a$ and $\beta_a$ are not enumerated one by one due to limited space, instead, the ranges of $\gamma_a$ for each kind of appliance are listed, which is presented in the last column of Table 1.

The mean $\bar{m}_h$ and standard deviation $\sigma_h$ of the reference distribution, together with the distance limits for the next 12 time slots are given in Table 2. Based on these data and adopting Algorithm 1, the robust REU threshold $\delta^{h*}$, representing the amount of energy dispatched to renewable energy plants for each time slot, is obtained. The results are demonstrated in the last column of Table 2 and are used to solve the main problem (45). Our experiments utilize MOSEK optimization toolbox 6.0 on an Intel-P4 2.4-GHz personal computer. To investigate the impacts
Table 1: Operation Data for Appliances in the microgrid

<table>
<thead>
<tr>
<th>Type of Appliance</th>
<th>Power Level (KW)</th>
<th>$T_a$</th>
<th>$\gamma_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC</td>
<td>2</td>
<td>1</td>
<td>0.3-0.4</td>
</tr>
<tr>
<td>AC</td>
<td>3.5</td>
<td>10</td>
<td>0.9-1.0</td>
</tr>
<tr>
<td>EWH</td>
<td>4.5</td>
<td>3</td>
<td>0.5-0.7</td>
</tr>
<tr>
<td>CD</td>
<td>5</td>
<td>3</td>
<td>0.3-0.4</td>
</tr>
<tr>
<td>DW</td>
<td>0.85</td>
<td>2</td>
<td>0.3-0.4</td>
</tr>
<tr>
<td>PHEV</td>
<td>7.3</td>
<td>7</td>
<td>0.6-0.7</td>
</tr>
</tbody>
</table>

Table 2: Parameters of Distribution Uncertainty Set and Corresponding Robust REU Threshold

<table>
<thead>
<tr>
<th>Time Slot</th>
<th>$\mu_h$</th>
<th>$\sigma_h$</th>
<th>$D_h$</th>
<th>$\delta^{h*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.678</td>
<td>0.9571</td>
<td>0.0162</td>
<td>8.419</td>
</tr>
<tr>
<td>2</td>
<td>14.757</td>
<td>0.4853</td>
<td>0.0181</td>
<td>11.453</td>
</tr>
<tr>
<td>3</td>
<td>14.743</td>
<td>0.8002</td>
<td>0.0025</td>
<td>11.531</td>
</tr>
<tr>
<td>4</td>
<td>14.392</td>
<td>0.1418</td>
<td>0.0182</td>
<td>13.423</td>
</tr>
<tr>
<td>5</td>
<td>14.655</td>
<td>0.4217</td>
<td>0.0126</td>
<td>12.137</td>
</tr>
<tr>
<td>6</td>
<td>14.171</td>
<td>0.9157</td>
<td>0.0019</td>
<td>10.630</td>
</tr>
<tr>
<td>7</td>
<td>14.706</td>
<td>0.7922</td>
<td>0.0055</td>
<td>10.995</td>
</tr>
<tr>
<td>8</td>
<td>14.031</td>
<td>0.9594</td>
<td>0.0109</td>
<td>8.577</td>
</tr>
<tr>
<td>9</td>
<td>14.276</td>
<td>0.6557</td>
<td>0.0191</td>
<td>9.716</td>
</tr>
<tr>
<td>10</td>
<td>14.046</td>
<td>0.0357</td>
<td>0.0192</td>
<td>13.797</td>
</tr>
<tr>
<td>11</td>
<td>14.097</td>
<td>0.8491</td>
<td>0.0031</td>
<td>10.565</td>
</tr>
<tr>
<td>12</td>
<td>14.823</td>
<td>0.9339</td>
<td>0.0194</td>
<td>8.294</td>
</tr>
</tbody>
</table>
of uninterruptible loads, the following cases are studied:

- **Case 1**: Only electric cookers are classified into the uninterruptible appliance set \( \mathcal{A}' \), i.e., \( \mathcal{A}' = \{ \text{EC} \} \).
- **Case 2**: On top of Case 1, air conditioners are added to the uninterruptible appliance set \( \mathcal{A}' \), i.e., \( \mathcal{A}' = \{ \text{EC, AC} \} \).
- **Case 3**: On top of Case 2, electric water heaters are added to the uninterruptible appliance set \( \mathcal{A}' \), i.e., \( \mathcal{A}' = \{ \text{EC, AC, EWH} \} \).
- **Case 4**: On top of Case 3, cloth dryers are added to the uninterruptible appliance set \( \mathcal{A}' \), i.e., \( \mathcal{A}' = \{ \text{EC, AC, EWH, CD} \} \).
- **Case 5**: On top of Case 4, dish washers are added to the uninterruptible appliance set \( \mathcal{A}' \), i.e., \( \mathcal{A}' = \{ \text{EC, AC, EWH, CD, DW} \} \).
- **Case 6**: On top of Case 5, PHEVs are added to the uninterruptible appliance set \( \mathcal{A}' \), i.e., \( \mathcal{A}' = \{ \text{EC, AC, EWH, CD, DW, PHEV} \} \).

Figure 5 demonstrates the energy cost for each case. Obviously, the energy cost goes up when the uninterruptible appliance set \( \mathcal{A}' \) is scaled up. We compare the costs of adjacent cases, and the difference between these costs is called cost gap. The largest cost gap is shown between Case 5 and Case 6 due to PHEVs’ high electric power consumption \( (P = 7.3 \text{ KW}) \) and relatively considerable scheduling elasticity \( (\gamma_a = 0.6 - 0.7) \).
5.4. The Price of User Elasticity

In this section, the effects of user elasticity on the energy cost of the microgrid system is explored. First it is assumed that all the appliances are uninterruptible. Since the minimum running time of each appliance $T_a$ is fixed, we extend or shrink the operation window $[\alpha_a, \beta_a]$ to relax or tighten the user elasticity. Note that, at one time, the operation window $[\alpha_a, \beta_a]$ of each appliance $a$ will expand or shrink one unit of time slot from both sides, i.e., the operation window will scale up or down two time slots. If one side of the operation window cannot be extended due to the finite length of time horizon, the operation window will only scale up on one side until it covers the whole time horizon. The operation window of each appliance is kept on extending or shrinking until all the operation windows cannot be changed. Then, how energy cost changes when operation windows vary is demonstrated. Note that the cases when operation windows are shrunk 6, 4, 2 time slots and extended 0, 2, 4 time slots are selected respectively. The results are presented in Fig. 6. In this figure, it is observed that when user elasticities are tightened, energy cost increases rapidly. Such phenomenon can be interpreted that the user elasticity can make a significant impact on the energy cost of the microgrid system. Compared with the the effects of interruptibility property, user elasticity has stronger influences on the expenditure of the whole system. This result may give MGCC an inspiration that it is worthy to provide more rewards to customers who agree to have more time flexibility than those who allow interruptions to some appliances. Moreover, it is also shown that when the
operation windows are shrunk by 6 time slots, nearly all the appliances’ elasticity reaches 1. This approximates the case when all the appliances operate at their desired time with no flexibility. Compared with this benchmark case, we observe that the proposed power consumption management scheme can reduce the energy cost significantly.

6. Conclusion

In this paper, a fundamental problem of using a microgrid system central controller to optimally schedule the demand and supply profiles so as to minimize the fuel consumption costs during the whole time horizon is studied. We focus on a scenario where the end-users control their power consumptions in accordance with the guideline that MGCC suggests. To tackle the randomness of renewable energy, a reference distribution is introduced and then a distribution uncertainty set is defined to confine the uncertainty. Such a novel model allows convenient handling of fluctuating renewable generation as long as the renewable energy generation profile is not too drastically different from the past observation or empirical knowledge. An optimization formulation of the problem was proposed and a two-stage algorithm approach was developed, to first transform and then solve the problem. Numerical results indicated that the proposed energy consumption management scheme can significantly cut down energy expenses. Effects of a few factors, including the reference distribution, the fault tolerant limit, the types and amount of uninterruptible loads, and the user elasticity etc. were carefully evaluated. Such evaluations, as we believe, help provide some useful insights for MGCC to develop more effective payback policies for their customers.

References


